On IFsgb - Closed Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract-In this paper, we introduces the concepts of intuitionistic fuzzy sgb closed and open sets in intuitionistic fuzzy topological spaces.

Index Terms- Intuitionistic fuzzy topology, intuitionistic fuzzy semi generalized b closed sets and intuitionistic fuzzy semi generalized b open sets.

1. Introduction
Fuzzy set was introduced by Zadeh[11] in1965 and fuzzy topology introduced by C.L.Chang[2] in 1968. After the introduction fuzzy set and fuzzy topology, there have been several generalizations of this notions. Intuitionistic fuzzy sets was introduced by Atanassov[1] in 1986 and Coker [3] introduced intuitionistic fuzzy topological spaces. In this paper, we introduced intuitionistic fuzzy semi generalized b closed sets and intuitionistic fuzzy semi generalized b open sets. The relation between intuitionistic fuzzy sgb closed sets and other intuitionistic fuzzy generalized closed sets are given. We also discusse some of its properties.

2. Preliminaries
Definition 2.1:[1]Let X be an non empty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form

A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}.

Where the functions \mu_A(x) : X \rightarrow [0,1] and \nu_A(x) : X \rightarrow [0,1] denote the degree of membership (namely \mu_A(x)) and the degree of non-membership (namely \nu_A(x)) of each element x \in X to the set A respectively and 0 \leq \mu_A(x) + \nu_A(x) \leq 1 for each x \in X.

Definition 2.2:[1] Let A and B be the IFSs of the forms A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} and B = \{ (x, \mu_B(x), \nu_B(x)) / x \in X \}. Then

\( \circ \quad A \subseteq B \) if and only if \mu_A(x) \leq \mu_B(x) and \nu_A(x) \leq \nu_B(x) for all x \in X,

\( \circ \quad A = B \) if and only if A \subseteq B and B \subseteq A,

\( \circ \quad A^c = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} \},

\( \circ \quad A \cap B = \{ (x, \mu_{A\cap B}(x), \nu_{A\cap B}(x)) / x \in X \} \},

\( \circ \quad A \cup B = \{ (x, \mu_{A\cup B}(x), \nu_{A\cup B}(x)) / x \in X \} \}.

For the sake of simplicity, the notation A = (x, \mu_A, \nu_A) shall be used instead of A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}. Also for the sake of simplicity, we shall use the notation A = (x, \mu_A, \nu_A) instead of A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}.

The intuitionistic fuzzy sets 0 = \{ (x, 0, 1) / x \in X \} and 1 = \{ (x, 1, 0) / x \in X \} are the empty set and the whole set of X, respectively.

Definition 2.3:[1] An intuitionistic fuzzy topology(IFT) on a non empty set X is a family \tau of IFSs in X satisfying the following axioms:

\( \circ \quad A \subseteq B \Rightarrow A^c \supseteq B^c \),

\( \circ \quad G_i \cap G_j \in \tau \) for any collection \{ G_i / i \in J \} \subseteq \tau.

In this case, the pair (X, \tau) is called an intuitionistic fuzzy topological space(IFTS) and any IFS in \tau is known as an intuitionistic fuzzy open set (IFOS) in X.

The complement A^c of a IFOS A in an IFTS (X, \tau) is called an intuitionistic fuzzy closed set(IFCS) in X.

Definition 2.4:[1] Let A and B be two IFSs of the form A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} and B = \{ (x, \mu_B(x), \nu_B(x)) / x \in X \}. Then

A \subseteq B \Rightarrow \exists C \subseteq B \cap C,

A \subseteq B \cap C \Rightarrow A \cup B \subseteq C,

A \subseteq B \cap C \Rightarrow A \cup B \subseteq C,

(\mu_{A\cap B})^y = A^c B^c and (\mu_{A\cup B})^y = A^c B^c,

((A^c)^c)^c = A,

(1-\gamma)^c = 0.\) and (0-\gamma)^c = 1.

Definition 2.5:[1] Let (X, \tau) be an IFTS and A = (x, \mu_A, \nu_A) be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

\( \circ \quad \text{int}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}, \)

\( \circ \quad \text{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}. \)

Proposition 2.6:[1] For any IFSs A and B in (X, \tau), we have
Proposition 2.7: [3] For any IFS A in (X, τ), we have

\[ \text{int}(A) \subseteq A, \]

\[ A \subseteq \text{cl}(A), \]

\[ A \text{ is an IFCS in } X \iff \text{cl}(A) = A, \]

\[ A \text{ is an IFOS in } X \iff \text{int}(A) = A, \]

\[ A \subseteq B \Rightarrow \text{int}(A) \subseteq \text{int}(B) \text{ and } \text{cl}(A) \subseteq \text{cl}(B), \]

\[ \text{int}(\text{int}(A)) = \text{int}(A), \]

\[ \text{cl}(\text{cl}(A)) = \text{cl}(A), \]

\[ \text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B), \]

\[ \text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B). \]

Proposition 2.8: [3] If A is an IFCS in (X, τ) then cl(A) = A and if A is an IFOS in (X, τ) then int(A) = A. The arbitrary union of IFCSs is an IFCS in (X, τ).

Definition 2.9: An IFS A = \{(x, μ_A(x), ν_A(x)) / x ∈ X\} in an IFTS (X, τ) is said to be

- intuitionistic fuzzy b-closed set [5] (IFbCS) if cl(int(A)) \cap int(cl(A)) \subseteq A,

- intuitionistic fuzzy α-closed set [4] (IFαCS) if cl(int(cl(A))) \subseteq A.

Definition 2.10: An IFS A = \{(x, μ_A(x), ν_A(x)) / x ∈ X\} in an IFTS (X, τ) is said to be

- intuitionistic fuzzy b-open set [5] (IFbOS) if A \subseteq int(cl(A)) \cup cl(int(A)),

- intuitionistic fuzzy α-open set [4] (IFαOS) if A \subseteq int(int(cl(A))).

Definition 2.11: An IFS A = \{(x, μ_A(x), ν_A(x)) / x ∈ X\} in an IFTS (X, τ) is said to be

- intuitionistic fuzzy generalized aclosed set [10] (IFGαCS) if acl(A) \subseteq U whenever A \subseteq U and U is an IFαCS in (X, τ),

- intuitionistic fuzzy generalized semi closed set [7] (IFGαsCS) if acl(A) \subseteq U whenever A \subseteq U and U is an IFsCS in (X, τ),

- intuitionistic fuzzy wclosed set [8] (IFwCS) if cl(A) \subseteq U whenever A \subseteq U and U is an IFOS in (X, τ),

- intuitionistic fuzzy πgeneralized beta closed sets [6] (IFπβCS) if βcl(A) \subseteq U whenever A \subseteq U and U is an IFπCS in (X, τ).

Definition 2.12: [9] Let (X, τ) be an IFTS and A = \{(x, μ_A(x), ν_A(x)) / x ∈ X\} be an IFS in (X, τ). Then the intuitionistic fuzzy b closure of A (bcl(A)) and intuitionistic fuzzy b closure of A (bint(A)) are defined as

\[ \text{bcl}(A) = \bigcup \{ G / G / G \text{ is an IFbOS in } X \text{ and } G \subseteq A \}, \]

\[ \text{bcl}(A) = \bigcap \{ K / K \text{ is an IFbCS in } X \text{ and } A \subseteq K \}. \]

Proposition 2.13: [10] Let (X, τ) be any IFTS. Let A and B be any two intuitionistic fuzzy sets in (X, τ). Then the intuitionistic fuzzy generalized b closure operator satisfies the following properties.

\[ \text{bcl}(0-) = 0- \text{ and } \text{bcl}(1-) = 1-, \]

\[ A \subseteq \text{bcl}(A), \]

\[ \text{bint}(A) \subseteq A, \]

\[ \text{bcl}(A) \subseteq A, \]

\[ A \subseteq B \Rightarrow \text{bcl}(A) \subseteq \text{bcl}(B), \]

\[ A \subseteq B \Rightarrow \text{bint}(A) \subseteq \text{bint}(B). \]

3. Intuitionistic Fuzzy sgb-Closed Sets

Definition 3.1: An IFS A is said to be an intuitionistic fuzzy semi generalized b-closed set (IFSGbCS) if bcl(A) \subseteq U whenever A \subseteq U and U is an IFsOS in (X, τ).

The collection of all intuitionistic fuzzy sgb-closed sets of an IFTS (X, τ) is denoted by IFSGbCS(X).

Example 3.2: Let X = \{a, b\} and let τ = \{0_, G , 1_\} be an IFT on (X, τ) where G = \{(x, (0.2, 0.3), (0.7, 0.6)\}. Consider an IFS A = \{(x, (0.6, 0.5), (0.3, 0.4)\}. Then A is an IFSGbCS in (X, τ).

Example 3.3: Let X = \{a, b\} and let τ = \{0_, G , 1_\} be an IFT on (X, τ) where G = \{(x, (0.2, 0.4), (0.7, 0.5)\}. Consider an IFS A = \{(x, (0.6, 0.7), (0.3, 0.2)\}. Then A is not an IFSGbCS in (X, τ).

Theorem 3.4: Every IFCS is an IFSGbCS.

Proof: Let AC \subseteq U and U is an IFsOS in (X, τ). Since A be an IFCS in (X, τ). We have, cl(A) = A. Since bcl(A) \subseteq cl(A) and A is an IFCS in (X, τ), cl(A) \subseteq cl(A) = A \subseteq U. Therefore A is an IFSGbCS in X.

The converse of Theorem 3.4 need not be true as seen from the following Example.

Example 3.5: Let X = \{a, b\} and let τ = \{0_, G , 1_\} be an IFT on (X, τ) where G = \{(x, (0.2, 0.3), (0.7, 0.6)\}. Consider an IFS A = \{(x, (0.6, 0.5), (0.3, 0.4)\}. Then A is an IFSGbCS but not an IFCS in (X, τ).

Theorem 3.6: Every IFαCS is an IFSGbCS.
**Proof:** Let \( A \subseteq U \) and \( U \) is an IFSOS in \((X, \tau)\). Since \( A \) be an IF\( \alpha \)CS in \((X, \tau)\). We have, \( \alpha cl(A) = A \). Since \( bcl(A) \subseteq \alpha cl(A) \) and \( A \) is an IF\( \alpha \)CS in \((X, \tau)\), \( bcl(A) \subseteq \alpha cl(A) = A \subseteq U \). Therefore \( A \) is an IFSGbCS in \((X, \tau)\).

The converse of Theorem 3.6 need not be true as seen from the following Example.

**Example 3.7:** Let \( X = \{a, b\} \) and let \( \tau = \{0, G, 1\} \) be an IFT on \((X, \tau)\) where \( G = \{(x, (0.2, 0.3), (0.7, 0.6)\) \}. Consider an IFS \( A = \{(x, (0.6, 0.5), (0.3, 0.4)\) \}. Then \( A \) is an IFSGbCS but not an IF\( \alpha \)CS in \((X, \tau)\).

**Theorem 3.8:** Every IFG\( \alpha \)CS is an IFSGbCS.

**Proof:** Let \( A \subseteq U \) and \( U \) is an IFSOS in \((X, \tau)\). Since \( A \) be an IFG\( \alpha \)CS in \((X, \tau)\). We have, \( \alpha cl(A) \subseteq U \). Since \( bcl(A) \subseteq \alpha cl(A) \) and \( A \) is an IFG\( \alpha \)CS in \((X, \tau)\), \( bcl(A) \subseteq \alpha cl(A) \subseteq U \). Therefore \( A \) is an IFSGbCS in \((X, \tau)\).

The converse of Theorem 3.8 need not be true as seen from the following Example.

**Example 3.9:** Let \( X = \{a, b\} \) and let \( \tau = \{0, G, 1\} \) be an IFT on \((X, \tau)\) where \( G = \{(x, (0.3, 0.4), (0.7, 0.5)\) \}. Consider an IFS \( A = \{(x, (0.3, 0.5), (0.6, 0.4)\) \}. Then \( A \) is an IFSGbCS but not an IFG\( \alpha \)CS in \((X, \tau)\).

**Theorem 3.10:** Every IF\( \alpha \)GSCS is an IFSGbCS.

**Proof:** Let \( A \subseteq U \) and \( U \) is an IFSOS in \((X, \tau)\). Since \( A \) be an IF\( \alpha \)GSCS in \((X, \tau)\). We have, \( \alpha cl(A) \subseteq U \). Since \( bcl(A) \subseteq \alpha cl(A) \) and \( A \) is an IF\( \alpha \)GSCS in \((X, \tau)\), \( bcl(A) \subseteq \alpha cl(A) \subseteq U \). Therefore \( A \) is an IFSGbCS in \((X, \tau)\).

The converse of Theorem 3.10 need not be true as seen from the following Example.

**Example 3.11:** Let \( X = \{a, b\} \) and let \( \tau = \{0, G, 1\} \) be an IFT on \((X, \tau)\) where \( G = \{(x, (0.3, 0.4), (0.7, 0.5)\) \}. Consider an IFS \( A = \{(x, (0.3, 0.5), (0.6, 0.4)\) \}. Then \( A \) is an IFSGbCS but not an IF\( \alpha \)GSCS in \((X, \tau)\).

**Theorem 3.12:** Every IFWCS is an IFSGbCS.

**Proof:** Let \( A \subseteq U \) and \( U \) is an IFSOS in \((X, \tau)\). Since \( A \) be an IFWCS in \((X, \tau)\). We have, \( cl(A) \subseteq U \). Since \( bcl(A) \subseteq cl(A) \) and \( A \) is an IFWCS in \((X, \tau)\), \( bcl(A) \subseteq cl(A) \subseteq U \). Therefore \( A \) is an IFSGbCS in \((X, \tau)\).

The converse of Theorem 3.12 need not be true as seen from the following Example.

**Example 3.13:** Let \( X = \{a, b\} \) and let \( \tau = \{0, G, 1\} \) be an IFT on \((X, \tau)\) where \( G = \{(x, (0.7, 0.8), (0.3, 0.2)\) \}. Consider an IFS \( A = \{(x, (0.6, 0.8), (0.4, 0.2)\) \}. Then \( A \) is an IFSGbCS but not an IFWCS in \((X, \tau)\).

**Theorem 3.14:** Every IFSGCS is an IFSGbCS.

**Proof:** Let \( A \subseteq U \) and \( U \) is an IFSOS in \((X, \tau)\). Since \( A \) be an IFSGCS in \((X, \tau)\). We have, \( scl(A) \subseteq U \). Since \( bcl(A) \subseteq scl(A) \) and \( A \) is an IFSGCS in \((X, \tau)\), \( bcl(A) \subseteq scl(A) \subseteq U \). Therefore \( A \) is an IFSGbCS in \((X, \tau)\).

The converse of Theorem 3.14 need not be true as seen from the following Example.

**Example 3.15:** Let \( X = \{a, b\} \) and let \( \tau = \{0, G, 1\} \) be an IFT on \((X, \tau)\) where \( G = \{(x, (0.1, 0.2), (0.5, 0.6)\) \}. Consider an IFS \( A = \{(x, (0.2, 0.2), (0.6, 0.7)\) \}. Then \( A \) is an IFSGbCS but not an IFSGCS in \((X, \tau)\).

**Theorem 3.16:** Every IF\( \pi \)G\( \beta \)CS is an IFSGbCS.

**Proof:** Let \( A \subseteq U \) and \( U \) is an IF\( \pi \)OS in \((X, \tau)\). Since \( A \) be an IF\( \pi \)G\( \beta \)CS in \((X, \tau)\). We have, \( bcl(A) \subseteq U \). Since \( \beta cl(A) \subseteq bcl(A) \) and \( A \) is an IF\( \pi \)G\( \beta \)CS in \((X, \tau)\), \( \beta cl(A) \subseteq bcl(A) \subseteq U \). Therefore \( A \) is an IF\( \pi \)G\( \beta \)CS in \((X, \tau)\).

The converse of Theorem 3.16 need not be true as seen from the following Example.

**Example 3.17:** Let \( X = \{a, b\} \) and let \( \tau = \{0, G, 1\} \) be an IFT on \((X, \tau)\) where \( G = \{(x, (0.2, 0.4), (0.7, 0.5)\) \}. Consider an IFS \( A = \{(x, (0.6, 0.7), (0.3, 0.2)\) \}. Then \( A \) is an IF\( \pi \)G\( \beta \)CS but not an IFSGbCS in \((X, \tau)\).

**Remark 3.18:** The following implications are true. None of them is reversible.
In this diagram by “A → B” we mean A implies B but not conversely.

**Remark 3.19:** Union of any two IFSGbC sets need not be an IFSGbC set as seen the following examples.

**Example 3.20:** Let $X = \{a, b\}$ and let $\tau = \{0, G, 1,\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.6, 0.8), (0.4, 0.2))$ and the IFS $B = (x, (0.6, 0.7), (0.4, 0.3))$. Then A and B are IFSGbCS but $A \cup B$ is not an IFSGbCS in $(X, \tau)$.

**Theorem 3.21:** If $A$ is an IFSGbCS in $(X, \tau)$ such that $A \subseteq B \subseteq \text{bcl}(A)$ then $B$ is an IFSGbCS in $(X, \tau)$.

**Proof:** Let $U$ be an IFOS and $B$ be an IFS in an IFTS in $(X, \tau)$ such that $B \subseteq U$. This implies $A \subseteq U$. Since $A$ is an IFSGbCS, $\text{bcl}(A) \subseteq U$. By hypothesis, we have $\text{bcl}(B) \subseteq \text{bcl}(\text{bcl}(A)) = \text{bcl}(A) \subseteq U$. Therefore $B$ is an IFSGbCS in $(X, \tau)$.

**Theorem 3.22:** If $A$ is IFbOS and IFSGbCS in an IFTS in $(X, \tau)$ then $A$ is an IFbCS in $(X, \tau)$.

**Proof:** Let $A$ be an IFbOS and IFSGbCS in $(X, \tau)$, $\text{bcl}(A) \subseteq A$. But $A \subseteq \text{bcl}(A)$. Hence $\text{bcl}(A) = A$. Therefore $A$ is an IFbCS in $(X, \tau)$.

4. Intuitionistic Fuzzy Semi Generalized $b$ Open Sets

**Definition 4.1:** An IFS $A$ is said to be an intuitionistic fuzzy semi generalized $b$-open set (IFSGbO) in $(X, \tau)$ if the complement $A^c$ is an IFSGbCS in $(X, \tau)$.

The collection of all intuitionistic fuzzy sgb-open sets of an IFTS $(X, \tau)$ is denoted by IFSGbO$(X)$.

**Example 4.2:** Let $X = \{a, b\}$ and let $\tau = \{0, G, 1.\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.2, 0.3), (0.7, 0.6))$. Consider an IFS $A = (x, (0.3, 0.4), (0.6, 0.5))$. Then $A$ is an IFSGbO in $(X, \tau)$.

**Example 4.3:** Let $X = \{a, b\}$ and let $\tau = \{0, G, 1.\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.2, 0.4), (0.7, 0.5))$. Consider an IFS $A = (x, (0.3, 0.2), (0.6, 0.7))$. Then $A$ is not an IFSGbO in $(X, \tau)$.

**Theorem 4.4:** Every IFOS, IFαOS, IFGαOS, IFαGSOS, IFwOS, IFSGOS is an IFSGbOS in $(X, \tau)$. But the converses are not true as seen from the following examples.

**Proof:** Straight forward.

**Example 4.5:** Let $X = \{a, b\}$ and let $\tau = \{0, G, 1.\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.2, 0.3), (0.7, 0.6))$. Consider an IFS $A = (x, (0.3, 0.4), (0.6, 0.5))$. Then $A$ is an IFSGbOS but not an IFOS and IFαOS in $(X, \tau)$. 
Example 4.6: Let $X = \{a, b\}$ and let $\tau = \{\emptyset, G, 1\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.3, 0.4), (0.7, 0.5))$. Consider an IFS $A = (x, (0.6, 0.4), (0.3, 0.5))$. Then A is an IFSGbOS but not an IFGαO and IFGSOS in $(X, \tau)$.

Example 4.7: Let $X = \{a, b\}$ and let $\tau = \{\emptyset, G, 1\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.7, 0.8), (0.3, 0.2))$ and the IFS $B = (x, (0.4, 0.2), (0.3, 0.5))$. Then A is an IFSGbOS but not an IFWO in $(X, \tau)$.

Example 4.8: Let $X = \{a, b\}$ and let $\tau = \{\emptyset, G, 1\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.1, 0.2), (0.5, 0.6))$ and the IFS $A = (x, (0.6, 0.7), (0.2, 0.2))$. Then A is an IFSGbO but not an IFSGO in $(X, \tau)$.

Remark 4.9: Intersection of any two IFSGbO sets need not be an IFSGbO set as seen in the following examples.

Example 4.10: Let $X = \{a, b\}$ and let $\tau = \{\emptyset, G, 1\}$ be an IFT on $(X, \tau)$ where $G = (x, (0.6, 0.8), (0.4, 0.2))$. Consider the IFS $A = (x, (0.9, 0.2), (0.1, 0.8))$ and the IFS $B = (x, (0.4, 0.2), (0.6, 0.7))$. Then A and B are IFSGbO but $A \cap B$ is not an IFSGbO in $(X, \tau)$.

Theorem 4.11: An IFS $A$ of an IFTS $(X, \tau)$ is an IFSGbOS if and only if $F \subseteq \operatorname{bint}(A)$ whenever $F$ is an IFCS and $F \subseteq A$.

Proof: Necessity: Suppose $A$ is an IFSGbOS in $(X, \tau)$. Let $F$ be an IFCS and $F \subseteq A$. Then $F^c$ is an IFOS in $(X, \tau)$ such that $A^c \subseteq F^c$. Since $A^c$ is an IFSGbCS, $\operatorname{bcl}(A^c) \subseteq F^c$. Hence $\operatorname{bint}(A)^c \subseteq F^c$. Therefore $F \subseteq \operatorname{bint}(A)$.

Sufficiency: Let $A$ be any IFS of $(X, \tau)$ and let $F \subseteq \operatorname{bint}(A)$ whenever $F$ is an IFCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and $F^c$ is an IFOS. By hypothesis, $\operatorname{bint}(A)^c \subseteq F^c$. Hence $\operatorname{bcl}(A^c) \subseteq F^c$. Thus $A$ is an IFSGbOS in $(X, \tau)$.

Theorem 4.12: If $A$ is an IFSGbOS in $(X, \tau)$ such that $\operatorname{bint}(A) \subseteq B \subseteq A$ then $B$ is an IFSGbOS in $(X, \tau)$.

Proof: By hypothesis, we have $\operatorname{bint}(A) \subseteq B \subseteq A$. This implies $A^c \subseteq B^c \subseteq (\operatorname{bint}(A))^c$. That is, $A^c \subseteq B^c \subseteq \operatorname{bcl}(A^c)$. Since $A^c$ is an IFSGbCS, by theorem 3.21, $B^c$ is an IFSGbCS. Therefore $B$ is an IFSGbOS in $(X, \tau)$.

References: