# Relation between Centroid and Moment of Inertia in the Aspect of Strength

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***Abstract*—In thispaper we describe about the relationship between the centroid and moment of inertia in the aspect of the strength of the section. Most of the engineers are well known the importance of the moment of the inertia. To increase the strength of a section, moment of inertia can be increased, but in some scenarios even by increasing moment of inertia of a section the strength can't be increased because of the centroid of the section. In geometrical property both centroid and moment of inertia have same weightage. The importance of centroid has beenemphasized in this paper.**

***IndexTerms*— Centroid,Moment of Inertia,Geometrical Property,Section Shape,Strength of materials.**

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## I. Introduction

Moment of inertia is the most important geometrical factor in sectional strength. Most of the engineers are well expertise in this. But mostly no one will aware about the centroid which plays the major role in the strength. Without knowing the importance of the centroid, engineers can't bring out the effective strength and shape for the materials. If the engineers don't have sufficient knowledge about the relation between centroid and moment of inertia, which will brings drastic damages in building and it will cause damages to life and property of the building users. To ensure the safe design and efficiency in designing, while selecting the higher moment of inertia, it is also need to check the centroid of the section. And the value of the centroid and moment of inertia is needed to be analyzed well so that the safety in designing can be ensured.

## II. Centroid

In a material which have uniform density, the center of mass will acts in its geometric center, this is known as centroid. In the point where a body's equilibrium force is balance to the resultant of earth's gravitational pull of the body is known as center of gravity. This concept of center of gravity is applicable only to bodies having weight. The term centroid is use to describe the analogous point in a geometric form such as a line or area where the entire area may be conceived to be concentrated and have the same moment of with the respect to any axis as the original distributed area has. [1]

In a symmetrical section, the centroid will be in the point where axis is will be coincided.Most of geometrical shapes in the structural elements like beam, column, and steel sections are symmetric, so centroid for the Non-Symmetrical Section was not discussed.

## III. Moment of Inertia

Moment of inertia is defined as, a quantity expressing a body's tendency to resist angular acceleration, which is the sum of the product of the mass of each particle in the body with the square of its distance from the axis of rotation. It is a measure of an object's resistance to changes to its rotation. It is usually quantified in $m^4$. Moment of inertia will be denoted in 'I'.

Usually structural engineers will consider moment of inertia as important factor in the aspect of strength of the section. Therefore to increase the strength of a section the moment of inertia will be increased.

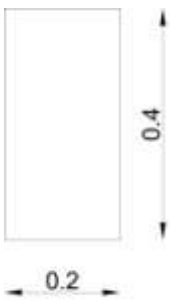
## IV. Relation between Centroid and Moment of Inertia

To explain the relation between centroid and moment of inertia, we have compared the three different shapes like rectangular, circular and triangular section for a beam element. The length of the beam is 3 meter and the end condition is simply supported. All the three shapes are compared with their size, area, moment of inertia, centroid and bending stress. The importance of both moment of inertia and centroid values can be noted when determining thebending stress values. When evaluating the centroid of the triangular section, there will be two different centroid values, the critical stress determination of the beam capacity will be usually in the basis of the largest centroid distance, so let's consider larger bending stress value for triangle in calculation of bending stress.

***Rectangular Section Property***

The size of the beam is 0.23m X 0.40 m and the calculated area of the section is 0.092Sq.m. The moment of the inertia is calculated using the standard formula rectangular and the value is 0.00123 $m^4$. And the centroid is 0.2m.

Table 1Rectangle Section Property

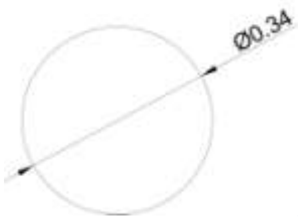



| Description | |
|---|---|
| Size | 0.23 M X 0.4 M |
| Area | 0.092 Sq.M |
| Moment of Inertia | 0.00123 $m^4$ |
| Centroid | 0.2 m |

***Circular Section Property***

The diameter of the circular section is 0.342m and the calculated area of the section is 0.092 Sq.m. The moment of the inertia is Calculated using the standard formula for circular and the value is 0.000671 $m^4$. And the centroid is 0.171 m.

Table 2Circle Section Property

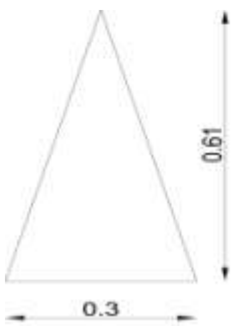



| Description | |
|---|---|
| Diameter | 0.342 M |
| Area | 0.092 Sq.M |
| Moment of Inertia | 0.000671 $m^4$ |
| Centroid | 0.171 m |

***Triangular Section Property***

The base breadth of the triangle is 0.3 m and the depth is 0.613 m. The calculated area of the section is 0.092 Sq.m. The moment of the inertia is calculated using the standard formula for triangle and the value is 0.001919 $m^4$. And the centroid value is 0.204 m from bottom and 0.409 m from top of the triangle.

Table 3Triangle Section Property




| Description | |
|---|---|
| Size | Base = 0.3 M, Depth = 0.613 M |
| Area | 0.092 Sq.M |
| Moment of Inertia | 0.001919 $m^4$ |
| Centroid | From bottom : 0.204 m<br>From Top : 0.409 m |

***Beam Details***

For all the section the span of the beam will be 3 meter. And the end condition will be simply supported. Here we omitted the material of the beam, because the discussion is only about the moment of inertia and centroid. These two properties are geometrical property so there is no need for the consideration of the material type and its properties.

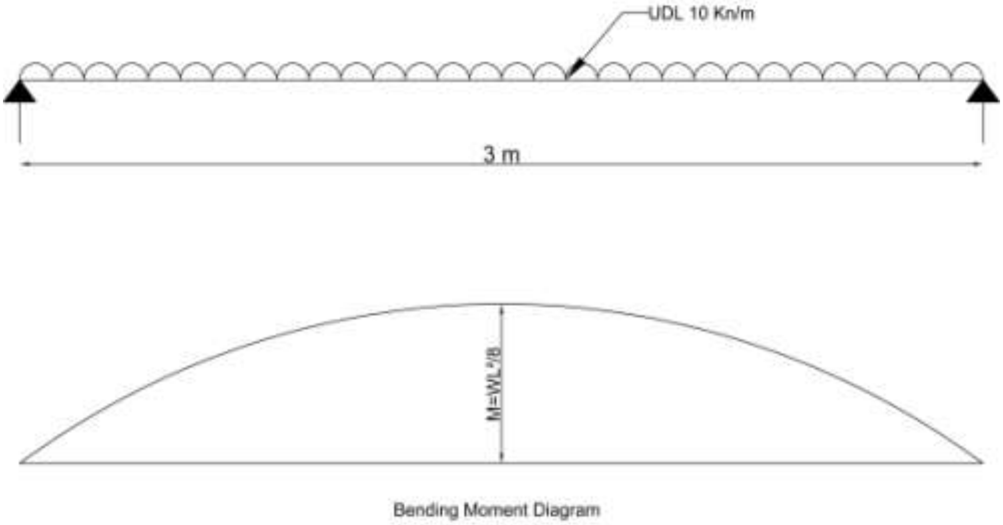
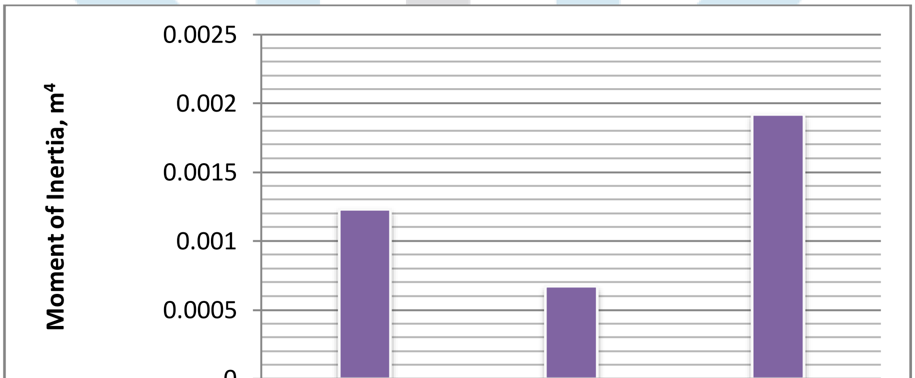



Fig 1 Beam with BMD Diagram

The beam will carry uniformly distributed load of 10 Kn/m over it full length. For the given load the bending moment is calculated. Beam using either a rectangular, circular or triangular section, the maximum bending moment present in the beam will be 11.25 Kn.m.

***Comparison of Moment of Inertia***

For rectangle, circle and triangle the calculated moment of inertia is compared to find which section will have more moment of inertia. From figure 2, it will be very clear that the triangular section will carry much higher load carrying capacity, other than two sections. But good practice for Civil/structural engineer is that the decision should not take with the only consideration of the moment of the inertia while selection of the section for a structural element.




| | Rectangle | Circle | Triangle |
|---|---|---|---|
| Moment of Inertia | 0.00123 | 0.000671 | 0.001919 |

Fig 2Comparison of Moment of Inertia for all sections

## V. BENDING STRESS FOR BEAM

Bending stress is the one of the basis analysis of beam element. Normally bending stresses are developed in response to the external bending moment existing in the beam. Bending Stresses will vary linearly at cross section of beam element. Bending stress can be calculated using the below mentioned formula. Where f is bending moment, $\bar{Y}$ is centroid and I is moment of inertia.

$$f = M\bar{Y}/I \qquad (1)$$

Bending stress is calculated for rectangular, triangular and circular section and the bending stress diagram is show in the figure 3. For triangular section there will two different bending stress values it is because of the two centroid values. In general for structural analysis always higher bending stress value is needed to consider, so we have consider the higher values.

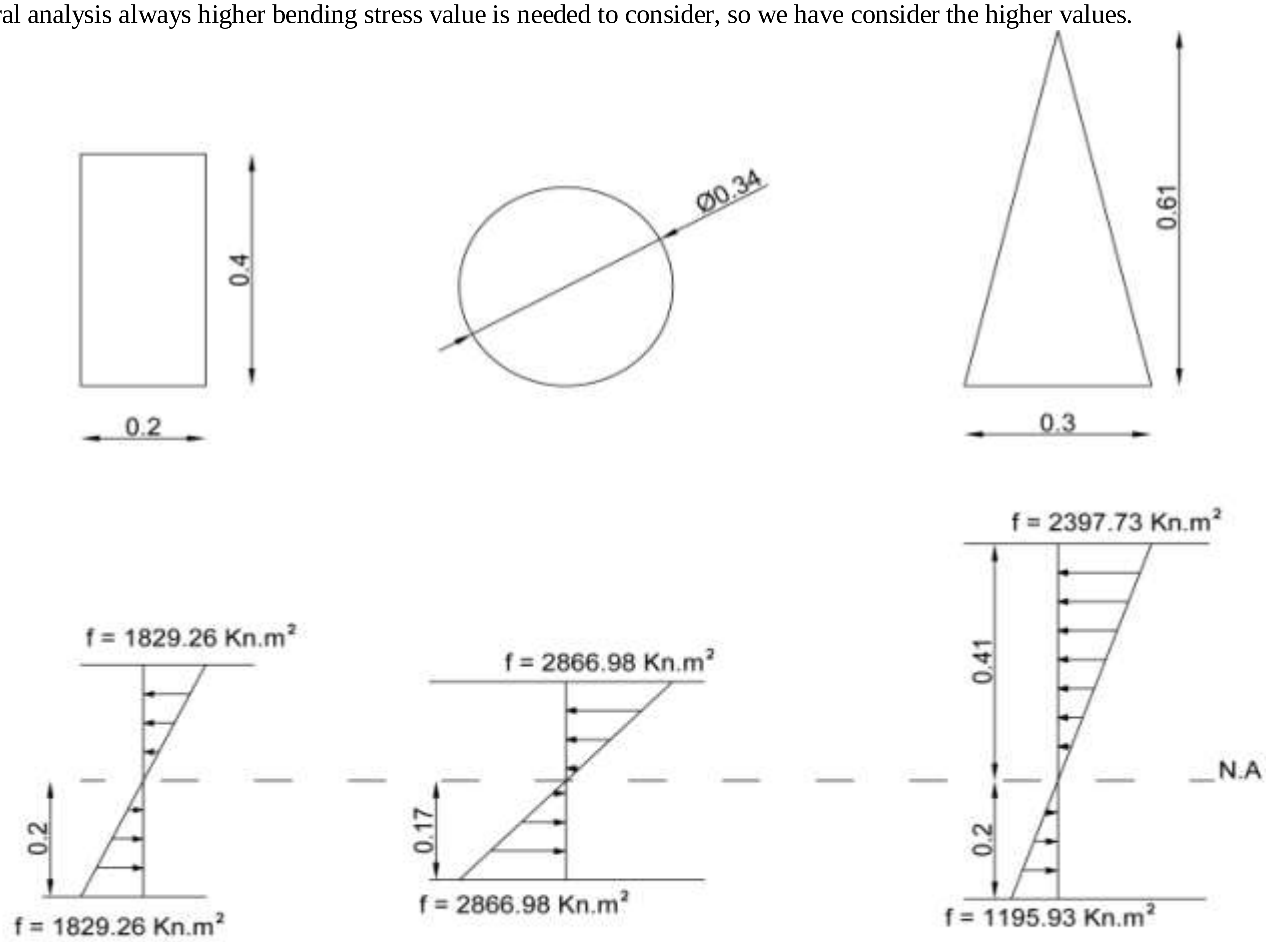



Fig 3Bending Stress for Various Sections.

From the above figure it's clear that smallest stress developed in the rectangular beam, which could thus carry the greatest external loading before the section gets overstressed. Note that the triangular section has higher moment of inertia value than any of the sections, but triangular section develops higher stress than the rectangular beam section, this is because of the lager centroid value of triangular section.

## VI. Conclusion

Centroid and moment of inertia is related very closely to any structural section. Both the parameter involved in determining the strength of the section. From the comparison of moment of inertial it seems like that the triangular section will carry much higher loads, but it's not true. Moment of inertia is not the only factor to determine the strength of the section. It is proved from the comparison of the bending stress, where rectangular section will carry much higher load than the triangular section which has higher value of moment of inertia than rectangle. Centroid value for rectangle is less than triangular section, which is the reason rectangular section has smaller bending stress. Circular section has smaller moment of inertia than other two sections, because more material is nearer the neutral axis in circular section. Engineers should consider the centroid with equal weightage of the moment of inertia, in structural analysis. So that drastic failure in structural members can be avoided and efficiency of the member will be increased in the aspect of load carrying and resistance in bending.

## References

[1] Daniel L.Schodek, Martin Bechthold, Structures, PHI Learning Private Limited, Sixth edition.
[2] R.Vaidyanathan, P.Perumal, Structural Analysis Volume 1, Laksmi Publication, Third Edition.
[3] R.Vaidyanathan, P.Perumal, Structural Analysis Volume 2, Laksmi Publication, Third Edition.
[4] U.C.Jindal, A Text Book on Engineering Mechanics, Made Easy Publication, 2013 Edition.
[5] M.A.Velusami, Engineering Mechanics, S.Chand& Company, Fourth Edition.
[6] Dr.K.S.Yadav, Engineering Mechanics (Theory of Practicals), Vei Publication, Third Edition.