ON SOFT CONTRA $\pi g*b*$ - CONTINUOUS FUNCTION AND SOFT ALMOST $\pi g*b*$- CONTINUOUS FUNCTION IN SOFT TOPOLOGICAL SPACES

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***Abstract*: The purpose of this paper is to introduce the concept of soft contra-continuity, soft contra $\pi g*b*$-continuity, soft almost continuity and soft almost $\pi g*b*$-continuity in soft topological spaces and some of their properties are also obtained. The relation of these mappings with other soft contra-continuous functions is also introduced.**

Keywords: soft set, soft topology, soft $\pi g*b*$-closed set, soft $\pi g*b*$-open set, soft contra-continuity, soft contra $\pi g*b*$-continuity, soft almost continuity, soft almost $\pi g*b*$-continuity

1. INTRODUCTION

Soft set theory was initiated by Molodtsov [1] as a new method for vagueness. He showed in his paper that this theory can be applied to several areas such as smoothness of functions, game theory, Riemann-integration, operation research, etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parameterization inadequacy syndrome of other theories developed for vagueness. Muhammad Shabir and Munazza Naz [3] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also defined some concepts of soft sets on soft topological spaces such as soft interior, soft closure, soft spaces and soft separation axioms. In 1996 Dontchev [4] introduced the notion of contra continuous functions. M.K Singal and Asha rani Singal [6] introduced the concept of almost continuity in topological spaces. Kharal et al. [5] introduced soft functions over classes of soft sets. Cigdem Gunduz Aras et al., studied and discussed the properties of soft continuous mappings. The purpose of this paper is to introduce the concepts of soft contra continuity, soft contra $\pi g*b*$-continuity, soft almost continuity and soft almost $\pi g*b*$continuity and to obtain some characterization of these mappings. The relation of such mappings with other soft contra-continuous functions is also introduced.

2. PRELIMINARIES

Definition : 2.1 [1] Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let A ⊆ E. Let $\tilde{X}$,be an initial universal set and E be the set of parameters. Let P($\tilde{X}$) denote the power set of $\tilde{X}$, and A $\tilde{\subseteq}$ E. The pair (F, A) is called a soft set over $\tilde{X}$, where F is a mapping given by F : A→ P($\tilde{X}$).

Definition 2.2. [2] A soft set (F,E) over $\tilde{X}$ is said to be

i) A null soft set, denoted by φ , if ∀ e∈ E , F(e) = φ .

ii) An absolute soft set, denoted by $\tilde{X}$, if e ϵ E , F(e) = $\tilde{X}$.

Definition 2.3. [2] Let τ be the collection of soft sets over $\tilde{X}$, then τ is said to be a soft Topology on $\tilde{X}$ if

i) $\tilde{X}$ are belongs to τ .
ii) The union of any number of soft sets in τ belongs to τ.
iii) The intersection of any two soft sets in τ belongs to τ.

The triplet ($\tilde{X}$, τ, E) is called a soft topological space over $\tilde{X}$ and any member of τ is known as soft open set in $\tilde{X}$. The complement of a soft open set is called soft closed set over $\tilde{X}$.

Definition 2.5 ([1]. A pair (F,A) is called a soft set over U, where F is a mapping given by F : A → P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular e ∈ A. F(e) may be considered the set of e-approximate elements of the soft set (F,A).

2. SOFT CONTRA $\pi g*b*$-CONTINUOUS FUNCTION

Definition : 2.1

Let f: (X,τ,E) and (Y,σ,E) be two soft topological spaces. A soft function f: $(X,\tau,E)\to(Y,\sigma,E)$ is said to be

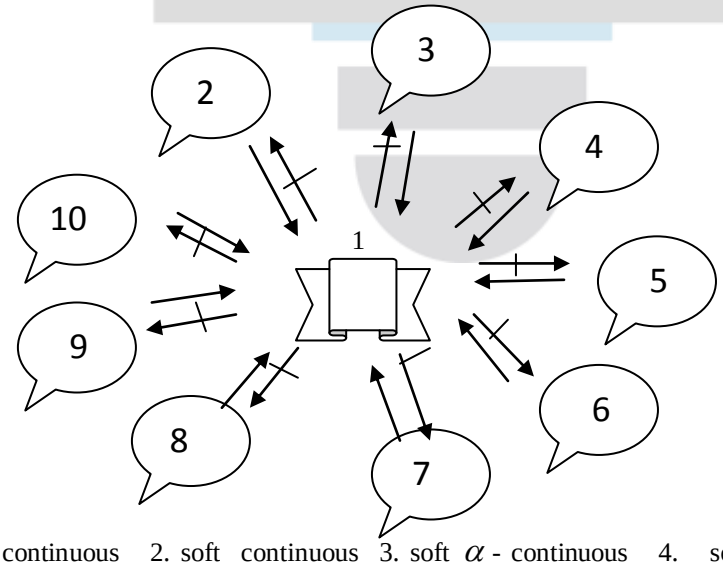
(i) soft contra-continuous if f^{-1}(G,E) is soft closed over X for every soft open set (G,E) over Y.
(ii) soft almost-continuous if f^{-1}(F,E) is soft closed over X for every soft regular closed set (G,E) over Y.
(i) soft contra α -continuous if f^{-1}(G,E) is soft α -closed over X for every soft open set (G,E) over Y.
(i) soft contra g-continuous if f^{-1}(G,E) is soft g-closed over X for every soft open set (G,E) over Y.
(i) soft contra πg -continuous if f^{-1}(G,E) is soft πg -closed over X for every soft open set (G,E) over Y.
(i) soft contra $\pi^{\wedge} g$ -continuous if f^{-1}(G,E) is soft $\pi^{\wedge} g$ closed over X for every soft open set (G,E) over Y.
(i) soft contra $\pi * g$ -continuous if f^{-1}(G,E) is soft $\pi * g$ closed over X for every soft open set (G,E) over Y.
(i) soft contra πgb -continuous if f^{-1}(G,E) is soft πgb closed over X for every soft open set (G,E) over Y.
(i) soft contra πgp -continuous if f^{-1}(G,E) is soft πgp -closed over X for every soft open set (G,E) over Y.
(i) soft contra πgr -continuous if f^{-1}(G,E) is soft πgr -closed over X for every soft open set (G,E) over Y.

Definition : 2.2
A function f: $(X,\tau,E) \to (Y,\sigma,E)$ is called soft contra $\pi g * b *$- continuous if f^{-1}(V) is $\pi g * b *$- closed in X for each open set V of Y.
Definition : 2.3
A map f: $(X,\tau,E) \to (Y,\sigma,E)$ is called soft almost contra $\pi g * b *$- continuous if the inverse image of every regular open set in Y is $\pi g * b *$- closed in X .
Theorem : 2.4
(i) Every soft contra continuous is soft contra $\pi g * b *$ -continuous .
(ii) Every soft contra α - continuous is soft contra $\pi g * b *$ - continuous .
(iii) Every soft contra g- continuous is soft contra $\pi g * b *$ -continuous .
(iv) Every soft contra πg - continuous is soft contra $\pi g * b *$ -continuous .
(v) Every soft contra $\pi^{\wedge} g$ - continuous set is soft contra $\pi g * b *$ -continuous .
(vi) Every soft contra $\pi * g$ -continuous is soft contra $\pi g * b *$ -continuous .
(vii) Every soft contra πgb -continuous is soft contra $\pi g * b *$ -continuous .
(viii)Every soft contra πgp -continuous is soft contra $\pi g * b *$ -continuous .
(ix) Every soft contra πgr -continuous is soft contra $\pi g * b *$ -continuous

Proof :
Let f: $(X,\tau,E) \to (Y,\sigma,E)$ is called soft contra – continuous. Let (G,B) be a soft open set in Y.
Then the inverse image f^{-1}(G,B) is soft closed in X.

Since every closed set is soft $\pi g * b *$ - closed , f^{-1}(G,B) is soft $\pi g * b *$ - closed in X.

Hence f is soft contra $\pi g * b *$ - continuous. The proof is obvious .



1. soft $\pi g * b *$ - continuous 2. soft continuous 3. soft α - continuous 4. soft g- continuous 5. soft πg - continuous 6. soft $\pi \wedge g$ - continuous 7. soft $\pi * g$ -continuous 8. soft πgb -continuous 9. soft πgp -continuous 10. soft πgr - continuous

Example : 2.5
Let X ={a,b,c}=Y, E = {e_1,e_2}. Let F_1, F_2,…, F_7 are functions from E to P(X) and are defined as follows :
$F_1(e_1)$ = {a}, $F_1(e_2)$ = {a}, $F_2(e_1)$ = {b}, $F_2(e_2)$ = {b}, $F_3(e_1)$ = {c}, $F_3(e_2)$ = { ϕ },
$F_4(e_1)$ = {a,b}, $F_4(e_2)$ = {a,b}, $F_5(e_1)$ = {a,c}, $F_5(e_2)$ = {a}, $F_6(e_1)$ = {b,c}, $F_6(e_2)$ = {b},
$F_7(e_1)$ = {X}, $F_7(e_2)$ ={b}.
Then τ = { ϕ, $\tilde{X}$, (F_1,E),(F_2,E),…,(F_7,E) is a soft topology and elements in τ are soft open sets. Let G_1,G_2 ,…,G_7 are functions from E to P(Y) and are defined as follows :
$G_1(e_1)$ = {c}, $G_1(e_2)$ = {b}, $G_2(e_1)$ = {c}, $G_2(e_2)$ = {a}, $G_3(e_1)$ = {c}, $G_3(e_2)$ = {a,b},
$G_4(e_1)$ = {a,c}, $G_4(e_2)$ = {a}, $G_5(e_1)$ = {a,c}, $G_5(e_2)$ = {a,b}, $G_6(e_1)$ = {a,c}, $G_6(e_2)$ = {b},
$G_7(e_1)$ = {a,c}, $G_7(e_2)$ = { ϕ }.
Then σ = { ϕ, $\tilde{Y}$,(G_1,E),(G_2,E),…,(G_7,E) is a soft topology on Y.
Let f : (X, τ , A) $\rightarrow$ (Y, σ ,B) be an identity map by f(a)=a, f(b)=b, f(c)=c.

Here the inverse image of the soft open set in (Y, σ) is $\pi g*b*$-closed and hence f is soft

contra $\pi g*b*$- continuous. (A,E)={{c},{a,b}},{{a,c},{a}},{{a,c},{a,b}},{{a,c},{b}} in (Y, σ) is not soft closed, soft α -closed, soft g –closed, soft π g- closed, soft $\pi \wedge g$ -closed, soft $\pi * g$ -closed, soft πgb -closed, soft πgr -closed, soft πgp -closed in X. Hence f is contra $\pi g*b*$- continuous not soft contra continuous, soft contra α - continuous , soft contra g – continuous , soft contra π g- continuous, soft contra $\pi \hat{} g$ - continuous, soft contra $\pi * g$ - continuous, soft contra πgb - continuous, soft contra πgr - continuous, soft contra πgp - continuous.

Theorem : 2.6
Let (X,τ,E) and (Y,σ,E) be two soft topological spaces, f : $(X,\tau,E) \rightarrow (Y,\sigma,E)$ be a mapping and $\pi g*b*-O((X,\tau,E))$ is closed under arbitrary union. Then the following conditions are equivalent :
(i) f : $(X,\tau,E) \rightarrow (Y,\sigma,E)$ be a soft contra $\pi g*b*$-continuous mapping
(ii) The inverse image of each soft closed set over Y is soft $\pi g*b*$-open over X.
Proof:
(i) $\rightarrow$ (ii)
Let (F,E) be a soft closed set over Y. Then Y-(F,E) is soft open over Y. By assumption, f^{-1}((Y-(F,E)) is soft $\pi g*b*$-closed over X. That is X-f^{-1}(F,E) is soft $\pi g*b*$-closed over X. Therefore f^{-1}(F,E) is soft $\pi g*b*$-open over X.
(ii) $\rightarrow$ (iii)
Let (G,E) be a soft open set over Y. Then Y-f^{-1}(G,E) is soft closed over Y. By assumption, Y-f^{-1}(F,E) = X-f^{-1}(G,E) is soft $\pi g*b*$- open over X. That is f^{-1}(F,E) is soft $\pi g*b*$-closed over X. Therefore f is soft contra $\pi g*b*$-continuous.
Theorem : 2.7
Let f : $(X,\tau,E) \rightarrow (Y,\sigma,E)$ be a soft contra $\pi g*b*$-continuous function and g : $(Y,\tau,E) \rightarrow (Z,\eta,E)$ be a soft continuous function then $g \circ f : (X,\tau,E) \rightarrow (Z,\eta,E)$ is soft contra $\pi g*b*$-continuous function.
Proof :
Let (G,E) be any soft open set over Z. since g is soft continuous g^{-1}(G,E) is soft open over Y and since f is soft contra $\pi g*b*$-continuous, f^{-1}(g^{-1}((G,E))) = (g ∘ f) $^{-1}$(G,E) is soft $\pi g*b*$-closed over X which implies g ∘ f is soft contra $\pi g*b*$-continuous.
Theorem : 2.8
Let f : $(X,\tau,E) \rightarrow (Y,\sigma,E)$ be a soft $\pi g*b*$- irresolute and g : $(Y,\tau,E) \rightarrow (Z,\eta,E)$ be a soft contra $\pi g*b*$-continuous function then $g \circ f : (X,\tau,E) \rightarrow (Z,\eta,E)$ is soft contra $\pi g*b*$-continuous function.
Proof :
Let (G,E) be any soft open set over Z. since g is soft contra $\pi g*b*$-continuous g^{-1}(G,E) is soft $\pi g*b*$-closed over Y and since f is soft $\pi g*b*$-irresolute, f^{-1}(g^{-1}((F,E))) = (g ∘ f) $^{-1}$(F,E) is soft $\pi g*b*$-closed over X and consequently g ∘ f is soft contra $\pi g*b*$-continuous.

3. Soft Almost $\pi g*b*$-continuous Function

Definition : 3.1

Let (X,τ,E) and (Y,σ,E) be two soft topological spaces and f : $(X,\tau,E)\to(Y,\sigma,E)$ be a soft functions, then

(i) f is said to be a almost continuous function if f^{-1}(F,E) is soft closed over X for every soft regular closed set (F,E) over Y.

(ii) f is said to be a almost $\pi g*b*$-continuous function if f^{-1}(F,E) is soft $\pi g*b*$-closed over X for every soft regular closed set (F,E) over Y.

Theorem : 3.2

Every soft $\pi g*b*$-continuous function is soft almost $\pi g*b*$-continuous.

Proof :

Let f : $(X,\tau,E)\to(Y,\sigma,E)$ be a soft $\pi g*b*$-continuous function and (F, E) be any soft regular closed set over Y. Since every soft regular closed set is soft closed, (F, E) is soft closed over Y. So by assumption f^{-1}(F, E) is soft $\pi g*b*$-closed over X. Therefore f is soft almost $\pi g*b*$-continuous.

Theorem : 3.3

Every soft $\pi g*b*$-irresolute function is soft almost $\pi g*b*$-continuous.

Proof:

Let f : $(X,\tau,E)\to(Y,\sigma,E)$ be a soft $\pi g*b*$-irresolute function and (F, E) be any soft regular closed set over Y. Since every soft regular closed set is soft $\pi g*b*$-closed, (F, E) is soft $\pi g*b*$-closed over Y. So by assumption f^{-1}(F, E) is soft $\pi g*b*$-closed over X. Therefore f is soft almost $\pi g*b*$-continuous.

Theorem : 3.4

Let f : (X,τ,E) and (Y,σ,E) be two soft topological spaces and f : $(X,\tau,E)\to(Y,\sigma,E)$ be a soft almost continuous function, then f is soft almost $\pi g*b*$-continuous function.

Proof:

Let (F, E) be any soft regular closed set over Y. So by assumption f^{-1}(F, E) is soft closed over X. But every soft closed set is soft $\pi g*b*$-closed, f^{-1}(F, E) is soft $\pi g*b*$-closed over X. Therefore f is soft almost $\pi g*b*$-continuous.

References

[1] D. Molodtsov, Soft set theory-First results, Computers and Mathematics with Applications, 37(1999), 19-31

[2] K.Kannan "Soft generalized closed sets in topological spaces", Journal of Theoretical and Applied Information Technology, Mar 2012, Vol.37,No 1.17-21.

[**3**] Muhammad Shabir, Munazza Naz, On soft topological spaces, Comput. Math. Appl., 61, (2011), 1786–1799.

[**4**] Dontchev J, Contra-continuous functions and strongly s-closed spaces, Int. Math Math Sci., 19 (1996), 303-10.

[**5**] Kharal A, Ahamed B, Mapping on soft classes, New Mathematics and Natural Computation, 2011; 7(3): 471-481

[**6**] M K Singal and Asha Rani Singal, Almost-continuous mappings, Yokohoma Math. J. 16, 63-73. 1968.

[7] Vaiyomathi.K and Athika.A, "On Soft πg*b*-closed sets in soft topological spaces, International Journal of Applied Research 2017; 3(9): 73-80