DIFFERENT FORMS OF Ω_{gb}^{+} - CONTINUITY IN SIMPLE EXETENSION TOPOLOGICAL SPACES

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***ABSTRACT*: In this paper we propose the concept of a new forms continuous functions named totally Ω_{gb}^{+}-continuous, totally Ω_{gb}^{+}-irresolute and Ω_{gb}^{+}-totally continuous functions in simple extended topological spaces (SETS) and study their relationships.**

***KEYWORDS*: Totally Ω_{gb}^{+}-continuous, Totally Ω_{gb}^{+}-irresolute and Ω_{gb}^{+}-totally continuous functions.**

1. INTRODUCTION

Continuity is one of the major research areas in Mathematical sciences. Many different forms of continuous functions have been introduced by Mathematicians over the years. Some of them are strongly continuous functions (Levine 1963), contra continuous functions (Dontchev1996), supra continuous functions (Mashhour 1983) and Jain introduced the totally continuous functions in 1980.

The concept of extending a topology by a non open set was proposed by Levine in 1963.A simple extension of a topology τ is defined as $\tau(B) = \{(B \cap O') \cup O / O,O' \in \tau\}$ by Levine. The purpose of the present paper is to study some new forms of totally Ω_{gb}^{+} -continuous totally Ω_{gb}^{+}- irresolute and Ω_{gb}^{+}-totally continuous functions in extended topological spaces and investigate some of their properties.

Throughout this paper X, Y and Z (or (X, τ^{+}),(Y, σ^{+}) and (Z, η^{+})) are simple extension topological space in which no separation axioms are assumed unless and otherwise stated. For any subset A of X, the interior of A is same as the interior in usual topology and the closure of A is newly defined in simple extension topological space.

2. PRELIMINARIES

Some definitions and basic concepts related to this paper.

Definition 2.1: A subset A of a topological space (X,τ) is said to be,

(i) regular open set [13], if $A = int(cl(A))$ and a b-open set [1],if $A \subseteq cl(int(A)) \cup int(cl(A))$.
(ii) a generalized closed (briefly g-closed) [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(iii)a generalized b-closed (briefly gb-closed) [5] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(iv)πgb-closed[12] if $bcl(A) \subset A$ whenever $A \subset U$ and U is π-open in (X,τ).

By πGBC(X,τ) we mean the family of all πgb-closed subsets of the space (X,τ)

Definition 2.2: A subset A of a topological space (X,τ^{+}) is said to be,

(i) regular^{+} open set[9] if $A = int(cl^{+}(A))$ and b^{+}-open set, if $A \subseteq cl^{+}(int(A)) \cup int(cl^{+}(A))$.
(ii) a generalized^{+} closed[9] (briefly g^{+}-closed) if $cl^{+}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(iii)a generalized b^{+}-closed[9] (briefly gb^{+}-closed) if $bcl^{+}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(iv)πgb^{+}-closed[10] if $bcl^{+}(A) \subset A$ whenever $A \subset U$ and U is π^{+}-open in (X,τ^{+}).

By $\pi GB^{+}C(X,\tau^{+})$ we mean the family of all πgb^{+}-closed subsets of the space (X,τ^{+}).

Definition 2.3:A function $f : X \rightarrow Y$ is

(i) totally- continuous function [3] if the inverse image of every open subset of (Y,σ) is a clopen subset of (X,τ) .
(ii) totally R^{*}-continuous[4] if $f^{-1}(V)$ every is R^{*}-clopen subset of X for every open subset of Y.
(iii) totally irresolute[14] if every $f^{-1}(V)$ is clopen in X for every open set V in Y.
(iv) quasi irresolute[11] if every $f^{-1}(V)$ is clopen in X for every clopen set V in Y.
(v) totally R^{*}-irresolute[4] if the inverse image of every R^{*}-open subset of Y is a R^{*}-clopen subset of Y.
(vi) quasi R^{*}-irresolute[4] if the inverse image of every R^{*}-clopen subset of Y is a R^{*}-clopen subset of X.

Definition 2.4[6]: Let S be a subset of a topological space (X, τ^+) we define the sets $\Omega_{gb}^+(S)$ and $\mho_{gb}^+(S)$ as follows, $\Omega_{gb}^+(S) = \cap\{G|G\in\pi GB^+O(X,\tau^+)$ and $S\subseteq G\}$, $\mho_{gb}^+(S) = \cup\{F|F\in \pi GB^+C(X,\tau^+)$ and $S\supseteq F\}$.

Definition 2.5[6]: A subset S of a space (X, τ^+) is called a Ω_{gb}^+ -set if $S= \Omega_{gb}^+(S)$

Definition 2.6[6]: A subset A of a space (X, τ^+) is called a Ω_{gb}^+-closed set if $A = S\cap C$ where S is Ω_{gb}^+-set and C is a closed set.

Definition 2.7: A function f: $(X,\tau^+)\rightarrow(Y,\sigma^+)$ is called
1. Ω_{gb}^+-continuous[7] if every $f^{-1}(V)$ is Ω_{gb}^+-closed in (X,τ^+) for every closed set V of (Y,σ^+).
2. Ω_{gb}^+-irresolute[7] if $f^{-1}(V)$ is Ω_{gb}^+-closed in (X,τ^+) for every Ω_{gb}^+-closed set V in (Y,σ^+).

Definition 2.8[7]: A topological space X is a Ω_{gb}^+ space if every Ω_{gb}^+- closed set is closed.

Definition 2.9[8]: A space (X,τ^+) is called a Ω_{gb}^+-locally indiscrete if every Ω_{gb}^+-open set in it is closed.

Definition 2.10[4]:A subset A of a topological space (X, τ) is called R^*-connected if X cannot be written as the disjoint union of two non-empty R^*-open subsets.

Definition 2.11[7]: (X, τ^+) is Ω_{gb}^+-T_0 if for each pair of distinct points x, y of X, there exists a Ω_{gb}^+-open set containing one of points but not the other.

Definition 2.12[7]: (X, τ^+) is Ω_{gb}^+-T_1 if for any pair of distinct points x,y of X, there is a Ω_{gb}^+-open set U in X such that $x\in U$ and $y \notin U$ and there is a Ω_{gb}^+-open set V in X such that $y \in V$ and $x\notin V$.

Definition 2.13[7]: (X, τ^+) is Ω_{gb}^+- T_2 if for each pair of distinct points x and y in X there exists a Ω_{gb}^+ -open set U and a Ω_{gb}^+ -open set V in X such that $x \in U$, $y\in V$ and $U\cap V= \phi$.

3. TOTALLY Ω_{gb}^+ -CONTINUOUS FUNCTION

Definition 3.1: A function $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is said to be totally$^+$- continuous function if the inverse image of every open subset of (Y,σ^+) is a clopen subset of (X,τ^+) .

Definition 3.2: A function $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ is said to be totally Ω_{gb}^+- continuous if the inverse image of every open subset in (Y ,σ^+) is a Ω_{gb}^+-clopen subset in (X ,τ^+).

Proposition 3.3 :A function $f : (X ,\tau^+) \rightarrow(Y ,\sigma^+)$ is totally Ω_{gb}^+ - continuous if and only if the inverse of every closed subset in (Y ,σ^+) is a Ω_{gb}^+-clopen subset in (X ,τ^+).

Proof: Assume that f is totally Ω_{gb}^+- continuous . Let A be any closed subset in Y. Then A^c is a open subset in Y. Since f is totally Ω_{gb}^+- continuous . Thus, $f^{-1}(A^c)$ is Ω_{gb}^+- clopen subset
in X(i.e., Ω_{gb}^+- open and Ω_{gb}^+-closed subset in X) . But $f^{-1}(A^c)=X-f^{-1}(A)$ and so $f^{-1}(A)$ is both Ω_{gb}^+-closed and Ω_{gb}^+-open subset . Hence,$f^{-1}(A)$ is Ω_{gb}^+-clopen subset in X.

Conversely, Let G be an open subset in Y. Then G^c is closed subset in Y.
By assumption $f^{-1}(G^c)$ is Ω_{gb}^+-clopen subset in X (i.e., Ω_{gb}^+- open and Ω_{gb}^+-closed subset in X). But $f^{-1}(G^c)=X-f^{-1}(G)$ and so $f^{-1}(G)$ is both Ω_{gb}^+-closed and Ω_{gb}^+-open subset.
Hence $f^{-1}(G)$ is Ω_{gb}^+-clopen subset in X. Therefore, f is totally Ω_{gb}^+- continuous function.

Proposition 3.4: Every totally$^+$-continuous function is totally Ω_{gb}^+-continuous.

Proof: Let $f : (X, \tau^+) \rightarrow (Y, \sigma^+)$ be a totally$^+$-continuous and A be an open subset in Y. Since f is totally$^+$- continuous function. Thus $f^{-1}(A)$ is clopen subset in X. Since every open set is Ω_{gb}^+-open and every closed set is Ω_{gb}^+-closed. This implies that $f^{-1}(A)$ is Ω_{gb}^+-clopen subset in X. Therefore f is a totally Ω_{gb}^+-continuous.

Example 3.5: Let X=Y={a,b,c} be two topology spaces with topologies $\tau = \{\Phi,X,\{b,c\}\}$, B={a}, $\tau^+ =\{ \Phi,X,\{a\},\{b,c\}\}$and $\sigma=\{\Phi,Y,\{a\}\}$, B={b}, $\sigma^+=\{ \Phi,Y,\{a\},\{b\},\{a,b\}\}$. Define $f:(X, \tau^+) \rightarrow (Y,\sigma^+)$ by f(a)=c, f(b)=a, f(c)=b. Then the function f is totally Ω_{gb}^+-continuous function but f is not totally$^+$-continuous function, since for open set A={a}. $f^{-1}(A)=f^{-1}(\{a\})=\{b\}$ is not clopen subset in X.

Proposition 3.6: Every totally Ω_{gb}^+-continuous function is Ω_{gb}^+-continuous

Proof: Let A be an open subset in Y. Since f is a totally Ω_{gb}^{+}-continuous function. Thus, $f^{-1}(A)$ is Ω_{gb}^{+}-clopen subset in X.(i.e., $f^{-1}(A)$ is both Ω_{gb}^{+}-open and Ω_{gb}^{+}-closed subset in X). Thus $f^{-1}(A)$ is Ω_{gb}^{+}-open subset in X. Therefore f is a Ω_{gb}^{+}-continuous.

Example 3.7: Let X=Y={a,b,c,d} be two topology spaces with topologies τ={Φ,X,{a},{c,d},{a,c,d}}, B={d}, τ^{+}={Φ,X,{a},{d},{a,d},{c,d},{a,c,d}} and σ={Φ,Y,{a},{a,b},{a,b,d}}, B={d}, σ^{+}={ Φ,Y,{a},{d},{a,b},{a,d},{a,b,d}}, then $\Omega_{gb}^{+}O(X,\tau^{+})$={X,Φ,{a},{c},{d},{a,b},{a,c},{a,d},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}$\Omega_{gb}^{+}C(X,\tau^{+})$={X,Φ,{a},{b},{c},{d},{a,b},{a,c},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d}} Define f:(X, τ^{+}) → (Y,σ^{+}) by f(a)=c, f(b)=b, f(c)=a,f(d)=d. Then the function f is Ω_{gb}^{+}-continuous function, but f is not totally Ω_{gb}^{+}-continuous function. Since for open set A={a,b} in Y. $f^{-1}(A)=f^{-1}(\{a,b\})$={b,c} is Ω_{gb}^{+}-closed but is not Ω_{gb}^{+}-open subset in X. Hence $f^{-1}(A)$ $f^{-1}(A)=f^{-1}(\{a,b\})$={b,c} is not Ω_{gb}^{+}-clopen subset in X.

Proposition 3.8: Let f:(X, τ^{+}) → (Y,σ^{+}) and g:(Y, τ^{+}) → (Z,σ^{+}) be any two function then g∘f:(X, τ^{+}) → (Z,σ^{+}) totally Ω_{gb}^{+}- continuous.

(i) If f is Ω_{gb}^{+}- irresolute and g is totally Ω_{gb}^{+}- continuous then g∘f is totally Ω_{gb}^{+}- continuous.
(ii) If f is totally Ω_{gb}^{+}- continuous and g is continuous then g∘f is totally Ω_{gb}^{+}-continuous.
(iii) If f and g are two totally^{+}-continuous functions then g∘f is totally Ω_{gb}^{+}-continuous.

Proof: (i) Let V be an open set in Z. Since g is totally Ω_{gb}^{+}-continuous, $g^{-1}(V)$ is Ω_{gb}^{+}-clopen in Y. Since f is Ω_{gb}^{+}-irresolute, $f^{-1}(g^{-1}(V))$ is Ω_{gb}^{+}-open and Ω_{gb}^{+}- closed in X. Since $(g\circ f)^{-1}(V)= f^{-1}(g^{-1}(V))$. Hence g∘f is totally Ω_{gb}^{+}-continuous.
(ii) Let V be an open set in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Also since f is totally Ω_{gb}^{+}-continuous, $f^{1}(g^{-1}(V))$ is Ω_{gb}^{+}-clopen in X. Hence g∘f is totally Ω_{gb}^{+}-continuous.
(iii) Let V be a Ω_{gb}^{+}-open subset in Z. Since g totally^{+}-continuous function. Thus $g^{-1}(V)$ is clopen subset in Y, this implies that $g^{-1}(V)$ is an open subset in Y. By hypothesis f is a totally^{+}-continuous function, then $f^{-1}(g^{-1}(V))$ is clopen in X. Since every open set is Ω_{gb}^{+}-open set and every closed set is Ω_{gb}^{+}-closed set. Thus $f^{1}(g^{-1}(V))$ is Ω_{gb}^{+}-clopen subset in X. But $f^{-1}(g^{-1}(V))= (g\circ f)^{-1}(V)$ and so $(g\circ f)^{-1}(V)$ is clopen subset in X.
Hence g∘f:(X, τ^{+}) → (Z,σ^{+}) totally Ω_{gb}^{+}- continuous.

Definition 3.9: A subset A of a topological space (X, τ^{+}) is called Ω_{gb}^{+}-connected if X cannot be written as the disjoint union of two non-empty Ω_{gb}^{+}-open subsets.

Proposition 3.10: If f is totally Ω_{gb}^{+}-continuous map from a Ω_{gb}^{+}-connected space X onto another space Y then Y is an indiscrete space.

Proof: Suppose Y is not an indiscrete space. Let A be a proper non-empty open subset of Y. Since f is totally Ω_{gb}^{+}-continuous. $f^{-1}(A)$ is proper non-empty Ω_{gb}^{+}-clopen subset of X. Then $X = f^{-1}(A) \cup (f^{-1}(A))^{c}$. Hence X is a union of two non-empty disjoint Ω_{gb}^{+}-open subsets. This is a contradiction. Thus Y is an indiscrete space.

Proposition 3.11:A space X is Ω_{gb}^{+}-connected if every totally Ω_{gb}^{+}-continuous function from a space X into any T_0 space Y then f is a constant map.

Proof: Suppose f:(X, τ^{+}) → (Y,σ^{+}) is totally Ω_{gb}^{+}-continuous function. Let Y be a T_0 space. Suppose f is not a constant map, Then we choose two points x and y in X such that f(x)≠ f(y).Since Y is a T_0 space and f(x) and f(y) are distinct points of Y, then there exist an open set G say in Y containing f(x) but not f(y).Now $f^{1}(G)$ is Ω_{gb}^{+}-clopen subset of X since f is totally Ω_{gb}^{+}-continuous. Thus $x\in f^{1}(G)$ and $y\notin f^{1}(G)$. Now $X = f^{1}(G) \cup (f^{1}(G))^{c}$ which is the union of two non-empty Ω_{gb}^{+}-clopen subsets of X. Hence X is not Ω_{gb}^{+}- connected, which is a contradiction to our hypothesis. Thus f is a constant map.

Proposition 3.12: Let f:(X, τ^{+}) → (Y,σ^{+}) be totally Ω_{gb}^{+}-continuous and Y is a T_1 space. If A is a non-empty Ω_{gb}^{+}- connected subset of X then f(A) is a singleton.

Proof: Suppose f(A) is not a singleton. Let $f(x_1) = y_1\in A$, $f(x_2) = y_2\in A$. since $y_1,y_2\in$ Y and Y is a T_1 space , then there exist an open set G in Y (say) containing y_1 but not y_2. Since f is totally Ω_{gb}^{+}-continuous, Then $f^{1}(G)$ is Ω_{gb}^{+}- clopen set containing x_1 but not x_2. Now $X = f^{-1}(G) \cup (f^{-1}(G))^{c}$. Hence X is the union of two non-empty Ω_{gb}^{+}- open subsets, which is a contradiction. Thus f(A) is a singleton.

Proposition 3.13: Let f:(X, τ^{+}) → (Y,σ^{+}) be totally Ω_{gb}^{+}-continuous injection. If Y is T_0 then X is Ω_{gb}^{+}-T_2 .

Proof: Let x and y be any two distinct points of X. Since f is an injection, then f(x) ≠ f(y). Since Y is T_0 , there exist an open subset V of Y containing f(x) but not f(y).Then $x\in f^{-1}(V)$ and $y\notin f^{-1}(V)$. Since f is totally Ω_{gb}^{+}-continuous, then $f^{-1}(V)$ is Ω_{gb}^{+}-clopen subset of X. Assume $A = f^{-1}(V)$ and $B = (f^{-1}(V))^{c}$. Here A and B are two disjoint Ω_{gb}^{+}- open subsets of X containing x and y respectively. Hence X is Ω_{gb}^{+}-T_2 space.

4. TOTALLY Ω_{gb}^{+} -IRRESOLUTE FUNCTION

Definition 4.1:A function $f:(X, \tau^{+}) \rightarrow (Y,\sigma^{+})$ is called totally Ω_{gb}^{+} - irresolute if the inverse image of every Ω_{gb}^{+}- open subset of Y is a Ω_{gb}^{+}-clopen subset of X.

Definition 4.2:A function $f:(X, \tau^{+}) \rightarrow (Y,\sigma^{+})$ is called quasi Ω_{gb}^{+}-irresolute if the inverse image of every Ω_{gb}^{+}-clopen subset of Y in the simple extended topological space is a Ω_{gb}^{+}- clopen subset of X.

Proposition 4.3: Every totally Ω_{gb}^{+}-irresolute function is totally Ω_{gb}^{+}-continuous function.

Proof: Let $f:X \rightarrow Y$ be a totally Ω_{gb}^{+}-irresolute function. Let V be a open subset of Y. Since f is totally Ω_{gb}^{+}-irresolute, $f^{-1}(V)$ is Ω_{gb}^{+}-clopen subset of X in the simple extended topological space. Therefore, f is totally Ω_{gb}^{+}-continuous function

Proposition 4.4: Every totally Ω_{gb}^{+}-irresolute function is quasi Ω_{gb}^{+}-irresolute function.

Proof: Let $f:X \rightarrow Y$ be a totally Ω_{gb}^{+}-irresolute function. Let V be a Ω_{gb}^{+}-clopen subset of Y in the simple extended topological space. Since f is totally Ω_{gb}^{+}-irresolute, $f^{-1}(V)$ is Ω_{gb}^{+}-clopen subset of X in the simple extended topological space. Therefore, f is quasi Ω_{gb}^{+}-irresolute function.

Proposition 4.5: If $f:X \rightarrow Y$ is totally Ω_{gb}^{+} -continuous function and Y is an Ω_{gb}^{+} -space, then f is totally Ω_{gb}^{+} -irresolute.

Proof: Let U be Ω_{gb}^{+}-open in Y. Since Y is an Ω_{gb}^{+}-space, U is open and f is totally Ω_{gb}^{+} -continuous, $f^{-1}(U)$ is Ω_{gb}^{+} -clopen in X in the simple extended topological space. Suppose U is Ω_{gb}^{+} -closed, then Y-U is Ω_{gb}^{+} -open in Y. By similar argument $f^{-1}(Y\text{-}U)$ is Ω_{gb}^{+} -clopen in X in the simple extended topological space.So $f^{-1}(Y)$- $f^{-1}(U)$ is Ω_{gb}^{+} -clopen in X in the simple extended topological space. $X - f^{-1}(U)$is Ω_{gb}^{+} -clopen in X in the simple extended topological space. Then $f^{-1}(U)$ is Ω_{gb}^{+} -clopen in X in the simple extended topological space and hence f is totally Ω_{gb}^{+} -irresolute

Proposition 4.6: If $f:X \rightarrow Y$ is totally Ω_{gb}^{+} -irresolute and $g:Y \rightarrow Z$ is Ω_{gb}^{+} -irresolute, then $g \circ f$ is totally Ω_{gb}^{+} -irresolute.

Proof: Let U be Ω_{gb}^{+}open subset of Z. Since g is Ω_{gb}^{+} -irresolute function, $g^{-1}(U)$ is Ω_{gb}^{+} -open subset of Y. Since f is totally Ω_{gb}^{+}-irresolute function, $f^{-1}(g^{-1}(U))$ is Ω_{gb}^{+} -clopen in X in the simple extended topological space. Hence $g \circ f$ is totally Ω_{gb}^{+}-irresolute.

Proposition 4.7: Let $f:X \rightarrow Y$ and $g:Y \rightarrow Z$ be two functions, such that f is quasi Ω_{gb}^{+} -irresolute and g is totally Ω_{gb}^{+} -irresolute, then $g \circ f$ is totally Ω_{gb}^{+}-irresolute.

Proof: Let U be Ω_{gb}^{+} -open subset of Z. Since g is totally Ω_{gb}^{+}-irresolute function, $g^{-1}(U)$ is Ω_{gb}^{+}-clopen subset of Y in the simple extended topological space. Since f is quasi Ω_{gb}^{+}-irresolute function, $f^{-1}(g^{-1}(U))$ is Ω_{gb}^{+}-clopen in X in the simple extended topological space. Hence $g \circ f$ is totally Ω_{gb}^{+} -irresolute and contra Ω_{gb}^{+} -irresolute.

Proposition 4.8: Let $f:X \rightarrow Y$ and $g:Y \rightarrow Z$ be two functions. Then, if f is totally Ω_{gb}^{+}-irresolute and g is Ω_{gb}^{+} -continuous then $g \circ f$ is totally Ω_{gb}^{+} -continuous.

Proof: Let U be open subset of Z. Since g is Ω_{gb}^{+} -continuous function, $g^{-1}(U)$ is Ω_{gb}^{+}-open subset of Y. Since f is totally Ω_{gb}^{+}-irresolute function. $f^{-1}(g^{-1}(U))$ is Ω_{gb}^{+}-clopen in X in the simple extended topological space. Hence $g \circ f$ is totally Ω_{gb}^{+}-continuous.

Proposition 4.9: The composition of two totally Ω_{gb}^{+} -irresolute functions is totally Ω_{gb}^{+} -irresolute.

Proof: Let U be Ω_{gb}^{+} -open subset of Z, then $g^{-1}(U)$ is Ω_{gb}^{+}-clopen subset of Y in the simple extended topological space. Since f is totally Ω_{gb}^{+} -irresolute function. $f^{-1}(g^{-1}(U))$ is Ω_{gb}^{+} -clopen in X in the simple extended topological space. Hence $g \circ f$ is totally Ω_{gb}^{+} -irresolute.

5. Ω_{gb}^{+} - TOTALLY CONTINUOUS FUNCTION

Definition 5.1: A function $f : (X, \tau^{+}) \rightarrow (Y, \sigma^{+})$ is said to be Ω_{gb}^{+}-totally continuous function if the inverse image of every Ω_{gb}^{+}-open subset in (Y,σ^{+}) is a clopen subset in (X,τ^{+}) .

Proposition 5.2:A function $f : (X ,\tau^{+}) \rightarrow (Y ,\sigma^{+})$ is Ω_{gb}^{+} -totally continuous if and only if the inverse of every Ω_{gb}^{+}-closed subset in (Y ,σ^{+}) is a clopen subset in (X ,τ^{+}).

Proof: Let F be any Ω_{gb}^{+}-closed set in Y. Then Y-F is Ω_{gb}^{+}-open set in Y. By definition $f^{-1}(Y\text{-}F)$ is clopen in X.(i.e) $X\text{-}f^{-1}(F)$ is clopen in X. This implies $f^{-1}(F)$ is clopen in X.

Conversely, If V is Ω_{gb}^{+}-open in Y. Then Y-V is Ω_{gb}^{+}-closed in Y. By assumption, $f^{-1}(Y\text{-}V)=X\text{-}f^{-1}(V)$ is clopen in X. This implies $f^{-1}(V)$ is clopen in X. Therefore f is Ω_{gb}^{+}-totally continuous function.

Proposition 5.3: Every Ω_{gb}^{+}-totally continuous function is totally^{+}-continuous.

Proof: Suppose $f : (X,\tau^{+}) \to (Y,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous and U is any open subset of Y. Since every open set is Ω_{gb}^{+}-open set, U is Ω_{gb}^{+}-open in Y and $f : (X,\tau^{+}) \to (Y,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous. Hence it follows $f^{-1}(U)$ is clopen in X.

Example 5.4: Let X=Y={a,b,c} be two topology spaces with topologies τ = {Φ,X,{a,c}}, B={b}, τ^{+} ={ Φ,X,{b},{a,c}}and σ={Φ,Y}, B={a}, σ^{+}={ Φ,Y,{a}}. Define $f:(X, \tau^{+}) \to (Y,\sigma^{+})$ by f(a)=c, f(b)=a, f(c)=b. Then the function f is totally^{+}-continuous function but f is not Ω_{gb}^{+}-totally continuous function, since for open set A={a,b}. $f^{-1}(A)=f^{-1}(\{a,b\})=\{b,c\}$ is not clopen subset in X.

Proposition 5.5: Every Ω_{gb}^{+}-totally continuous function is Ω_{gb}^{+}-continuous.

Proof: Suppose $f : (X,\tau^{+}) \to (Y,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous and U is any open subset of Y. Since every open set is Ω_{gb}^{+}-open set, U is Ω_{gb}^{+}-open in Y and $f : (X,\tau^{+}) \to (Y,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous. Thus, $f^{-1}(U)$ is clopen in X. Since every open set is Ω_{gb}^{+}-open and every closed set is Ω_{gb}^{+}-closed. This implies that $f^{-1}(U)$ is Ω_{gb}^{+}-open in X. Therefore f is Ω_{gb}^{+}-continuous.

Example 5.6: Let X=Y={a,b,c} be two topology spaces with topologies τ = {Φ,X,{c}}, B={a,b}, τ^{+} ={ Φ,X,{c},{a,b}}and σ={Φ,Y,{b,c}}, B={b}, σ^{+}={ Φ,Y,{b},{b,c}}. Define $f:(X, \tau^{+}) \to (Y,\sigma^{+})$ by f(a)=b, f(b)=a, f(c)=c. Then the function f is Ω_{gb}^{+}-continuous function but f is not Ω_{gb}^{+}-totally continuous function, since for open set A={b}. $f^{-1}(A)=f^{-1}(\{b\})=\{a\}$ is not clopen subset in X.

Proposition 5.7: Let $f : (X,\tau^{+}) \to (Y,\sigma^{+})$ be a function where X and Y are simple extension topological spaces. Then the following condition are equivalent.

(i) f is Ω_{gb}^{+}-totally continuous

(ii) For each x∈X and each Ω_{gb}^{+}-open set V in Y with f(x)∈V, there is clopen set U in X such that x∈U and f(U)⊂ V.

Proof: (i) ⇒ (ii) Suppose f is Ω_{gb}^{+}-totally continuous and V be any Ω_{gb}^{+}-open set in Y containing x. so that $x\in f^{-1}(V)$. Since f is Ω_{gb}^{+}-totally continuous, $f^{-1}(V)$ is clopen in X. Let $U=f^{-1}(V)$ then U is clopen set in X and x∈U. Also $f(U)=f(f^{-1}(V))\subset V$. This implies f(U)⊂V.

(ii) ⇒ (i) Let V be Ω_{gb}^{+}-open in Y. Let $x\in f^{-1}(V)$ be any arbitrary point. This implies f(x)∈V. Therefore by (ii) there is clopen set $f(U_x)\subset Y$ containing x such that $f(U_x)\subset V$ which implies $U_x\subset f^{-1}(V)$ is clopen neighbourhood of x. Since x is arbitrary. It implies $f^{-1}(V)$ is clopen neighbourhood of each of its points. Hence it is clopen set in X. Therefore f is Ω_{gb}^{+}-totally continuous.

Proposition 5.8: For a function $f : (X,\tau^{+}) \to (Y,\sigma^{+})$ the following properties hold

(i) If f is continuous and X is locally indiscrete then f is totally^{+}-continuous.

(ii) If f is totally continuous and Y is Ω_{gb}^{+}-space then f is Ω_{gb}^{+}-totally continuous.

Proof: (i) Let V be open in Y. Since f is continuous and X is locally indiscrete, $f^{-1}(V)$ is open and closed in X. Hence $f^{-1}(V)$ is clopen in X. Therefore f is totally continuous. (ii) Let V be Ω_{gb}^{+}-open in Y. Then Y-V is Ω_{gb}^{+}-closed in Y. Since Y is Ω_{gb}^{+}-space,Y-V is closed in Y, which implies V is open in Y. Since f is totally continuous $f^{-1}(V)$ is clopen in X. Therefore f is Ω_{gb}^{+}-totally continuous.

Proposition 5.9: The composition of two Ω_{gb}^{+}-totally continuous function is Ω_{gb}^{+}- totally continuous.

Proof: Let V be Ω_{gb}^{+}-closed set in Z. Since g is Ω_{gb}^{+}-totally continuous function $g^{-1}(V)$ is clopen in Y. Then $g^{-1}(V)$ is closed in Y which is Ω_{gb}^{+}-closed set in Y. Since f is Ω_{gb}^{+}-totally continuous $f^{-1}(g^{-1}(V))$ is clopen in X. But $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(A)$ and so $(g\circ f)^{-1}(A)$ is clopen in X. Therefore g∘f is Ω_{gb}^{+}-totally continuous.

Proposition 5.10: If $f:(X, \tau^{+}) \to (Y,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous and $g:(Y, \tau^{+}) \to (Z,\sigma^{+})$ is Ω_{gb}^{+}-irresolute then $g\circ f:(X, \tau^{+}) \to (Z,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous.

Proof: Let V be a Ω_{gb}^{+}-open set in Z. Since g is Ω_{gb}^{+}-irresolute, $g^{-1}(V)$ is Ω_{gb}^{+}-open set in Y. Since f is Ω_{gb}^{+}-totally continuous $f^{-1}(g^{-1}(V))$ is clopen in X. Therefore g∘f is Ω_{gb}^{+}-totally continuous.

Proposition 5.11: If $f:(X, \tau^{+}) \to (Y,\sigma^{+})$ is Ω_{gb}^{+}-totally continuous and $g:(Y, \tau^{+}) \to (Z,\sigma^{+})$ is Ω_{gb}^{+}-continuous then $g\circ f:(X, \tau^{+}) \to (Z,\sigma^{+})$ is totally^{+}- continuous.

Proof: Let V be a open set in Z. Since g is Ω_{gb}^{+}-continuous, $g^{-1}(V)$ is Ω_{gb}^{+}-open set in Y. Since f is Ω_{gb}^{+}-totally continuous $f^{-1}(g^{-1}(V))$ is clopen in X. Therefore g∘f is Ω_{gb}^{+}-totally continuous.

Proposition 5.12: If f:(X, τ^+) → (Y,σ^+) is Ω_{gb}^+-closed map and g:(Y, τ^+) → (Z,σ^+) be any function. If g∘f:(X, τ^+) → (Z,σ^+) is Ω_{gb}^+-totally continuous then g is Ω_{gb}^+-irresolute.

Proof: Let g∘f:(X, τ^+) → (Z,σ^+) be Ω_{gb}^+-totally continuous and V be Ω_{gb}^+-open set in Z. Since g∘f is Ω_{gb}^+-totally continuous, $(g\circ f)^{-1}(V)= f^{-1}(g^{-1}(V))$ is clopen in X. Since f is Ω_{gb}^+- closed map, $f(f^{-1}(g^{-1}(V)))$ is clopen in Y. Then $g^{-1}(V)$ is Ω_{gb}^+-open in Y. Hence g is Ω_{gb}^+-irresolute.

Proposition 5.13: If f:(X, τ^+) → (Y,σ^+) is a totally$^+$-continuous and g:(Y, τ^+) → (Z,σ^+) is Ω_{gb}^+-continuous then g∘f:(X, τ^+) → (Z,σ^+) is Ω_{gb}^+-totally continuous.

Proof: Let A be Ω_{gb}^+-open subset in Z. Since g is Ω_{gb}^+-totally continuous function. Thus $g^{-1}(A)$ is clopen subset in Y. This implies that $g^{-1}(A)$ is an open subset in Y. By hypothesis f is a totally$^+$-continuous function, then $f(f^{-1}(g^{-1}(A))$ is closed subset in X. Therefore g∘f is Ω_{gb}^+-totally continuous.

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