Theoretical analysis and experimental validation of efficiency of counter flow heat exchanger and parallel flow heat exchanger

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***Abstract* - A unique kind of approach of heat exchanger is heat exchanger efficiency. It gives us a clear understanding for analyzing heat exchanger by giving us approximately best solution and giving as a data if further improvements are required. This new concept is easy and flexible to select type of the exchanger, the number of exchangers and its efficiency by using a simple general expression. Various parameters are determined by single equation but our aim is to validate the results theoretically as well as experimentally with the help of efficiency approach. Different kind of heat exchangers, including the network problems without any data books, charts or complicated expressions can be analyzed.**

In this study we prove this concept using efficiency approach. We have two problems were selected one each for counter flow and other is parallel flow system. Calculating theoretical and experimental values of heat exchanger and it comparative analysis is a complicate task. Various software helped in making faster and simpler calculations for efficiency values. From heat energy balance outlet fluid temperatures of the system is calculated. Final values and variation in efficiency with irreversibility is studied.

***Index terms:* heat exchanger, efficiency, counter flow /parallel flow, fin analogy number.**

I. INTRODUCTION

Efficiency for heat exchanger is the ratio of the actual rate of heat transfer in the heat exchanger and the optimal rate of heat transfer[1], given by

$$\eta = \frac{q}{q_{opt}} = \frac{q}{UA(\bar{T} - \bar{t})} \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

Where $\bar{T}$ and $\bar{t}$ are the arithmetic mean temperatures of the hot and cold fluids.

Recently the concept of efficiency was introduced for heat exchangers.[1] Efficiency provides a clear and intuitive measure of how well a system is performing by showing how close it comes to the best that it can be and if further improvements are feasible and justified. The new approach “The heat exchanger efficiency” [1] is far more general compared to the traditional approaches. To provide the flexibility to select efficiency of the individual heat exchanger, the overall system efficiency, the number of heat exchangers, as well as type of exchanger, by utilizing a single equation. All the parameters are determined by only single set of equation but our aim is to validate the results theoretically as well as experimentally with the help of efficiency approach. Ahmed Fakheri shown that the optimum heat transfer rate takes place in a balanced counter flow heat exchanger. [2] General expression of heat exchanger efficiency in terms of fin analogy number for counter flow heat exchanger [2] is given by

$$\eta = \frac{\tanh(Fa)}{Fa} \ \ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

This Equation (2) is identical to the efficiency of a constant area fin with insulated tip.

Where the Fin analogy number

$$Fa = \frac{NTU}{2N}(1 + mC_r^n)^{1/n} \ldots\ldots\ldots \ (3)$$

Where the value of m and n are exponents and depends on types of exchangers.

Fa is the non-dimensional group that characterizes the performance of heat exchangers. In this equation, NTU=Number of transfer units is based on the total area of the heat exchangers and N is the number of heat exchangers connected in series. Efficiency depends only on a universal non dimensional parameter i.e. fin analogy number, Fa. Additionally, the relevant driving temperature potential in heat exchanger appears to be arithmetic mean temperature difference, and not LMTD [2] which is simplifies our analysis and it can be shown by Sir Ahmed Fakheri. Traditionally, two different methods have been used for analyzing heat exchangers: the Log mean temperature difference (LMTD); and Effectiveness NTU-method (ε-NTU). The LMTD method is generally used for solving heat exchanger problems where the inlet and the exit temperatures are known and the size of the heat exchanger is to be determined i.e. called sizing problems. The reverse problem is called the rating problem where the size of the heat exchanger and the inlet temperatures are known and the heat transfer rate and the fluid exit temperatures are to be determined. The rating problem is typically analyzed using ε-NTU approach. But the third concept Heat exchanger efficiency can be used to conveniently analyze different heat exchangers design problems, including the network of heat exchangers without the need of charts, or complicated performance expressions.

Thus following are the objectives of this project work.

- Theoretical analysis of efficiency of counter flow heat exchanger and parallel flow heat exchanger.
- To solve rating and sizing problems without the need for charts or complicated performance expressions by a convenient approach “heat exchanger efficiency”
- To validate experimentally the efficiency of counter flow heat exchanger and parallel flow heat exchanger and find irreversibility.
- To show irreversibility on the graphs for both cases ($C_c < C_h$ & $C_c > C_h$) of counter flow heat exchanger and parallel flow heat exchanger.

II. LITERATURE REVIEW

The project as the title suggests is an attempt to carry the theoretical analysis and experimental validation of efficiency of counter flow heat exchanger and parallel flow heat exchanger. The first question which arises is why Efficiency Approach is chosen over traditional approach!! The answer being Efficiency Approach has become increasingly popular in recent years from the viewpoints of elimination of need of chart or complicated performances and expressions. The optimum heat transfer rate takes place in a balanced counter flow heat exchanger and by using this optimum rate of heat transfer, the concept of heat exchanger efficiency is introduced as the ratio of the actual to optimum heat transfer rate [1].Ahmad Fakheri shown that the arithmetic mean temperature difference(AMTD), which is the difference between the average temperatures of hot and cold fluids, can be used instead of the log mean temperature difference (LMTD) in heat exchanger analysis.[2] F.P. Incropera analyses that AMTD and the heat exchanger efficiency allow the direct comparison of the different types of heat exchangers.[3] Boca Raton shown that by generalizing the definition of fin analogy number (Fa), very accurate results can be obtained by using a single algebraic expression can be used to assess the performance of a variety of heat exchangers [4]. Concept of heat exchanger efficiency provides a new way for the design and analysis of heat exchangers and heat exchanger networks.[4] S. Ahmad, B. Linnhoff, R. Smith shown that for parallel flow, counter flow, and shell and tube heat exchanger the efficiency is only a function of a single non dimensional parameter called Fin Analogy Number, Fa. [5].

III. METHODOLOGY:

As there is no direct relation for calculation of Efficiency as a function of fin analogy number, Efficiency Approach helps in analyzing performance of counter flow heat exchanger and similarly parallel flow heat exchanger. Outlet temperature of fluid can be calculated by using heat energy balancing for different inlet temperatures of heat exchanger and capacity ratios. Once effectiveness and NTU values are obtained, fin analogy number can easily be calculated using efficiency approach. Finally efficiency for both counter flow and parallel flow heat exchanger can be calculated and compared theoretically vs. experimentally.

IV. ANALYSIS

The LMTD correction factor, the heat exchanger effectiveness, and heat exchanger efficiency are all derived from the same basic set of equations and therefore can be related to each other.

For heat exchangers,

$$q = UA\eta\,(\bar{T}-\bar{t}) = UAF\,(LMTD) = \varepsilon C_{min}(T_1-t_1) = C_h(T_1-T_2)$$

$$= C_c(t_2-t_1) = C_{min}\Delta T_{min} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

To find a relation between F and ε,

$$\frac{UA\ F\ LMTD}{C_{min}} = \varepsilon\,(T_1-t_1) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

This can be rearranged into

$$NTUF\frac{\frac{(T_1-T_2)(t_2-t_1)}{(t_2-t_1)\,(T_1-t_1)}-\frac{(t_2-t_1)}{(T_1-t_1)}}{\ln\frac{(T_1-t_2)}{(T_2-t_1)}} = \varepsilon \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Suppose that

$$P = \frac{(t_2\text{-}t_1)}{(T_1\text{-}t_1)} = \frac{(t_2\text{-}t_1)}{(T_1\text{-}t_1)}\frac{C_c}{C_{min}}\frac{C_{min}}{C_c} = \frac{q}{q_{max}}\frac{C_{min}}{C_c} = \varepsilon\frac{C_{min}}{C_c} \quad \text{……………..(7)}$$

$$R = \frac{(T_1\text{-}T_2)}{(t_2\text{-}t_1)} = \frac{(T_1\text{-}T_2)}{(t_2\text{-}t_1)}\frac{C_h}{C_c}\frac{C_c}{C_h} = \frac{C_c}{C_h} \quad \text{……………………………..... (8)}$$

Also then $P = \varepsilon C_r$ & $R = 1/\varepsilon$

Then eq. (6) rearranged into

$$F = \frac{1}{NTU}\frac{\ln\frac{(1-\varepsilon C_r)}{1-\varepsilon}}{1-C_r} \text{……………………………………….……. (9)}$$

Regardless of which fluid has the lower heat capacity. Heat exchanger efficiency is related to heat exchanger effectiveness through [4] this equation is given by

$$\eta = \frac{1}{NTU}\frac{1}{\frac{1}{\varepsilon}-\frac{(1+C_r)}{2}} \quad \text{……………………………….………..……… (10)}$$

Equation (10) is a general expression that can be used to determine the efficiency of any heat exchanger, when its effectiveness is known. Eliminating effectiveness between Eqns. (9) and (10) results in the following expression relating F to heat exchanger efficiency:

$$\eta = \frac{\tanh[\frac{FNTU(1-C_r)}{2}]}{\frac{NTU(1-C_r)}{2}} \quad \text{……………………..………………. (11)}$$

For example, In a counter flow heat exchanger, optimum rate flow shown by Ahmed Fakheri and so that value of LMTD correction factor F=1, and therefore Eq.(11) reduces to Eq. (2)

for efficiency of a counter flow heat exchanger which is again given by

$$\eta = \frac{\tanh[\frac{NTU(1-C_r)}{2}]}{\frac{NTU(1-C_r)}{2}} \quad \text{………………………………..………... (12)}$$

(Expression for counter flow heat exchanger efficiency)

Table 4.1
The values of m and n for different heat exchangers are shown:

Heat exchanger	m	n
Counter flow	-1	1
Parallel flow	1	1

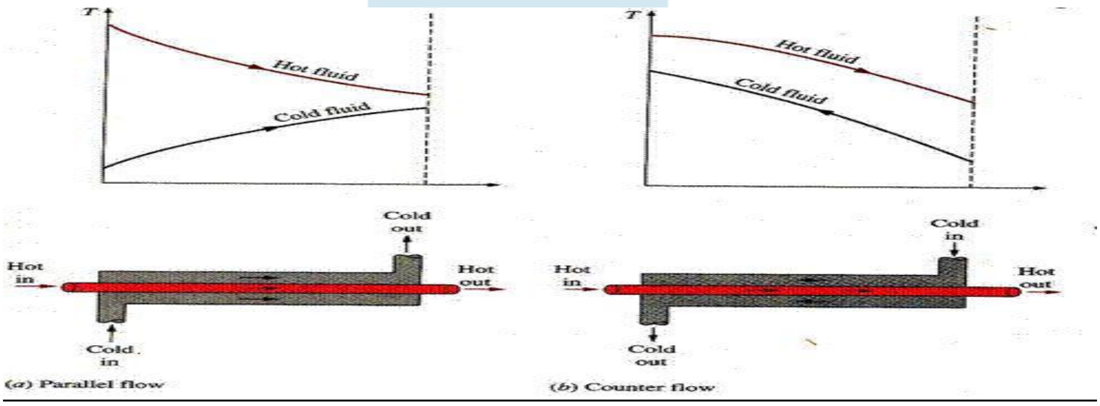

Figure 4.1: Parallel flow & Counter flow heat exchanger

Table 4.2 shows comparative analysis between theoretical values and experimental values of efficiency and effectiveness for counter flow heat exchanger in case where heat capacity of cold fluid is greater than heat capacity of hot fluid.

Table 4.2

Comparative analysis for counter flow HE ($C_c > C_h$)

S. no.	C_r	T_{hi}	t_{ci}	ε_T	ε_P	η_T	η_P
1	0.5	83	42	0.83	0.95	0.89	0.70
2	0.4	85	37	0.68	0.94	0.95	0.71
3	0.3	95	33	0.59	0.90	0.96	0.76
4	0.2	75	34	0.73	0.98	0.90	0.54
5	0.1	89	33	0.54	0.95	0.96	0.63
6	0.5	91	34	0.77	0.91	0.93	0.79
7	0.4	83	32	0.88	0.90	0.81	0.78
8	0.3	81	33	0.76	0.98	0.90	0.54
9	0.2	88	30	0.78	0.91	0.88	0.72
10	0.1	90	31	0.85	0.86	0.80	0.78
11	0.5	83	28	0.65	0.93	0.96	0.76
12	0.4	94	33	0.78	0.92	0.90	0.75
13	0.3	88	33	0.67	0.95	0.94	0.67
14	0.2	74	32	0.60	0.88	0.95	0.77
15	0.1	91	34	0.70	0.88	0.90	0.76
16	0.5	72	33	0.87	0.92	0.85	0.77
17	0.4	93	34	0.89	0.90	0.80	0.79
18	0.3	96	32	0.73	0.91	0.92	0.75

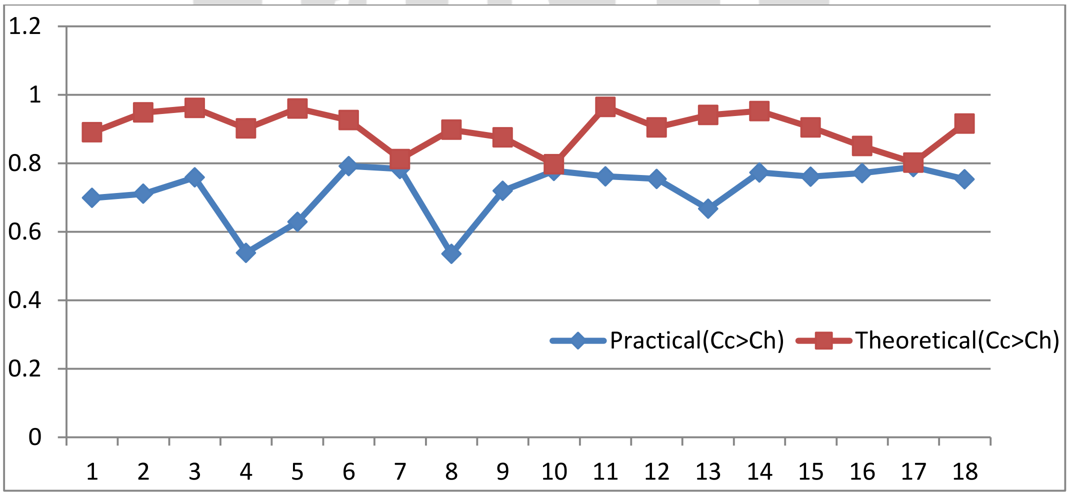

Figure 4.2: Graph (Cc > Ch) for efficiency of Counter flow HE

Hence we gain the graphical difference between theoretical and practical efficiency of counter flow heat exchanger in case where heat capacity of cold fluid is greater than heat capacity of hot fluid. This is represented by Irreversibility which is shown in figure 4.2, where theoretical efficiency is shown by red line where as practical efficiency is shown by blue line, difference between both the lines is Irreversibility.

Table 4.3 shows comparative analysis between theoretical values and experimental values of efficiency and effectiveness for counter flow heat exchanger in case where heat capacity of cold fluid is less than heat capacity of hot fluid.

Table 4.3

Comparative analysis for counter flow HE ($C_c < C_h$)

S.no.	C_r	T_{hi}	t_{ci}	ε_T	ε_P	η_T	η_P
1	0.5	89	32	0.84	0.93	0.88	0.76
2	0.4	73	33	0.83	0.98	0.87	0.58
3	0.3	91	32	0.64	0.95	0.95	0.66
4	0.2	93	32	0.54	0.89	0.96	0.77
5	0.1	91	32	0.78	0.95	0.86	0.62
6	0.5	94	34	0.85	0.93	0.87	0.75
7	0.4	90	32	0.84	0.93	0.86	0.73
8	0.3	93	34	0.69	0.93	0.93	0.70
9	0.2	88	33	0.84	0.96	0.83	0.59
10	0.1	89	32	0.72	0.89	0.90	0.73
11	0.5	71	32	0.85	0.92	0.88	0.77
12	0.4	86	34	0.87	0.94	0.83	0.70
13	0.3	95	32	0.70	0.89	0.93	0.78
14	0.2	88	34	0.76	0.94	0.89	0.65
15	0.1	92	32	0.78	0.87	0.86	0.77
16	0.5	93	32	0.87	0.93	0.85	0.75
17	0.4	82	33	0.84	0.94	0.86	0.71
18	0.3	86	31	0.76	0.91	0.90	0.75
19	0.2	73	32	0.80	0.90	0.85	0.74

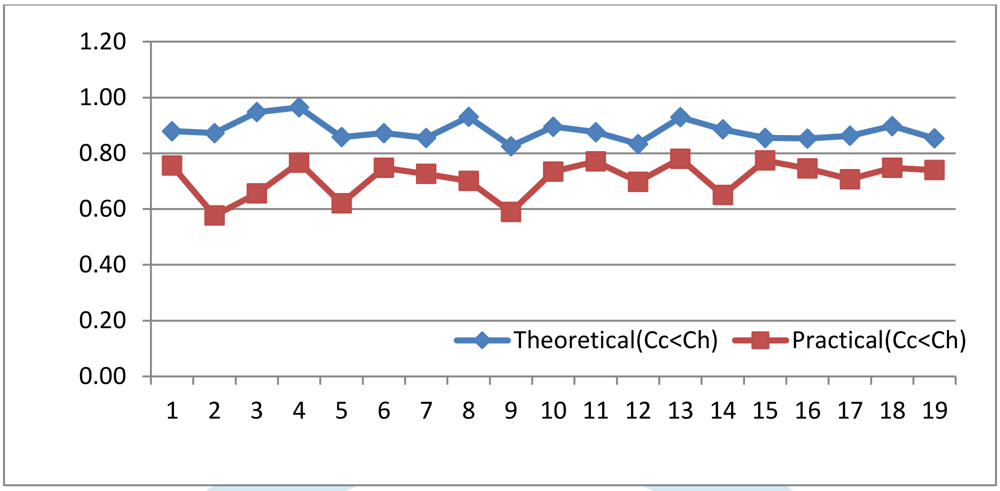

Figure 4.3: Graph (Cc < Ch) for efficiency of Counter flow Heat Exchanger

Hence we gain the graphical difference between theoretical and practical efficiency of counter flow heat exchanger in case where heat capacity of cold fluid is less than heat capacity of hot fluid. This is represented by Irreversibility which is shown in figure 4.3, where theoretical efficiency is shown by red line where as practical efficiency is shown by blue line, difference between both the lines is Irreversibility.

Table 4.4 shows comparative analysis between theoretical values and experimental values of efficiency and effectiveness for parallel flow heat exchanger in case where heat capacity of cold fluid is greater than heat capacity of hot fluid.

Table 4.4

Comparative analysis for parallel flow HE (C_c>C_h)

S.no.	C_r	T_{hi}	t_{ci}	ε_T	ε_P	η_T	η_P
1	0.5	90	34	0.86	0.96	0.87	0.65
2	0.4	90	30	0.88	0.98	0.82	0.53
3	0.3	81	31	0.80	0.96	0.87	0.62
4	0.2	76	30	0.76	0.91	0.88	0.72
5	0.1	93	30	0.79	0.97	0.85	0.56
6	0.5	73	29	0.82	0.95	0.90	0.69
7	0.4	95	30	0.81	0.98	0.89	0.52
8	0.3	94	29	0.87	0.92	0.80	0.72
9	0.2	94	30	0.78	0.97	0.87	0.57
10	0.1	93	32	0.82	0.90	0.83	0.72
11	0.5	93	31	0.84	0.95	0.88	0.70
12	0.4	94	32	0.85	0.95	0.85	0.67
13	0.3	92	31	0.82	0.92	0.86	0.73
14	0.2	94	32	0.81	0.97	0.85	0.57
15	0.1	86	31	0.55	0.89	0.96	0.74
16	0.5	95	31	0.88	0.97	0.85	0.63
17	0.4	94	32	0.69	0.97	0.95	0.61
18	0.3	85	32	0.82	0.92	0.86	0.72
19	0.2	93	34	0.85	0.93	0.81	0.68
20	0.1	92	33	0.68	0.90	0.92	0.73
21	0.5	89	34	0.84	0.96	0.88	0.65
22	0.4	89	34	0.86	0.95	0.84	0.69
23	0.3	92	33	0.85	0.90	0.83	0.77

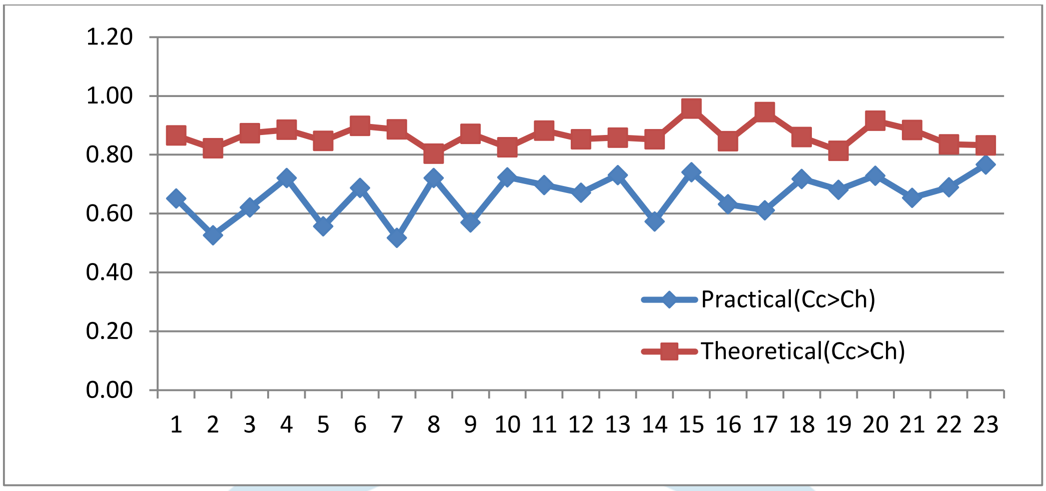

Figure 4.4: Graph ($C_c > C_h$) for efficiency of Parallel flow HE

Here we gain the graphical difference between theoretical and practical efficiency of parallel flow heat exchanger in case where heat capacity of cold fluid is greater than heat capacity of hot fluid. This is represented by Irreversibility which is shown in figure 4.4, where theoretical efficiency is shown by red line where as practical efficiency is shown by blue line, difference between both the lines is Irreversibility.

Table 4.5 shows comparative analysis between theoretical values and experimental values of efficiency and effectiveness for parallel flow heat exchanger in case where heat capacity of cold fluid is less than heat capacity of hot fluid.

Table 4.5

Comparative analysis for parallel flow HE ($C_c<C_h$)

S.no.	C_r	T_{hi}	t_{ci}	ε_T	ε_P	η_T	η_P
1	0.5	91	40	0.88	0.94	0.84	0.73
2	0.4	47	28	0.84	0.95	0.86	0.68
3	0.3	79	29	0.78	0.88	0.89	0.79
4	0.2	73	30	0.84	0.93	0.82	0.69
5	0.1	60	34	0.81	0.88	0.84	0.75
6	0.5	77	33	0.84	0.95	0.88	0.69
7	0.4	80	34	0.89	0.89	0.80	0.80
8	0.3	89	34	0.78	0.93	0.89	0.71
9	0.2	55	33	0.77	0.91	0.88	0.73
10	0.1	61	34	0.70	0.89	0.90	0.74
11	0.5	84	33	0.86	0.94	0.86	0.73
12	0.4	81	35	0.76	0.93	0.91	0.72
13	0.3	49	32	0.65	0.88	0.95	0.79
14	0.2	57	33	0.83	0.88	0.83	0.78
15	0.1	60	34.2	0.69	0.84	0.91	0.80
16	0.5	56	36	0.75	0.95	0.94	0.70
17	0.4	66	36.5	0.80	0.90	0.89	0.79
18	0.3	43	33	0.70	0.90	0.93	0.76
19	0.2	45	33	0.50	0.83	0.97	0.83
20	0.1	47	34	0.69	0.85	0.91	0.80
21	0.5	44	33.8	0.90	0.90	0.81	0.81
22	0.4	52.8	34.4	0.74	0.85	0.93	0.85

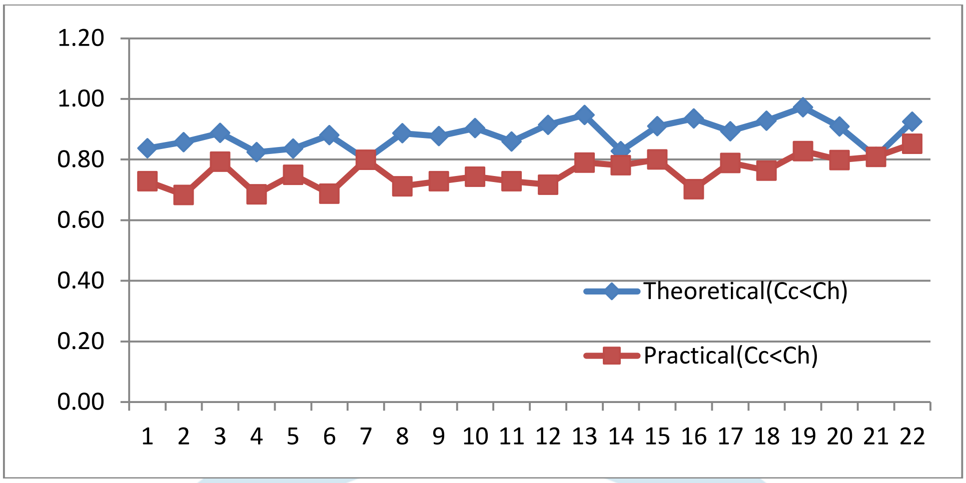

Figure 4.5: Graph (Cc <C_h) for efficiency of Parallel flow HE

Hence we gain the graphical difference between theoretical and practical efficiency of parallel flow heat exchanger in case where heat capacity of cold fluid is less than heat capacity of hot fluid. This is represented by Irreversibility which is shown in figure 4.5, where theoretical efficiency is shown by red line where as practical efficiency is shown by blue line, difference between both the lines is Irreversibility.

V RESULTS & CONCLUSION

The differences between the experimental and theoretical values of efficiencies in the graphs are known as Irreversibility's and from this we can also find out entropy change of universe. (Irreversibility is equal to Temperature of Surroundings Multiplied by Entropy change of Universe.) Because heat input in the form of temperature is same in the both cases. Heat exchanger efficiency is a convenient approach for heat exchanger analysis and can be used to solve rating and sizing problems as well as network of heat exchangers without the need for charts or complicated performance expressions. The concept of heat exchanger efficiency is extended to the heat exchanger networks. The new approach that can be used for determining the overall efficiency and effectiveness of heat exchangers connected in series regardless of the heat exchanger type.

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