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Formulation of solutions of two special classes of congruence of higher degree of even composite modulus

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***ABSTRACT*: In this paper, solutions of two special classes of congruence of higher degree of even composite modulus are obtained. Solutions obtained are verified correct and tested true by citing examples.**

***Keywords*: Binomial expansion, congruence of higher degree, Composite modulus, Euler's function.**

INTRODUCTION

Up to now, I have formulated so many congruence of composite modulus. All the papers are liked by readers and my enthusiasm to formulate more congruence goes on increasing; in this regard, here I am considering a class of congruence of higher degree of composite modulus to formulate. In the literature of mathematics, I found that no attempt had been made by the previous mathematicians to formulate the congruence. To quest my thirst, I tried my best to put my effort in the form of this paper. Here lies the need of my research.

PROBLEM STAMENT

To formulate the congruence $x^{\emptyset(2^m)} \equiv 1 \pmod{2^m}$ & $x^{\frac{1}{2}\emptyset(2^m)} \equiv 1 \pmod{2^m}$where m is a positive integer.

$\emptyset$ is Euler's function.

We show that both the congruence have the same solutions [1].

ANALYSIS & RESULT (Formulation)

Here, $\emptyset(2^m) = 2^{m-1}$ [2].

Then the congruence under consideration becomes $x^{2^{m-1}} \equiv 1 \pmod{2^m}$

If $2^{m-1} = a$, then $2^m = 2a$. The congruence reduces to the form: $x^a \equiv 1 \pmod{2a}$.

Let u be a solution of it. Then u must be an odd positive integer [2].

So, let $u = 2b + 1$, an odd positive integer.

Then, $u^a \equiv 1 \pmod{2a}$

But $(2b+1)^a = (2b)^a + a.(2b)^{a-1} + \cdots \ldots \ldots \ldots . + a.2b + 1$

$= 2a(T) + 1$

$\equiv 1 \pmod{2a}$

Therefore, $(2b+1)^{2^{m-1}} \equiv 1 \pmod{2^m}$ i.e. $u^{\emptyset(2^m)} \equiv 1 \pmod{2^m}$

So, $x \equiv u = 2b + 1 \pmod{2^m}$ is a solution of (1).

But u is any odd positive integer.

Hence, every such odd positive integer less than2^m is a solution.

Now, consider the second congruence as in above. Put $a = 2c$. Then $2a = 4c$ with $c = 2^{m-2}$

$$(2b+1)^c = \cdots \ldots \ldots . \equiv 1 \pmod{4c} \equiv 1 \pmod{2a} \equiv 1 \pmod{2^m}$$

Thus, every odd positive integer is a solution of the congruence as before.

Therefore, we can conclude that both the congruence have the same solutions.

Illustration by example:

Consider the congruence $x^8 \equiv 1 \pmod{16}$ with $16 = 2^4$.

We see that $\emptyset(2^m) = \emptyset(2^4) = \emptyset(16) = 8$.

Then given congruence is of the type: $x^{\emptyset(2^m)} \equiv 1 \pmod{2^m}$.

Every odd positive integer less than $2^m = 16$ are the solutions.

Therefore, the required solutions as per above formula are: $x \equiv 1, 3, 5, 7, 9, 11, 13, 15 \pmod{16}$.

Verification:

We show that these are true.

Consider $x = 7$.

Then $7^8 = 7.7.7.7.7.7.7.7$

$= 49.49.49.49$

$\equiv 1.1.1.1 \pmod{16}$

$\equiv 1 \pmod{16}$

Similarly, others can be verified.

Again consider the congruence $x^4 \equiv 1 \pmod{16}$ with $m = 4$.

It can be shown that it is a congruence of the type: $x^{\frac{1}{2}\emptyset(2^m)} \equiv 1 \pmod{2^m}$.

So, every odd positive integer less than 2^m are the solutions.

We see that the solutions are $x \equiv 1, 3, 5, 7, 9, 11, 13, 15 \pmod{16}$ which are also tested true as can be seen below:

Consider $x = 11$

Then $11^4 = 11.11.11.11$

$\equiv 5.5.5.5 \pmod{16}$

$\equiv 9.9 \pmod{16}$

$\equiv 1 \pmod{16}$.

Thus, $x \equiv 11 \pmod{16}$ is a solution of the above congruence. Similarly, others can be verified.

CONCLUSION

Thus, two special classes of congruence are considered for solutions. These congruence are:

$x^{\emptyset(2^m)} \equiv 1 \pmod{2^m}$ & $x^{\frac{1}{2}\emptyset(2^m)} \equiv 1 \pmod{2^m}$, where m is appositive integer; $\emptyset$ is Euler's function. It is found that every odd positive integer less than 2^m are the solutions.

Merit of the paper

Formulation of solutions (though not a formula) is the merit of this paper. Solutions are the odd positive integers less than the modulus of the congruence. Previously, it was a boring, long & complicated procedure to find all the solutions. It was not an easy task.

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