On IFsgb - Closed Sets in Intuitionistic Fuzzy Topological Spaces

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***Abstract*-In this paper, we introduces the concepts of intuitionistic fuzzy sgb closed and open sets in intuitionistic fuzzy topological spaces.**

***Index Terms*- Intuitionistic fuzzy topology, intuitionistic fuzzy semi generalized b closed sets and intuitionistic fuzzy semi generalized b open sets.**

1. Introduction

Fuzzy set was introduced by **Zadeh**[11] in1965 and fuzzy topology introduced by **C.L.Chang**[2] in 1968. After the introduction fuzzy set and fuzzy topology, there have been several generalizations of this notions. Intuitionistic fuzzy sets was introduced by Atanassov[1] in 1986 and **Coker** [3] introduced intuitionistic fuzzy topological spaces. In this paper, we introduced intuitionistic fuzzy semi generalized b closed sets and intuitionistic fuzzy semi generalized b open sets. The relation between intuitionistic fuzzy sgb closed sets and other intuitionistic fuzzy generalized closed sets are given. We also discusse some of its properties.

2. Preliminaries

Definition 2.1:[1]Let X be an non empty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

Where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Definition 2.2:[1] Let A and B be the IFSs of the forms $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- ✓ $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$ for all $x \in X$,
- ✓ $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- ✓ $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,
- ✓ $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
- ✓ $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$shall be used instead of $A= \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x,(\mu_A, \mu_B), (\nu_A ,\nu_B) \rangle$ instead of $A = \langle x,(A/\mu_A, B/\mu_B), (A/\nu_A ,B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim= \{\langle x, 0,1\rangle / x \in X\}$ and $1_\sim= \{\langle x, 1,0\rangle / x \in X\}$ are the empty set and the whole set of X, respectively.

Definition 2.3:[3]An intuitionistic fuzzy topology(IFT) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

- ✓ $0_\sim, 1_\sim \in \tau$,
- ✓ $G_1 \cap G_2 \in \tau$ for any $G_1,G_2 \in \tau$,
- ✓ $\cup G_i \in \tau$ for any arbitrary family $\{G_i / I \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4:[1] Let A and B be any two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then$A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq B \cap C$,

- ✓ $A \subseteq C$ and $B \subseteq C \Rightarrow A \cup B \subseteq C$,
- ✓ $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$,
- ✓ $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$,
- ✓ $((A)^c)^c = A$,
- ✓ $(1_\sim)^c = 0_\sim$ and $(0_\sim)^c = 1_\sim$.

Definition 2.5: [3]Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

- ✓ $int(A) = \cup \{G / G$ is an IFOS in X and $G \subseteq A\}$,
- ✓ $cl(A) = \cap \{K / K$ is an IFCS in X and $A \subseteq K\}$.

Proposition 2.6:[3]For any IFSs A and B in (X, τ), we have

- ✓ int(A) ⊆ A,
- ✓ A ⊆ cl(A),
- ✓ A is an IFCS in X ⟺ cl(A) = A,
- ✓ A is an IFOS in X ⟺int(A) = A,
- ✓ A ⊆ B ⇒int(A) ⊆int(B) and cl(A) ⊆ cl(B),
- ✓ int(int(A)) = int(A),
- ✓ cl(cl(A)) = cl(A),
- ✓ cl(A⋃B) = cl(A) ⋃ cl(B),
- ✓ int(A⋂B) = int(A) ⋂int(B).

Proposition 2.7: [3]For any IFS A in (X, τ), we have

- ✓ $int(0_\sim) = 0_\sim$ and $cl(0_\sim) = 0_\sim$,
- ✓ $int(1_\sim) = 1_\sim$ and $cl(1_\sim) = 1_\sim$,
- ✓ $(int(A))^c = cl(A^c)$,
- ✓ $(cl(A))^c = int(A^c)$.

Proposition 2.8:[3] If A is an IFCS in (X, τ) then cl(A) = A and if A is an IFOS in (X, τ) then int(A) = A. The arbitrary union of IFCSs is an IFCS in (X, τ).

Definition 2.9: An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x ∈ X} in an IFTS (X, τ) is said to be

- ✓ intuitionistic fuzzy b- closed set [5] (IFbCS) if cl(int(A)) ⋂int(cl(A)) ⊆ A,
- ✓ intuitionistic fuzzy α-closed set [4] (IFαCS) if cl(int(cl(A))) ⊆ A.

Definition 2.10: An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x ∈ X} in an IFTS (X, τ) is said to be

- ✓ intuitionistic fuzzy b open set [5] (IFbOS) if A ⊆int(cl(A)) ⋃ cl(int(A)),
- ✓ intuitionistic fuzzy α-open set [4] (IFαOS) if A ⊆int(cl(int(A))).

Definition 2.11: An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x ∈ X} in an IFTS (X, τ) is said to be

- ✓ intuitionistic fuzzy generalized αclosed set [10] (IFGαCS) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFαOS in (X,τ),
- ✓ intuitionistic fuzzy α generalized semi closed set [7] (IFαGSCS) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X,τ),
- ✓ intuitionistic fuzzy wclosed set [8] (IFwCS) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X,τ),
- ✓ intuitionistic fuzzy πgeneralized beta closed sets[6] (IFπGβCS) if βcl(A)⊆ U whenever A ⊆ U and U is an IFπOS in (X,τ).

Definition 2.12: [9]Let (X, τ) be an IFTS and A = ⟨ x, μ_A, ν_A⟩ be an IFS in (X, τ). Then the intuitionistic fuzzy b closure of A (bcl(A)) and intuitionistic fuzzy b interior of A (bint(A)) are defined as

- ✓ bint(A) = ⋃ { G / G is an IFbOS in X and G ⊆ A},
- ✓ bcl (A) = ⋂ { K / K is an IFbCS in X and A ⊆ K }.

Proposition 2.13: [9]Let (X, τ) be any IFTS. Let A and B be any two intuitionistic fuzzy sets in (X, τ). Then the intuitionistic fuzzy generalized b closure operator satisfies the following properties.

- ✓ $bcl(0_\sim) = 0_\sim$ and $bcl(1_\sim) = 1_\sim$,
- ✓ A ⊆bcl(A),
- ✓ bint(A) ⊆ A,
- ✓ If A is an IFbCS then A = bcl(bcl(A)),
- ✓ A ⊆ B ⇒bcl(A) ⊆bcl(B),
- ✓ A ⊆ B⇒bint(A) ⊆bint(B).

3. Intuitionistic Fuzzy sgb-Closed Sets

Definition 3.1: An IFS A is said to be an intuitionistic fuzzy semi generalized b-closed set (IFSGbCS) if bcl(A) ⊆U whenever A ⊆ U and U is an IFSOS in (X,τ).

The collection of all intuitionistic fuzzy sgb-closed sets of an IFTS (X,τ) is denoted by IFSGbC(X).

Example 3.2: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X, τ) where G = ⟨x, (0.2, 0.3), (0.7, 0.6) ⟩. Consider an IFS A = ⟨x, (0.6, 0.5), (0.3, 0.4) ⟩. Then A is an IFSGbCS in (X, τ).

Example 3.3: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X ,τ) where G = ⟨x, (0.2, 0.4), (0.7, 0.5) ⟩. Consider an IFS A = ⟨x, (0.6, 0.7), (0.3, 0.2) ⟩. Then A is not an IFSGbCS in (X, τ).

Theorem 3.4: Every IFCS is an IFSGbCS.

Proof: Let A⊆ U and U is an IFSOS in (X, τ). Since A be an IFCS in (X, τ). We have, cl(A) = A. Since bcl(A) ⊆ cl(A) and A is an IFCS in (X, τ), bcl(A) ⊆ cl(A) = A ⊆ U. Therefore A is an IFSGbCS in X.

The converse of Theorem 3.4 need not be true as seen from the following Example.

Example 3.5: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X, τ) where G = ⟨x, (0.2, 0.3), (0.7, 0.6) ⟩. Consider an IFS A = ⟨x, (0.6, 0.5), (0.3, 0.4) ⟩. Then A is an IFSGbCS but not an IFCS in (X, τ).

Theorem 3.6: Every IFαCS is an IFSGbCS.

Proof: Let A⊆ U and U is an IFSOS in (X, τ). Since A be an IFαCS in (X, τ). We have, αcl(A) = A. Since bcl(A) ⊆ αcl(A) and A is an IFαCS in (X, τ), bcl(A) ⊆ αcl(A) = A ⊆ U. Therefore A is an IFSGbCS in (X, τ).

The converse of Theorem 3.6 need not be true as seen from the following Example.

Example 3.7: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.2, 0.3), (0.7, 0.6) ⟩. Consider an IFS A = ⟨x, (0.6, 0.5), (0.3, 0.4) ⟩. Then A is an IFSGbCS but not an IFαCS in (X, τ).

Theorem 3.8: Every IFGαCS is an IFSGbCS.

Proof: Let A⊆ U and U is an IFSOS in (X, τ). Since A be an IFGαCS in (X, τ). We have, αcl(A) ⊆ U. Since bcl(A) ⊆ αcl(A) and A is an IFGαCS in (X, τ), bcl(A) ⊆ αcl(A) ⊆ U. Therefore A is an IFSGbCS in (X, τ).

The converse of Theorem 3.8 need not be true as seen from the following Example.

Example 3.9: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.3, 0.4), (0.7, 0.5) ⟩. Consider an IFS A = ⟨x, (0.3, 0.5), (0.6, 0.4) ⟩. Then A is an IFSGbCS but not an IFGαCS in (X, τ).

Theorem 3.10: Every IFαGSCS is an IFSGbCS.

Proof: Let A⊆ U and U is an IFSOS in (X, τ). Since A be an IFαGSCS in (X, τ). We have, αcl(A) ⊆ U. Since bcl(A) ⊆ αcl(A) and A is an IFαGSCS in (X, τ), bcl(A) ⊆ αcl(A) ⊆ U. Therefore A is an IFSGbCS in (X, τ).

The converse of Theorem 3.10 need not be true as seen from the following Example.

Example 3.11: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.3, 0.4), (0.7, 0.5) ⟩. Consider an IFS A = ⟨x, (0.3, 0.5), (0.6, 0.4) ⟩. Then A is an IFSGbCS but not an IFαGSCS in (X, τ).

Theorem 3.12: Every IFWCS is an IFSGbCS.

Proof: Let A⊆ U and U is an IFSOS in (X, τ). Since A be an IFWCS in (X, τ). We have, cl(A) ⊆ U. Since bcl(A) ⊆ cl(A) and A is an IFWCS in (X, τ), bcl(A) ⊆ cl(A) ⊆ U. Therefore A is an IFSGbCS in (X, τ).

The converse of Theorem 3.12 need not be true as seen from the following Example.

Example 3.13: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.7, 0.8), (0.3, 0.2) ⟩. Consider an IFS A = ⟨x, (0.6, 0.8), (0.4, 0.2) ⟩. Then A is an IFSGbCS but not an IFWCS in (X, τ).

Theorem 3.14: Every IFSGCS is an IFSGbCS.

Proof: Let A⊆ U and U is an IFSOS in (X, τ). Since A be an IFSGCS in (X, τ). We have, scl(A) ⊆ U. Since bcl(A) ⊆scl(A) and A is an IFSGCS in (X, τ), bcl(A) ⊆scl(A) ⊆ U. Therefore A is an IFSGCS in (X, τ).

The converse of Theorem 3.14 need not be true as seen from the following Example.

Example 3.15: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.1, 0.2), (0.5, 0.6) ⟩. Consider an IFS A = ⟨x, (0.2, 0.2), (0.6, 0.7) ⟩. Then A is an IFSGbCS but not an IFSGCS in (X, τ).

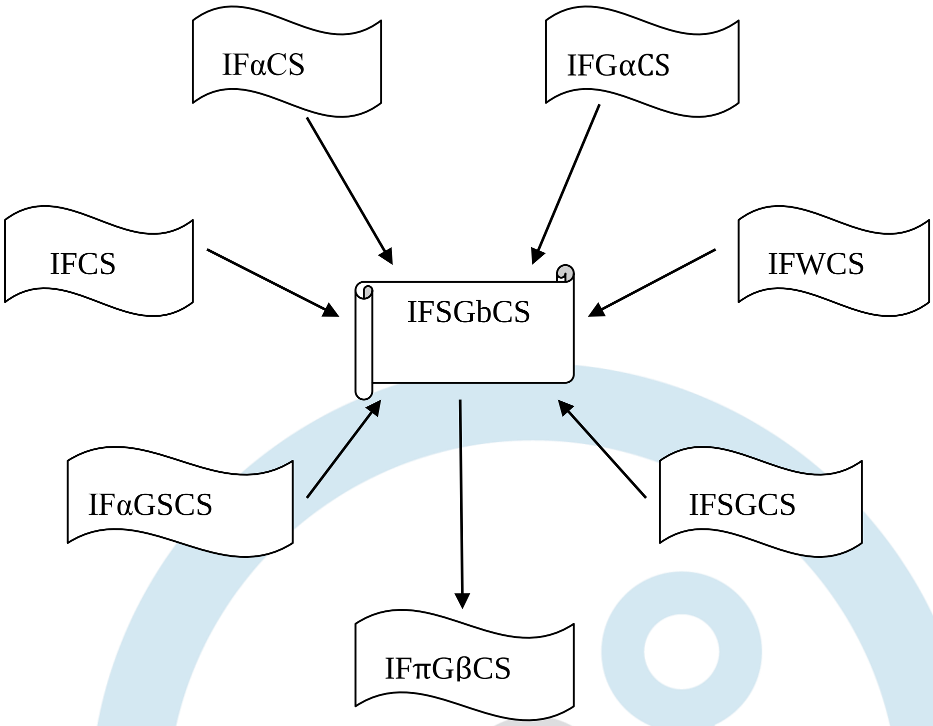
Theorem 3.16: Every IFSGbCS is an IFπGβCS.

Proof: Let A⊆ U and U is an IFπOS in (X, τ). Since A be an IFSGbCS in (X, τ). We have, bcl(A) ⊆ U. Since βcl(A) ⊆bcl(A) and A is an IFSGbCS in (X, τ), βcl(A) ⊆bcl(A) ⊆ U. Therefore A is an IFπGβCS in (X, τ).

The converse of Theorem 3.16 need not be true as seen from the following Example.

Example 3.17: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.2, 0.4), (0.7, 0.5) ⟩. Consider an IFS A = ⟨x, (0.6, 0.7), (0.3, 0.2) ⟩. Then A is an IFπGβCS but not an IFSGbCS in (X, τ).

Remark 3.18: The following implications are true. None of them is reversible.



In this diagram by "A → B" we mean A implies B but not conversely.

Remark 3.19: Union of any two IFSGbC sets need not be an IFSGbC set as seen the following examples.

Example 3.20: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X, τ) where G = ⟨x, (0.6, 0.8), (0.4, 0.2) ⟩. Consider the IFS A = ⟨x, (0.1, 0.8), (0.9, 0.2) ⟩ and the IFS B = ⟨x, (0.6, 0.7), (0.4, 0.3) ⟩. Then A and B are IFSGbCS but A ∪ B is not an IFSGbCS in (X, τ).

Theorem 3.21: If A is an IFSGbCS in (X,τ) such that A ⊆ B ⊆bcl(A) then B is an IFSGbCS in (X,τ).

Proof: Let U be an IFSOS and B is an IFS in an IFTS in (X,τ) such thst B ⊆ U. This implies A ⊆ U. Since A is an IFSGbCS, bcl(A) ⊆ U. By hypothesis, we have bcl(B) ⊆bcl(bcl(A)) = bcl(A) ⊆ U. Therefore B is an IFSGbCS in (X,τ).

Theorem 3.22: If A is IFbOS and IFSGbCS in an IFTS in (X,τ) then A is an IFbCS in (X,τ).

Proof: Let A be an IFbOS and IFSGbCS in (X,τ), bcl(A) ⊆ A. But A ⊆bcl(A). Hence bcl(A) = A. Therefore A is an IFbCS in (X,τ).

4. Intuitionistic Fuzzy Semi Generalized b Open Sets

Definition 4.1: An IFS A is said to be an intuitionistic fuzzy semi generalized b-open set (IFSGbOS) in (X,τ) if the complement A^c is an IFSGbCS in (X,τ).
The collection of all intuitionistic fuzzy sgb-open sets of an IFTS (X,τ) is denoted by IFSGbO(X).

Example 4.2: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X, τ) where G = ⟨x, (0.2, 0.3), (0.7, 0.6) ⟩. Consider an IFS A = ⟨x, (0.3,0.4), (0.6, 0.5) ⟩. Then A is an IFSGbOS in (X, τ).

Example 4.3: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X ,τ) where G = ⟨x, (0.2, 0.4), (0.7, 0.5) ⟩. Consider an IFS A = ⟨x, (0.3, 0.2), (0.6, 0.7) ⟩. Then A is not an IFSGbOS in (X, τ).

Theorem 4.4: Every IFOS, IFαOS, IFGαOS, IFαGSOS, IFwOS, IFSGOS is an IFSGbOS in (X,τ). But the converses are not true as seen from the following examples.

Proof: Straight forward.

Example 4.5: Let X = {a, b} and let τ = {$0_\sim$, G , $1_\sim$} be an IFT on (X, τ) where G = ⟨x, (0.2, 0.3), (0.7, 0.6) ⟩. Consider an IFS A = ⟨x, (0.3, 0.4), (0.6, 0.5) ⟩. Then A is an IFSGbOS but not an IFOS and IFαOS in (X, τ).

Example 4.6: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.3, 0.4), (0.7, 0.5) ⟩. Consider an IFS A = ⟨x, (0.6, 0.4), (0.3, 0.5) ⟩. Then A is an IFSGbOS but not an IFGαOS and IFαGSOS in (X, τ).

Example 4.7: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.7, 0.8), (0.3, 0.2) ⟩. Consider an IFS A = ⟨x, (0.4, 0.2), (0.6, 0.8) ⟩. Then A is an IFSGbOS but not an IFWOS in (X, τ).

Example 4.8: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.1, 0.2), (0.5, 0.6) ⟩. Consider an IFS A = ⟨x, (0.6, 0.7), (0.2, 0.2) ⟩. Then A is an IFSGbOS but not an IFSGOS in (X, τ).

Remark 4.9: Intersection of any two IFSGbO sets need not be an IFSGbO set as seen in the following examples.

Example 4.10: Let X = {a, b} and let τ = {0~ , G , 1~} be an IFT on (X, τ) where G = ⟨x, (0.6, 0.8), (0.4, 0.2) ⟩. Consider the IFS A = ⟨x, (0.9, 0.2), (0.1, 0.8) ⟩ and the IFS B = ⟨x, (0.4, 0.2), (0.6, 0.7) ⟩. Then A and B areIFSGbOS but A ⋂ B is not an IFSGbOS in (X, τ).

Theorem 4.11: An IFS A of an IFTS (X, τ) is an IFSGbOS if and only if F ⊆bint(A) whenever F is an IFCS and F ⊆ A.

Proof: Necessity: Suppose A is an IFSGbOS in (X,τ). Let F be an IFCS and F ⊆ A. Then F^c is an IFOS in (X,τ) such that A^c⊆ F^c. Since A^c is an IFSGbCS, bcl(A^c) ⊆ F^c. Hence $(\text{bint}(A))^c$⊆ F^c. Therefore F ⊆bint(A).

Sufficiency: Let A be any IFS of (X,τ) and let F ⊆bint(A) whenever F is an IFCS and F ⊆ A. Then A^c⊆ F^c and F^c is an IFOS. By hypothesis, $(\text{bint}(A))^c$⊆ F^c. Hence bcl(A^c) ⊆ F^c . Thus A is an IFSGbOS in (X,τ)
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Theorem 4.12: If A is an IFSGbOS in (X, τ) such that bint(A) ⊆ B ⊆ A then B is an IFSGbOS in (X, τ).

Proof: By hypothesis, we have bint(A) ⊆ B ⊆ A. This implies A^c⊆B^c⊆ $(\text{bint}(A))^c$. That is, A^c⊆B^c⊆bcl(A^c). Since A^c is an IFSGbCS, by theorem 3.21, B^c is an IFSGbCS. Therefore B is an IFSGbOS in (X,τ).

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