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Intuitionistic Fuzzy gsr Cokernal Compact Spaces

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***Abstract* – In this paper, we introduced and studied the concepts of intuitionistic fuzzy gsr- cokernal compact spaces, intuitionistic fuzzy gsr C- compact continuous function and intuitionistic fuzzy gsr R-compact spaces.**

***Index Terms*- Intuitionistic fuzzy-gsr-C-compact set, Intuitionistic fuzzy-gsr-C-cocompact set, Intuitionistic fuzzy gsr cokernal compact spaces.**

1. INTRODUCTION

The concept of generalized closed sets in general topology was brought into light by Levine [5].Later its properties such as continuity, connectedness and compactness were studied. All these properties of generalized closed sets were extended to intuitionistic fuzzy set by Atanassov [2]. Generalized closed sets were further developed as gsr closed sets in soft topological spaces by Mohana et.al [6]. The same gsr closed sets in intuitionistic fuzzy topology was discussed by Anitha and Mohana and [1]. In this paper, IFgsr cokernal compact spaces evolved as a result of the idea proposed by Roja et.al [8] who introduced cokernal compact spaces in intuitionistic topology and the notion of Igsr cokernal compact spaces was introduced and investigated by Mohana and Stephy Stephen [10].

2. PRELIMINARIES

Definition 2.1. [4] An intuitionistic topology (IT in short) on a nonempty set X is a family τ of IS's in X containing $\underset{\sim}{\varphi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. The pair (X,τ) is called an intuitionistic topological space (ITS in short). Any intuitionistic set in τ is known as intuitionistic open set (IOS) in X and the complement of IOS is called intuitionistic closed set (ICS) in X.

Definition 2.2. [2] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = $\{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy set in X.

Definition 2.3. [2] $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > / x \in X\}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{< x, \nu_A(x) , \mu_A(x) > / x \in X\}$
- $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > / x \in X\}$
- $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu B(x) > / x \in X\}$

Definition 2.4. [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

- $0\sim, 1\sim \in \tau$
- $G_1 \cap G_2 \in \tau$, for any $G_1 , G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{ G_i / i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS(X,τ) is called an intuitionistic fuzzy closed set(IFCS in short) in X.

Definition 2.5.[1] An IFS A in an IFTS (X,τ) is said to be an Intuitionistic fuzzy gsr-closed sets (IFGSRCS in short) if scl(A) $\subseteq$ U and U is an IFROS in (X, τ).

Definition 2.6.[10] Let (X,τ) be an intutionistic topological space. Then $A = < X, A_1, A_2 > \in \tau$ is said to be intuitionistic –gsr *C*-compact (IgsrC-compact) set if every $A \subseteq \cup_{i\in\tau} A_i^c$ where A_i^c is Igsr-closed set in (X,τ).
The complement of an intuitionistic-gsr-*C*-compact set is an intuitionistic-gsr-*C*-cocompact (IgsrC-cocompact) set.

Definition 2.7.[10] Let (X, τ) be an intuitionistic topological space and $A = < X, A_1, A_2 >$ be an intuitionistic set in (X, τ). Then the Igsr*C*-compact kernel of A and Igsr*C*-compact cokernal of A are denoted and defined by

$IgsrK_c^{\circ}(A) = \cup$ { K= < X, K_1, K_2 > : K is an Igsr$C$-compact set in (X,τ) and K ⊆ A }

$IgsrCK_c^{\neg}(A) = \cap$ { K= < X, K_1, K_2 > : K is an Igsr$C$-cocompact set in (X,τ) and A ⊆ K }

Remark 2.8.[10] Let (X,τ) be an intuitionistic topological space and A = < X, A_1, A_2 > be an intuitionistic set of X. Then

i) $IgsrCK_c^{\neg}(A)$ = A ⇔ A is an Igsr$C$-cocompact set.

ii) $IgsrK_c^{\circ}(A)$ = A ⇔ A is an Igsr$C$-compact set.

Definition 2.9.[10] An intuitionistic topological space (X, τ) is said to be an Igsr cokernal compact space if the Igsr$C$-compact cokernal of every Igsr$C$-compact set. That is., $IgsrCK_c^{\neg}(A) \subseteq \cup A_i^c$.

3. INTUITIONISTIC FUZZY GSR COKERNAL COMPACT SPACES

Definition 3.1: Let (X,) be an intutionistic fuzzy topological space.
Then A = < x, $\mu_A(x)$, $\nu_A(x)$ > ∊ τ is said to be intuitionistic fuzzy –gsr C-compact (IFgsrC-compact) set if every $A \subseteq \cup_{i\epsilon\tau} A_i^c$ where A_i^c is IFgsr-closed set in (X,τ).

The complement of an intuitionistic Fuzzy-gsr-C-compact set is an intuitionistic Fuzzy-gsr-C-cocompact (IFgsrC-cocompact) set.

Definition 3.2: Let (X, τ) be an intuitionistic fuzzy topological space and A = < x, $\mu_A(x)$, $\nu_A(x)$ > be an intuitionistic fuzzy set in (X, τ). Then the IFgsr$C$-compact kernel of A and IFgsr$C$-compact cokernal of A are defined by
IFgsrK_c^o(A) = ∪{ K= < x, $\mu_K(x)$, $\nu_K(x)$ >: K is an IFgsr$C$-compact set in (X,τ) and K ⊆ A }
IFgsr$CK_c^{\neg}$(A)= ∩{ K= < x, $\mu_K(x)$, $\nu_K(x)$ >: K is an IFgsr$C$-cocompact set in (X,τ) and A ⊆ K }
cocompact set in (X,τ) and A ⊆ K }.

Remark 3.3: Let (X,) be an intuitionistic fuzzy topological space and A = < x, $\mu_A(x)$, $\nu_A(x)$ > be an intuitionistic fuzzy set of X. Then

- IFgsr$CK_c^{\neg}$(A)= A if and only if A is an IFgsr$C$-cocompact set.
- IFgsrK_c^o(A) = A if and only if A is an IFgsr$C$-compact set.

Definition 3.4: An intuitionistic fuzzy topological space (X, τ) is said to be an IFgsr cokernal compact space if the IFgsr$C$-compact cokernal of every IFgsr$C$-compact set. That is., IFgsr$CK_c^{\neg}(A) \subseteq \cup A_i^c$.

Example 3.5: Let X = {a,b}, then the intuitionistic fuzzy set A = < X, (0.5,0.4),(0.2,0.3)>, B = < X, (0.4,0.5),(0.3,0.4)>, C = < X, (0.6,0.5),(0.1,0.4) >, D = < X, (0.3,0.3),(0.6,0.5)>, E = <X, (0.2,0.4),(0.4,0.5)>, F = < X, (0.1,0.3),(0.6,0.5)>
Then the family τ = {φ, X, A, B, C, D, E, F } is an intuitionistic fuzzy topology on X.

IFgsr$C$-compact set = { < X, (0.5,0.4),(0.2,0.3)>, < X, (0.4,0.5),(0.3,0.4)>, < X,(0.6,0.5),(0.1,0.4) >, < X, (0.3,0.3),(0.6,0.5)>, <X, (0.2,0.4),(0.4,0.5)>, < X, (0.1,0.3),(0.6,0.5)> }.

IFgsr$C$-cocompact set ={< X, (0.2,0.3),(0.5,0.4)>, < X, (0.3,0.4),(0.4,0.5)>, < X, (0.1,0.4),(0.6,0.5) >, < X, (0.6,0.5),(0.3,0.3)>, <X, (0.4,0.5),(0.2,0.4)>, < X, (0.6,0.5),(0.1,0.3)>}.
Then IFgsrK_c°(A) = ∩ K. Therefore, (X, τ) is IFgsr cokernal compact space.

Proposition 3.6: Let (X,T) be any intuitionistic fuzzy topological space. Let A = < x, $\mu_A(x)$, $\nu_A(x)$ > be an IFgsr$C$-compact set in X. Then the following conditions hold:

i) $\overline{\text{IFgsrCK}_c^{\neg}(\text{A})}$ = IFgsr$K_c^o(\bar{A})$.

ii) $\overline{IFgsrK_c^{o}(\text{A})}$ = IFgsr$CK_c^{\neg}(\bar{A})$.

Proof: i) IFgsr$CK_c^{\neg}$ (A) = ∩ {K = < x, $\mu_K(x)$, $\nu_K(x)$ > : K is an IFgsr$C$-cocompact set in (X,τ) and A ⊆ K . Taking complements on both sides,
$\overline{\text{IFgsrCK}_c^{\neg}(\text{A})}$ = ∪ { $\bar{K}$: $\bar{K}$ is an IFgsr$C$-compact set in (X,τ) and $\bar{K} \subseteq \bar{A}$}
= IFgsr$K_c^o(\bar{A})$.
ii) IFgsrK_c^o(A) = ∪ { K = < x, $\mu_K(x)$, $\nu_K(x)$ > : K is an IFgsr$C$-compact set in (X,τ) and K ⊆ A }
Taking complements on both sides,
$\overline{IFgsrK_c^{o}(\text{A})}$ ∩ { $\bar{K}$: $\bar{K}$ is an IFgsr$C$-compact set in (X,τ) and $\bar{K} \supseteq \bar{A}$}

= IFgsrC$K_c^{\neg}$ ($\bar{A}$).

Proposition 3.7: Let (X, τ) be an intuitionistic fuzzy topological space. Then the following statements are equivalent:

i) (X, τ) is an IFgsr cokernal compact space.
ii) For each IFgsr*C*-cocompact set A, IFgsrK_c^o(A) is an IFgsr*C*-cocompact set.
iii) For each IFgsr*C*-compact set A, we have IFgsrC$K_c^{\neg}$ ($\overline{\text{IFgsrC}K_c^{\neg}(A)}$) = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$
iv) For every pair of IFgsr*C*-compact sets A and B with $\bar{B}$ = IFgsrC$K_c^{\neg}$ (A), we have IFgsrC$K_c^{\neg}$ (B) = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$.

Proof: (i) ⇒ (ii)

Let A be an IFgsrC-cocompact set in (X, τ). Then $\bar{A}$ is an IFgsrC-cocompact set in (X, τ). By assumption, we have IFgsrC$K_c^{\neg}$ ($\bar{A}$) is an IFgsrC-compact set in (X, τ). Now,. IFgsrC$K_c^{\neg}$ ($\bar{A}$) = $\overline{IFgsrK_c{}^o(A)}$. Therefore, IFgsrK_c^o(A) is an IFgsrC-cocompact set in (X, τ). Hence (i) ⇒ (ii).

(ii) ⇒ (iii)

Let A be an IFgsrC-compact set in (X, τ). Then $\bar{A}$ is an IFgsrC-cocompact set in (X, τ).
Given that IFgsrK_c^o($\bar{A}$) = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$ is an IFgsrC-cocompact set.
Now, = IFgsrC$K_c^{\neg}$ ($\overline{\text{IFgsrC}K_c^{\neg}(A)}$)
= IFgsrC$K_c^{\neg}$ (IFgsrK_c^o($\bar{A}$))= $\overline{\text{IFgsrC}K_c^{\neg}(A)}$. Hence (ii) ⇒ (iii).

(iii) ⇒ (iv)

Let A and B be any two IFgsrC-compact set in (X, τ) such that by (iii), IFgsrC$K_c^{\neg}$ ($\overline{\text{IFgsrC}K_c^{\neg}(A)}$) = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$ which implies that IFgsrC$K_c^{\neg}$ (B) = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$. Hence (iii) ⇒ (iv).

(iv) ⇒ (i)

Let A and B be any two IFgsrC-compact sets in (X, τ) such that B = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$
Given that, IFgsrC$K_c^{\neg}$ (B) = $\overline{\text{IFgsrC}K_c^{\neg}(A)}$.
i.e., $\overline{\text{IFgsrC}K_c^{\neg}(A)}$ is an IFgsrC-cocompact set in (X, τ).
This implies that IFgsrC$K_c^{\neg}$ (A) is an IFgsrC-compact set in (X, τ). Thus, (X, τ) is an IFgsrC-compact cokernal compact space. Hence (iv) ⇒ (i).

Proposition 3.8: Let (X, τ) be an intuitionistic fuzzy topological space. Then (X, τ) is an IFgsr cokernal compact space iff for each IFgsrC-compact set A and IFgsrC-cocompact set B such that A⊆ B, IFgsrC$K_c^{\neg}$ (A) ⊆ IFgsrK_c^o(B).

Proof: Let (X, τ) be an IFgsr cokernal compact space. Let A be an IFgsrC-compact set and B is an IFgsrC-cocompact set in (X, τ) such that $A \subseteq B$.
Then by (ii) of proposition 3.7, IFgsrK_c^o(B) is an IFgsrC-cocompact set in (X, τ).
Therefore , IFgsrC$K_c^{\neg}$ (IFgsrK_c^o(B)) = IFgsrK_c^o(B). Since A is an IFgsrC-compact set and A⊆ B,
A ⊆ IFgsrK_c^o(B). Now,

IFgsrC$K_c^{\neg}$ (A) ⊆ IFgsrC$K_c^{\neg}$(IFgsrK_c^o(B)) = IFgsrK_c^o(B)
⇒ IFgsrC$K_c^{\neg}$ (A) ⊆ IFgsrK_c^o(B)

Conversely, let B be an IFgsrC-cocompact set in (X, τ), then IFgsrK_c^o(B) is an IFgsrC-compact set and IFgsrK_c^o(B) ⊆ B . By assumption, IFgsrC$K_c^{\neg}$(IFgsrK_c^o(B)) = IFgsrK_c^o(B). Also, IFgsrK_c^o(B) ⊆ IFgsrC$K_c^{\neg}$(IFgsrK_c^o(B)), implies IFgsrK_c^o(B) ⊆ IFgsrC$K_c^{\neg}$(IFgsrK_c^o(B)) . Therefore, IFgsrK_c^o(B) is an IFgsr-closed set in (X, τ). By (ii) of proposition , (X, τ) is an IFgsr cokernal compact set.

Definition 3.9: Let (X, τ) and (Y, δ) be any two IFgsr cokernal compact spaces. A function f : (X, τ) → (Y, δ) is called an IFgsr C-compact open function if f(A) is an IFgsr C-compact set in (Y, δ) for each IFgsr C-compact set A in (X, τ) .

Proposition 3.10: Let (X, τ) and (Y, δ) be any two IFgsr cokernal compact spaces. A function f : (X, τ) → (Y, δ) be an IFgsr C-compact open and surjective function. Then for each f^{-1}(IFgsrC$K_c^{\neg}$ (A)) ⊆ IFgsrC$K_c^{\neg}$ (f^{-1}(A) intuitionistic set A in (Y, δ).

Proof: Let A be an intuitionistic fuzzy set in (Y, δ) and B = f^{-1}($\bar{A}$). Then,
IFgsrK_c^o(f^{-1}($\bar{A}$)) = IFgsrK_c^o(B) is an IFgsr C-compact set in (X, τ). Now, IFgsrK_c^o(B) ⊆ B. Hence f (IFgsrK_c^o(B)) ⊆ f (B). i.e., IFgsrK_c^o(f (IFgsrK_c^o(B))) ⊆ IFgsrK_c^o(f (B)).
Since f is an IFgsr C-compact open function, (f (IFgsrK_c^o(B)) is an IFgsr C-compact set in (Y, δ).
Therefore, (f (IFgsrK_c^o(B)) ⊆ IFgsrK_c^o(f (B)) = IFgsrK_c^o($\bar{A}$).
Hence, IFgsrK_c^o(f^{-1}($\bar{A}$)) ⊆ f^{-1}(IFgsrK_c^o($\bar{A}$)).
This implies that $\overline{\text{IFgsr}K_c^o(f^{-1}(\bar{A}))}$ ⊇ $\overline{f^{-1}(\text{IFgsr}K_c^o(\bar{A}))}$
implies IFgsrC$K_c^{\neg}$ (f^{-1}(A)) ⊇ f^{-1}(IFgsrC$K_c^{\neg}$ ($\bar{A}$)) ⟹ f^{-1}(IFgsrC$K_c^{\neg}$ ($\bar{A}$)) ⊆ IFgsrC$K_c^{\neg}$ (f^{-1}(A)) . Hence the proof.

Definition 3.11: Let (X, τ) and (Y, δ) be any two IFgsr cokernal compact spaces. A function f: (X, τ) → (Y, δ) is called an IFgsr C-compact continuous function if $f^{-1}(A)$ is IFgsr C-compact set in (X, τ) for every IFgsr C-compact set A in (Y, δ).

Remark 3.12: Let (X, τ) and (Y, δ) be any two IFgsr cokernal compact spaces. Let f: (X, τ) → (Y, δ) be any function. Then the following statements are equivalent:

(i) f: (X, τ) → (Y, δ) is an IFgsr C-compact continuous function.

(ii) IFgsrCK$_c$¬ (f^{-1}(A) ⊆ f^{-1}(IFgsrCK$_c$¬ (A)) for each IFgsr C-compact set A in (Y, δ).

Proof: (i)⇒(ii)

(IFgsrCK$_c$¬ (f^{-1}(A)) ⊆ (IFgsrCK$_c$¬ (f^{-1}(IFgsrCK$_c$¬ (A)))

Given f: (X, τ) → (Y, δ) is an IFgsr C-compact continuous function. Let A = < x, μ_A(x), ν_A(x) > be an IFgsrC-compact set in (Y, δ). Let (IFgsrCK$_c$¬ (A) is an Igsr C-compact set in (Y, δ) and hence f^{-1}(IFgsrCK$_c$¬ (A)) is an IFgsr C-compact set in (X, τ).

Therefore, (IFgsrCK$_c$¬ (f^{-1}(IFgsrCK$_c$¬ (A))) = f^{-1}(IFgsrCK$_c$¬ (A))

Since, A ⊆ (IFgsrCK$_c$¬ (A) , f^{-1}(A) = f^{-1}(IFgsrCK$_c$¬ (A)).

Therefore, (IFgsrCK$_c$¬ (f^{-1}(A)) ⊆ (IFgsrCK$_c$¬ (f^{-1}(IFgsrCK$_c$¬ (A)))

= f^{-1}(IFgsrCK$_c$¬ (A))

i.e., (IFgsrCK$_c$¬ (f^{-1}(A)) ⊆ f^{-1}(IFgsrCK$_c$¬ (A)).

(ii)⇒(i)

Given that ($IFgsrCK_c$¬ ($f^{-1}(A)$) ⊆ f^{-1}(IFgsrCK$_c$¬ (A)), for each IFgsr C-compact set in (Y, δ). Let A be an IFgsr C-cocompact set in (Y, δ). It is enough to show that f^{-1} (A)is an IFgsr C-compact set in (X, τ). Since A = IFgsrCK$_c$¬ (A), f^{-1}(A) = f^{-1}(IFgsrCK$_c$¬ (A)) but it is given that (IFgsrCK$_c$¬ (f^{-1}(A)) ⊆ f^{-1}(IFgsrCK$_c$¬ (A)). Hence IFgsrCK$_c$¬ (f^{-1}(A)) ⊆ f^{-1}((A) ⊆ f^{-1}(IFgsrCK$_c$¬ (A)). Thus f^{-1} (A) = f^{-1}(IFgsrCK$_c$¬ (A)), i.e., $f^{-1}(A)$ is an IFgsr C-cocompact set in (X, τ). This proves that f is an IFgsr C-compact continuous function.

Proposition 3.13: Let (X, τ) and (Y, δ) be any two IFgsr cokernal compact spaces. Let f: (X, τ) → (Y, δ) be a bijective function. Then f is an IFgsr C-compact continuous function if for every intuitionistic fuzzy set A in (X, τ), f (IFgsrCK$_c$¬ (A)) ⊆ IFgsrCK$_c$¬ (f(A)) .

Proof: Let us assume that f is an IFgsr C-compact continuous function and A be an intuitionistic fuzzy set in (X, τ). Hence, f^{-1}(IFgsrCK$_c$¬ (A)) is an IFgsr C-cocompact set in (X, τ).

By remark 3.12, IFgsrCK$_c$¬ (f^{-1} (f(A)) ⊆ f^{-1}(IFgsrCK$_c$¬ (f(A))).

Since f is an injective function, IFgsrCK$_c$¬ (A) ⊆ f^{-1}(IFgsrCK$_c$¬ (f(A))).

Taking f on both sides, f (IFgsrCK$_c$¬ (A)) ⊆ f (f^{-1}(IFgsrCK$_c$¬ (f (A)))). Since f is a surjective function, f (IFgsrCK$_c$¬ (A)) ⊆ IFgsrCK$_c$¬ (f(A)).

Proposition 3.14: Let (X, τ) and (Y, δ) be any two IFgsr cokernal compact spaces. Let f: (X, τ) → (Y, δ) be any function. Then the following statements are equivalent:

(i) f :(X, τ) → (Y, δ) is an IFgsr C-cocompact continuous function.

(ii) IFgsrCK$_c$¬ (f(A)) ⊆ f (IFgsrCK$_c$¬ (A)), for each IFgsr C-compact set A = < x, μ_A(x), ν_A(x) > in (X, τ).

Proof: (i)⇒(ii)

Let A = < x, μ_A(x), ν_A(x) > be an IFgsr C-compact set in (X, τ). Clearly IFgsrCK$_c$¬ (A), is an IFgsr C-cocompact set in (X, τ). Since f is an IFgsr C-cocompact function,

f (IFgsrCK$_c$¬ (A)) is an IFgsr C-cocompact set in (Y, δ).

Thus IFgsrCK$_c$¬ (f(A)) ⊆ IFgsrCK$_c$¬ (f(IFgsrCK$_c$¬ (A)))

= f (IFgsrCK$_c$¬ (A)). Hence (i)⇒(ii).

(ii)⇒(i)

Let A be any IFgsr C-cocompact set in (X, τ). Then A = IFgsrCK$_c$¬ (A).

By (ii), IFgsrCK$_c$¬ (f(A)) ⊆ f (IFgsrCK$_c$¬ (A)).

Thus f (A) = IFgsrCK$_c$¬ (f(A)) and hence f(A) is an IFgsr C-cocompact set in (Y, δ). Therefore, f is an IFgsr C-cocompact function. Hence (ii)⇒(i).

Definition 3.15: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. A function f: (X, τ) → (Y, δ) is called an IFgsr C-compact irresolute function if f^{-1}(A) is IFgsr C-compact set in (X, τ) for each IFgsr C-compact set A in (Y, δ).

Proposition 3.16: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. A function f: (X, τ) → (Y, δ) is an IFgsr C-compact irresolute function if and only if

f (IFgsrCK$_c$¬ (A)) ⊆ IFgsrCK$_c$¬ (f(A)), for every IFgsr C-compact set in (X, τ).

Proof: Suppose that f is an IFgsr C-compact irresolute function and let A be an IFgsr C-compact set in (X, τ). Then, IFgsrCK$_c$¬ (f(A)) is an IFgsr C-cocompact set in (Y, δ). By assumption, f^{-1}(IFgsrCK$_c$¬ (f(A))) is an IFgsr C-compact set (X, τ). Now, A ⊆ f^{-1}(f (A)) ⊆ f^{-1}(IFgsrCK$_c$¬ (f(A))). Now, A ⊆ f^{-1}(IFgsrCK$_c$¬ (A))
IFgsrCK$_c$¬ (A) ⊆ IFgsrCK$_c$¬(f^{-1}(IFgsrCK$_c$¬ (f(A)))).
IFgsrCK$_c$¬ (A) ⊆ f^{-1}(IFgsrCK$_c$¬ (f(A))). i.e., f(IFgsrCK$_c$¬ (A)) ⊆ IFgsrCK$_c$¬ (f(A)) .
Conversely , suppose that A is an IFgsr C-cocompact set in (Y, δ).Then IFgsrCK$_c$¬ (A) = A. Now, by assumption, f (IFgsrCK$_c$¬ (f^{-1}(A))) ⊆ IFgsrCK$_c$¬ (f(f^{-1}(A)))
= IFgsrCK$_c$¬ (A) = A.
This implies that, IFgsrCK$_c$¬ (f^{-1}(A)) ⊆ f^{-1}(A) but, IFgsrCK$_c$¬ (f^{-1}(A)) ⊇ f^{-1}(A).
Hence IFgsrCK$_c$¬ (f^{-1}(A)) = f^{-1}(A). i.e., $f^{-1}(A)$ is an IFgsr C-cocompact set in (X, τ).
Hence f is an IFgsr C-compact irresolute function.

4. PROPERTIES OF IFGSR R-COMPACT SPACES

Definition 4.1: Let (X, τ) be an IFgsr cokernal compact space and let A = < x, μ_A(x), ν_A(x) > be any intuitionistic fuzzy set in (X,). Then A is said to be an IFgsr RC-compact if A = IFgsrK$_c^o$(IFgsrCK$_c$¬ (A)).

Definition 4.2: Let (X, τ) be an IFgsr cokernal compact space and let A = < x, μ_A(x), ν_A(x) > be any intuitionistic fuzzy set in (X, τ). Then A is said to be an IFgsr RC-cocompact if A = IFgsrCK$_c$¬(IFgsrK$_c^o$(A)).

Remark 4.3: Every IFgsr RC-compact is an IFgsr C-compact.

Proposition 4.4: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. A function f: (X, τ) → (Y, δ) is an IFgsr C-compact continuous function of (X, τ) into an IFgsr cokernal compact space (Y, δ) and if V = < x, μ_V(x), ν_V(x) >is an IFgsr RC-compact in (Y, δ), then $f^{-1}(V)$ is an IFgsr RC-compact in (X, τ).

Proof: Since V is an IFgsr RC-compact in (Y, δ), it follows that V is an IFgsr C-compact in (Y,).
Since f is Igsr continuous, f^{-1}(V) is an IFgsr C-compact in (X, τ).
That is, IFgsrK$_c^o$(f^{-1}(V)) = f^{-1}(V) ----------------------------1
Since (Y, δ) is an IFgsr cokernal compact space and since V is an IFgsr RC-compact in (Y,),
V = IFgsrK$_c^o$(IFgsrCK$_c$¬(V))
= IFgsrCK$_c$¬(IFgsrK$_c^o$(IFgsrCK$_c$¬(V)))
That is, V = IFgsrCK$_c$¬(V)------------------------------------2
IFgsrCK$_c$¬(f^{-1}(V)) ⊆ f^{-1}(IFgsrCK$_c$¬(V)) because f is an IFgsr C-compact continuous function. Therefore, IFgsrK$_c^o$(IFgsrCK$_c$¬(f^{-1}(V))) ⊆ IFgsrK$_c^o$(f^{-1}(IFgsrCK$_c$¬(V))).
From 2, it follows that
IFgsrK$_c^o$(f^{-1}(IFgsrCK$_c$¬(V)))⊆IFgsrK$_c^o$(f^{-1}(V))--------------3
From 1 and 3, it follows that
IFgsrK$_c^o$(IFgsrCK$_c$¬(f^{-1}(V))) ⊆ f^{-1}(V) ------------------------4
Sincef^{-1}(V) ⊆ IFgsrK$_c$¬(f^{-1}(V)) , then IFgsrK$_c^o$(f^{-1}(V)) ⊆ IFgsrK$_c^o$(IFgsrCK$_c$¬(f^{-1}(V)))
From 1, it follows that
f^{-1}(V) ⊆ IFgsrK$_c^o$(IFgsrCK$_c$¬(f^{-1}(V))) -----------------------5
Therefore, from 4 and 5, it follows that IFgsrK$_c^o$(f^{-1}(V)) = IFgsrK$_c^o$(IFgsrCK$_c$¬(f^{-1}(V)))
Hence f^{-1}(V) is an IFgsr RC-compact in (X, τ).

Definition 4.5: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. If f: (X, τ) → (Y, δ) be a function, then f is said to be IFgsr C-compact function if the image of each IFgsr C-compact set in (X,τ) is an IFgsr C-compact set in (Y, δ).

Definition 4.6: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. If f: (X, τ) → (Y, δ) be a function, then f is said to be IFgsr C-cocompact function if the image of each IFgsr C-cocompact set in (X, τ) is an IFgsr C-cocompact set in (Y, δ).

Proposition 4.7: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. If f: (X, τ) → (Y, δ) is an IFgsr continuous bijective function of an IFgsr cokernal compact space (X, τ) into a space (Y, δ). If V = < x, μ_V(x), ν_V(x) > is an IFgsr RC-compact set in (X, τ) , then f(V) is an IFgsr RC-compact set in (Y, δ).

Proof: Since V is an IFgsr RC-compact set in (X, τ) and since (X, τ) is an IFgsr cokernal compact space, V = IFgsrK$_c^o$(IFgsrK$_c$¬(V)) = IFgsrK$_c$¬(V). That is, V = IFgsrK$_c$¬(V). Since f is an IFgsr C-compact continuous bijective function, f(V) = f(IFgsrK$_c$¬(V)) ⊆ IFgsrK$_c$¬(f(V)).
Since f is an intuitionistic fuzzy -gsr continuous function,
f(V) = IFgsrK$_c^o$(f(V)) ⊆ IFgsrK$_c^o$(IFgsrK$_c$¬(f(V)))
i.e., f(V) ⊆ IFgsrK$_c^o$(IFgsrK$_c$¬(f(V))) ------------6
Now IFgsrK$_c^o$(IFgsrK$_c$¬(f(V))) ⊆ IFgsrK$_c$¬(f(V)))

Since f is an IFgsr C-compact bijective function, f is an IFgsr C-cocompact function. Hence, $\text{IFgsrK}_c\neg(f(\text{V})) \subseteq f(\text{IFgsrK}_c\neg(\text{V}))$ $= f(\text{V})$then,

$\text{IFgsrK}_c^o(\text{IFgsrK}_c\neg(f(\text{V}))) \subseteq f(\text{V})$ -------------7

From 6 and 7, it follows that $\text{IFgsrK}_c^o(\text{IFgsrK}_c\neg(f(\text{V}))) = f(\text{V})$

i.e., $f(\text{V})$ is IFgsr RC-compact set in (Y, δ).

Definition 4.8: Let (X, τ) be an intuitionistic fuzzy topological space. If a family $\{G_i = \langle x, \mu_{G_i}(x), \nu_{G_i}(x) \rangle; i \in J\}$ of IFgsr RC-compact (X, τ) satisfies the condition $\cup\{G_i ; i \in J\} = X$, then it is called IFgsr RC-compact cover of X.

Definition 4.9: An intuitionistic fuzzy topological space (X, τ) is said to be IFgsr RC-compact space if and only if every IFgsr RC-compact cover of (X, τ) has a finite subfamily, the IFgsr C-compact cokernals of whose members cover the space (X, τ).

Proposition 4.10: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. Let f: (X, τ) → (Y, δ) be an IFgsr C-compact function of an IFgsr RC-compact space (X, τ) onto an IFgsr cokernal compact space (Y, δ), then (Y, δ) is an IFgsr R-compact space.

Proof: Let $V_j = \langle x, \mu_{v_j}(x), \nu_{v_j}(x) \rangle$ be an IFgsrRC-compact cover of (Y, δ). Since f is an IFgsrC-compact continuous function and (Y, δ) is an IFgsr cokernal compact space, from Proposition 4.4, $\{f^{-1}(V_j) : j \in J\}$ is an IFgsr RC-compact cover of (X, τ). Since (X, τ) is an IFgsr R-compact space, there exists a finite subfamily such that $f^{-1}(V_{j_1}),\ldots\ldots f^{-1}(V_{j_n})$ such that

$$\tilde{X} = \cup_{i=1}^{n} \text{IFgsrK}_c\neg(f^{-1}(V_{j_i}))$$

Thus, $\tilde{Y} = f(\cup_{i=1}^{n} \text{IFgsrK}_c\neg(f^{-1}(V_{j_i}))) \subseteq \cup_{i=1}^{n} \text{IFgsrK}_c\neg(V_{j_i})$

Hence $\tilde{Y} = \cup_{i=1}^{n} \text{IFgsrK}_c\neg(V_{j_i})$.

Proposition 4.11: Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces.
Let f: (X, τ) → (Y, δ) be an IFgsr C-compact continuous bijective function of an IFgsr cokernal compact space (X, τ) onto an IFgsrR-compact space (Y, δ), then (X, τ) is an IFgsrR-compact space.

Proof: Let $V_\alpha = \langle x, \mu_{v_\alpha}(x), \nu_{v_\alpha}(x) \rangle$ be an IFgsr RC-compact cover of (X, τ).
From Proposition 4.7, $\{f^{-1}(V_j) : j \in J\}$ is an IFgsr RC-compact cover of (Y, δ).
Since (Y, δ) is an IFgsr R-compact space, there exists $f^{-1}(V_{\alpha_1}),\ldots\ldots f^{-1}(V_{\alpha_n})$ such that

$$\tilde{Y} = \cup_{i=1}^{n} \text{IFgsrK}_c\neg(f^{-1}(V_{\alpha_i}))$$

Thus, $\tilde{X} = f^{-1}(\cup_{i=1}^{n} \text{IFgsrK}_c\neg(f^{-1}(V_{\alpha_i})))$.
Since f is an IFgsrC-cocompact function, $\text{IFgsrK}_c\neg(f^{-1}(V_{\alpha_i})) = f(\text{IFgsrK}_c\neg(V_{\alpha_i}))$.
Thus, $\tilde{X} = \cup_{i=1}^{n} f^{-1}(f(\text{IFgsrK}_c\neg(V_{\alpha_i}))) = \cup_{i=1}^{n} \text{IFgsrK}_c\neg(V_{\alpha_i})$.
Therefore, (X, τ) is an IFgsrR-compact space.

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