Application of Sliding Mode Technology to Buck Converter Fed by PV Array

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***Abstract*: This paper presents the averaging model of a buck converter derived from the generalized state-space averaging method. The sliding mode control is used to regulate the output voltage of the converter while the output of the solar panel is fed as input to the buck converter and taken into consideration in the model. The results show a good agreement in both transient and steady-state responses. The mathematical model of buck converter with SMC to regulate the output voltage of the system is derived. And the results of simulation are presented.**

***Keywords*—Photovoltaic cell, Generalised state-space averaging method, Buck converter, Sliding mode control, Modeling, Simulation.**

I. INTRODUCTION

Nowadays, power electronic converters are widely used in many applications. The mathematical models of these converters are very important to study the system dynamic behavior. However, the power converter models are normally time-varying due to the switching actions. The analysis of the system with time-varying models is very complicated. Moreover, such model consumes the more simulation time because of the switching devices. Thus, there are many techniques that are used to eliminate the converter switching behavior to achieve the time-invariant model suitable for the system analysis and design. In this paper, the generalized state-space averaging (GSSA) method is selected to analyze the buck converter in which this method is well known for analyzing the DC/DC converter [2]-[4]. Even though the output energy of PV arrays changes frequently by the surroundings we can get regulated output voltage. In addition, the sliding mode control (SMC) [5] is applied to regulate the output voltage of the system because of its good dynamic response, robustness, and simple implementation. This paper aims to derive the mathematical model of buck converter with SMC. The averaging model of power converter can be used for studies of dynamic behavior of complex power system. The proposed paper is investigated in the presence of variation in irradiance supplied to PV Array.

The paper is structured as follows. In Section II, the PV array operating characteristics are explained. The considered power system definition is explained in Section III. An introduction to GSSA method is provided in Section IV. Deriving the dynamic model including the SMC is described in Section V. In Section VI, the simulation results of the proposed model are illustrated. Finally, Section VII concludes and discusses the advantages of the system dynamic behavior and optimal SMC design.

II. POWER SYSTEM CONSIDERED

The studied power system in the paper is depicted in Fig. 2. It consists of PV Array as a voltage source, a buck converter feeding a resistive load, and SMC to keep the output voltage constant, which is regulated by the Vref. For model derivation, the buck converter is assumed to operate under the continuous conduction mode (CCM). The switching signal of the MOFTET for the open-loop control is shown in Fig. 3.

The power converter is controlled by SMC to regulate the output voltage. The switching signal *u(t)* becomes u_{eq} depicted in Fig. 4. The u_{eq} is the control signal from SMC depending on the V_{ref}. The dynamic model of buck converter is derived by using the general state-space averaging technique.

III. THE PV ARRAY OPERATING CHARACTREISTICS

The output energy of the PV arrays is influenced by the surroundings, such as the surrounding temperature, the solar radiation and the terminal voltage of PV arrays etc, the PV arrays characteristic curve is shown as Fig. 1.

Mathematical model of the solar array can be expressed as equation (1)

$$I = Iph - Io\left[\exp\left(\frac{V+I.Rs}{V_T}\right)\right] - 1 \qquad (1)$$

Where,I is the Cell current,
I_{ph} is the Insolation current,
I_0 is the Reverse saturation current,
V is the Cell voltage,
Rs is the Series resistance,
R_p is the Parallel resistance,
V_T is the Thermal voltage (KT/q),

K is the Boltzmann constant (1.3806e-23 J),
T is the Temperature in Kelvin,
q is the Charge of an electron (1.602e-19 C)

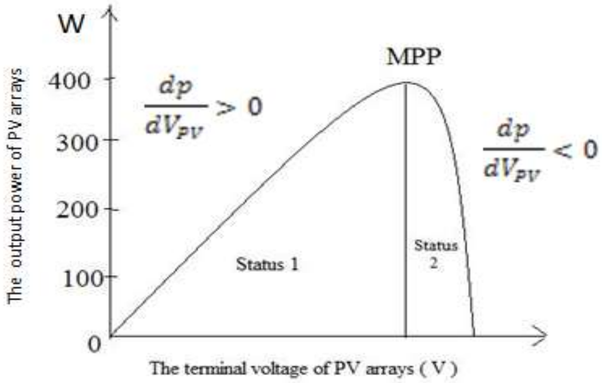

Fig. 1: The PV array characteristic curve

IV. GENERAL STATE-SPACE AVERAGING

It is a general circuit analysis program which treats arbitrary circuits that are switched between two modes. The approach is to obtain the state equations for each mode and combine those using switching times to obtain a single set of state equations. The nominal equations are solved to give equilibrium operating point duty cycle and state variables [12].

In most approaches to modeling switched-mode circuits, a particular configuration (usually consisting of 2 energy storage elements) is assumed of power stage. The state equations are derived for each mode in the form.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad \text{.......} (2)$$

And an averaged set of matrices is generated by averaging the elements of the A, B, C, and D matrices. The simplest approach weighs each by the fraction of time the circuit is in that mode. These matrices are generally used to find the circuit poles and zeros with which frequency response studies can be made.

Averaging method is based on the fact that the waveform *X(0)* can be approximated on the interval *(t-T,t]* to arbitrary accuracy with a Fourier series representation of the form.

$$x(t,s) = x(t-T+s) = \sum_{k=0} \langle x \rangle_k (t) \; e^{jk\omega_s (t-T+s)} \text{.....} (3)$$

Where the sum is over all integers k, $\omega_s = 2\pi/T$, $s \in (0, T]$, and the $\langle x \rangle_k (t)$ are complex Fourier coefficients. These Fourier coefficients are functions of time and are determined by

$$\langle x \rangle_k (t) = \frac{1}{T} \int_0^t x(t-T+s) \; e^{jk\omega_s (t-T+s)} dt \text{..............} (4)$$

V. DERIVING DYNAMIC MODEL OF BUCK CONVERTER

As mentioned in Section III, the buck converter model can be derived by using the GSSA technique. As the GSSA method itself an alternative method to eliminate the time-varying switching functions to achieve a time-invariant power converter model. The GSSA approach uses the time-dependent coefficients of the complex Fourier series as the state variables. The overview this approach [1]-[3] is as follows:

In general, a periodic waveform with period T can be represented by the complex Fourier series of the form

$$f(t) = \sum_{k=0}^{\infty} \langle x \rangle_k (t) \quad e^{jkw_s t} \text{.......} (5)$$

Where $\omega_s = 2\pi/T$ and $\langle x \rangle_k (t)$ is the complex Fourier coefficients.
The GSSA approach uses the $\langle x \rangle_k (t)$ of the waveform asthe state variables of the system. These coefficients can be determined by

$$\langle x \rangle_k (t) = \frac{1}{T} \int_{t-T}^t f(t) \; e^{jkw_s t} d \text{.......} (6)$$

Necessary properties of the $\langle x \rangle_k (t)$ for modeling the power systems using the GSSA technique are as follows:

-differentiation with respect to time

$$\frac{d\langle x \rangle_k}{dt} = \left\langle \frac{dx}{dt} \right\rangle_k - jk\omega_s \langle x \rangle_k \text{.......} (7)$$

-the convolution relationship:

$\langle xy\rangle_k = \sum_i \langle x\rangle_{k-i}\langle y\rangle_i$ .... (8)

if f(t) is real (real-value periodic waveform),

$\langle x\rangle_{-k} = \overline{\langle x\rangle}_k = \langle x\rangle_k^*$ .... (9)

In equations (5) and (6), the value of k depends on the accuracy level. Theoretically, if k approaches infinity, the approximation error approaches zero. If the waveform can be assumed to have no ripple, it can be set to $k = 0$ called zero order approximation [1]-[3]. On the other hand, if the waveform is similar to a sinusoidal signal, k can normally be set to -1, 1. This particular case is referred to as the first harmonic approximation [2]. In this section, the details of how to derive the zero-order approximation model of the considered system by using the GSSA method are fully described.

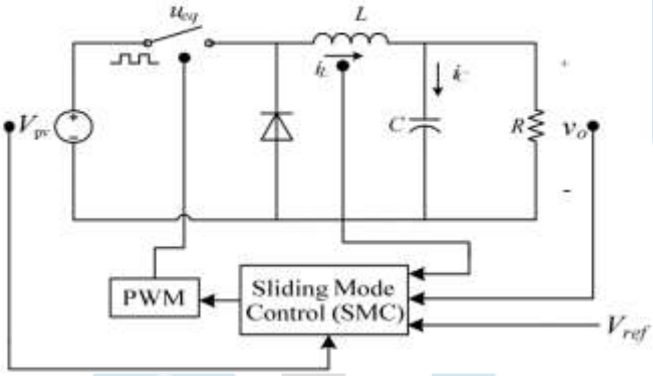

Fig. 2: Power system considered with SMC

Applying KVL and KCL to the circuit in Fig. 2 without considering the SMC, the time-varying differential equations can be written by:

$$\begin{cases} \frac{di_L}{dt} = \frac{1}{L}\left[V_{PV}u(t) - v_0\right] \\ \frac{dv_o}{dt} = \frac{1}{C}\left[i_L - \frac{v_0}{R}\right] \end{cases} \text{ .... (10)}$$

According to (6) with the switching signal of the buck converter as depicted in Fig. 3, the zero-order of the $u(t)$ can be expressed as:

$\langle u\rangle_0 = \frac{1}{T_s}\int_0^{dT_s} 1e^0 dt = \frac{1}{T_s}[t]_{t=0}^{t=dt_s} = d$.......................... (11)

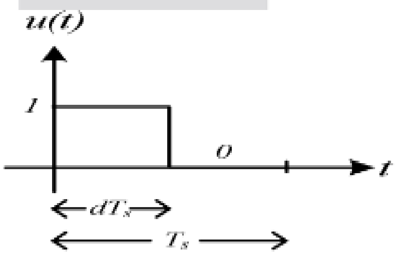

Fig. 3: The switching signal for the open-loop operation

For the voltage source V, due to the DC voltage source, the zero-order of V_{in} can be given in

$\langle V_{PV}\rangle_0 = V_{PV}$... (12)

From (1), the zero-order approximations of the actual states (k = 0) are denoted as:

$$\begin{cases} i_L = \langle i_L \rangle_0 \\ v_0 = \langle v_0 \rangle_0 \end{cases} \quad (13)$$

The Fourier expansion leads to 2 state variables denoted by x_k as follows:

$$\begin{cases} x_1 = \langle i_L \rangle_0 = i_L \\ x_2 = \langle v_0 \rangle_0 = v_0 \end{cases} \quad (14)$$

Applying the property (7) into (10) for k = 0, the $\frac{d\langle v_0 \rangle_0}{dt}$ and $\frac{d\langle i_L \rangle_0}{dt}$ can be expressed as:

$$\begin{cases} \frac{d\langle i_L \rangle_0}{dt} = \frac{1}{L}\left[\langle V_{PV} u(t) \rangle_0 - \langle v_0 \rangle_0\right] \\ \frac{d\langle v_0 \rangle_0}{dt} = \frac{1}{C}\left[\langle i_L \rangle_0 - \frac{\langle v_0 \rangle}{R}\right] \end{cases} \quad (15)$$

Using (8), (11), and (12), $\langle V_{PV} u(t) \rangle_0$ in (15) can be expressed as:

$$\langle V_{PV} u(t) \rangle_0 = \langle V_{PV} \rangle_0 \langle u \rangle_0 = dV_{PV} \quad (16)$$

Therefore, (15) can be rewritten in terms of the state variables defined in (10) as:

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix}\begin{bmatrix} i_L \\ v_0 \end{bmatrix} + \begin{bmatrix} \frac{dV_{PV}}{L} \\ 0 \end{bmatrix} \quad (17)$$

Where i_L and v_0 are the circuit state-variables and d is the duty cycle of the buck converter. Note that before applying the GSSA method, the dynamic model of the considered system is a time-varying as given in (10). Using the GSSA method, (10) becomes (17) that is the averaging model of the buck converter under the open-loop operation with the zero-order approximation.

For SMC, the sliding surface can be set as the linear combination of the state-variables because it is simple for implementation. The sliding surface equation can be expressed as:

$$s = J^T X = ax_1 + bx_2 + mx_3 = 0 \quad (18)$$

Note that a, b, and m are the coefficients of SMC, while x1, x2, and x3 are defined as:

$$\begin{cases} x_1 = i_{ref} - i_L \\ x_2 = V_{ref} - v_0 \\ x_3 = \int [x_1 + x_2] dt \end{cases} \quad (19)$$

Where $i_{ref} = K[V_{ref} - v_0]$ and K is the gain to amplify the voltage error.
Applying (19) into (18), the sliding surface equation becomes

$$s = a(i_{ref} - i_L) + b(V_{ref} - v_0) + m\int [K(V_{ref} - v_0) - i_L + V_{ref} - v_0] dt \quad (20)$$

Then,

$$\frac{ds}{dt} = [-a \quad -b]\frac{d}{dt}\begin{bmatrix} i_L \\ v_0 \end{bmatrix} - \frac{aKi_L}{C} + \frac{aKv_0}{RC} + m[(K+1)(V_{ref} - v_{0)} - i_L] = 0 \quad (21)$$

Substituting $\frac{d}{dt}\begin{bmatrix} i_L \\ v_0 \end{bmatrix}$ from (17) into (21) and replacing d by u_{eq} yields:

$$\frac{ds}{dt} = \frac{av_0}{L} - \frac{aV_{in}u_{eq}}{L} - \frac{bi_L}{C} + \frac{bv_0}{RC} - \frac{aKi_L}{C} + \frac{aKv_0}{RC} + m[(K+1)(V_{ref} - v_{0)} - i_L] = 0 \quad (22)$$

From (32), u_{eq} of SMC can be calculated by:

$$u_{eq} = \frac{aRCv_0 - bL(Ri_L - v_0) - aLK(Ri_L - v_0)}{aRCV_{in}} + \frac{mRLC[(K+1)(V_{ref} - v_0) - i_L]}{aRCV_{in}} \quad (23)$$

The block diagram for u_{eq} calculation based on (23) is depicted in Fig. 4. For deriving the dynamic model of the power system including the controllers of buck converter, the schematic of the SMC as shown in Fig. 5 is considered. It can be seen that the SMC parameters are represented by a, b, m, and K. In addition, when the buck converter is regulated by SMC, the d in (17) becomes u_{eq} as given in (23). Therefore, the dynamic model of the system on sliding surface can be expressed as:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (24)$$

Where state variables: $X = [i_L \quad v_0]^T$, input: $u = [V_{ref}]$, output: $y = [i_L \quad v_0]^T$
The details of A, B, C, and D in (24) are as follows:

$$\begin{cases} A = \begin{bmatrix} -\left(\frac{b+aK+mC}{aC}\right) & \frac{b+aK-mRC(K+1)}{aRC} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \\ B = \begin{bmatrix} \frac{m(K+1)}{a} \\ 0 \end{bmatrix} \\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{cases} \quad (25)$$

In (24) and (25), the SMC parameters are occurred in the dynamic model. Therefore, the proposed averaging model can represent the dynamic behavior of the system under the SMC [7].

The final equation (23) which provides the required duty cycle for the exact reference voltages considered and the simulation controller block is constructed by tools in simulink library.

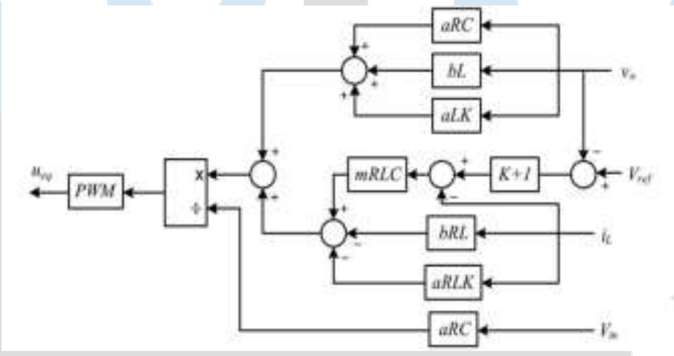

Fig. 4: The block diagram of u_{eq} calculation

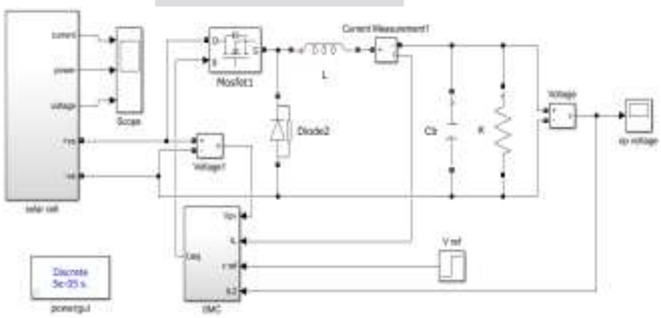

Fig. 5: PV Array simulation system

VI. THE SIMULATION RESULTS

The set of parameters for the system in Fig.1 is given as follows:
Vin = 60 V, R = 30 Ω, L = 15 mH, C = 125 µF, Ts = 0.1 ms, a = 3, b = 25, m = 2500, K = 2000.

The considered variations in irradiance over the simulation of 20 seconds which depicts the practical variations in surrounding solar radiation shown in fig. 6.

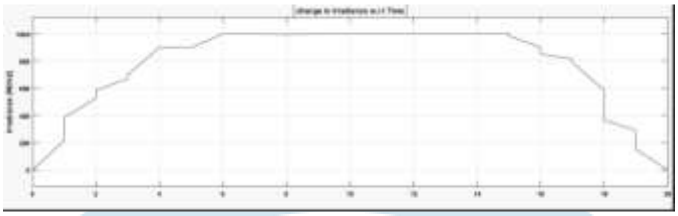

Fig. 6: change in Irradiance w.r.t Time.

The system in Fig. 9 was simulated for a regulated output voltage which is provided by a step change of V_{ref} from 40V to 90V that occurs at 5 seconds.

The system in fig. was simulated to a step change of V_{ref} from 40V to 90V that occurs at 0.06 second. The duty cycle, output voltage and inductor current responses for changing the V_{ref} from 40V to 90V are depicted in Fig. 7, Fig. 8

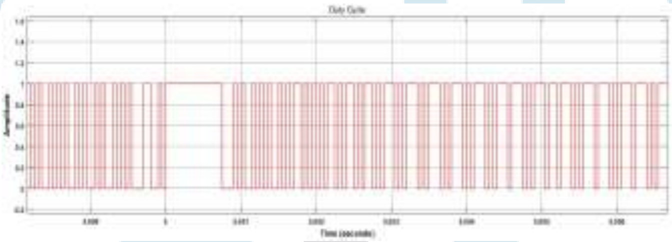

Fig. 7: Duty cycle for changing V_{ref} from 40V to 70V.

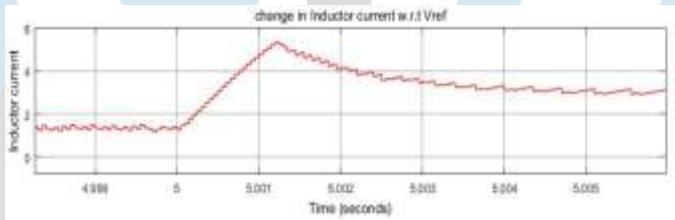

Fig. 8: The Inductor current response for changing V_{ref} from 40V to 70V.

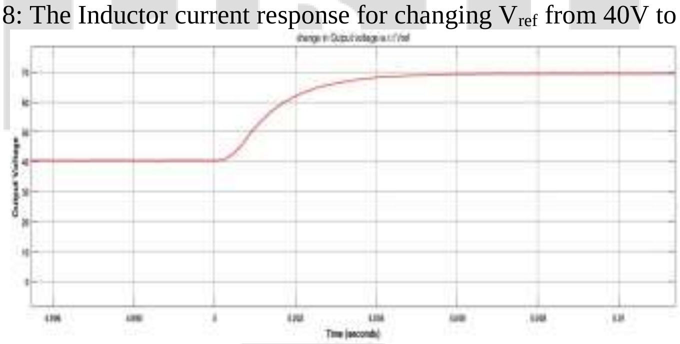

Fig. 9: The output voltage response for changing V_{ref} from 40V to 70V.

The variations in the output voltages of the buck converter with the change in the irradiance levels which depicts the practical variations in the solar radiations shown in Fig. 10.

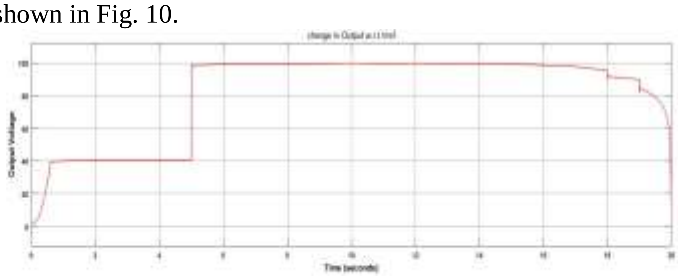

Fig. 10: The output voltage response for the varying Irradiance.

The comparison with the PV Array output voltages with the variations in the irradiance levels and the buck converter output voltages under the simulation time 20 seconds shown in Fig. 11.

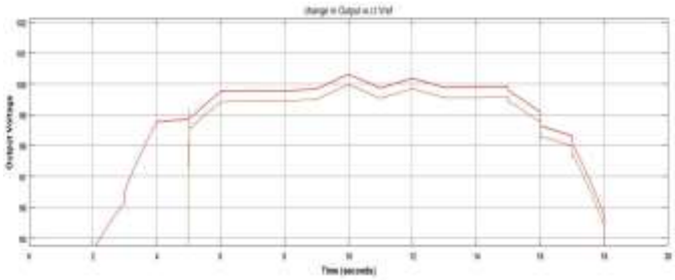

Fig. 11: comparison between PV Array output and converter output voltage

VII. CONCLUSION

This paper has explained how to derive the dynamic model of buck converter with SMC by using GSSA technique. An excellent agreement between the averaging model based on SMC and the full switching model is obtained. It confirms that the reported model in the paper can provide a high accuracy representation of the real system. The dynamic model from the paper can be used for the simulation with faster computational time. The result obtained from this method is feasible.

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