REENGINEERING CORDIAL LABELING OF ONE POINT UNION OF SOME GRAPHS

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***Abstract:** A graph H in which a vertex is distinguished from other vertices is called a rooted graph and the vertex is called the root of H. Let H be a rooted graph. The graph H(n) obtained by identifying the roots of n copies of H is called the one-point union of n copies of the graph H.*

A function from vertex set of a graph to the set {0,1}, which assigns the label $|f(u) - f(v)|$ or each edge uv, is called a cordial labelling of the graph if the number of vertices labelled 0 and number of vertices labelled 1 differ by at most 1, and similar condition is satisfied by the edges of the graph. In this paper we discuss cordial labelling of one point union of grid graph, cycle with one chord and cycle with twin chords.

Keywords: Cordial graph, One Point Union

1 INTRODUCTION

Let f be a function from vertex set V of a finite, undirected graph H to the set {0,1} and for each edge $e = uv$, assign the label $|f(u) - f(v)|$. Then f is called a cordial labeling of graph H if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and similarly the number of edges labelled 0 and the number of edges labeled 1 differ by at most 1. In this paper C_n denotes cycle with n vertices and P_n x P_n denotes grid graph with n^2 vertices. For graph theoretical terminology and notations we follow Gross and Yellen[5].

The concept of cordial graphs was introduced by Cahit[1]. Shee and Ho[6] prove that the one-point union of n copies of flag Fl_m (with the common point being the root) is cordial. Selvaraju[7] proved that the one-point union of any number of copies of a complete bipartite graph is cordial. Benson and Lee[8] investigated the regular windmill graphs $K_m^{(n)}$ and determined precisely which ones are cordial for m < 14. A dynamic survey of graph labeling is published and updated every year by Gallian[3]. In this paper we prove that the one point union of grid graph, cycle with one chord and cycle with twin chords are cordial graphs.

II. MAIN RESULTS

Theorem 3.1 The one point union of grid P_n x P_n is cordial.

Proof: Let H be the one point union of k copies $H_1, H_2, ..., H_k$ of grid graph P_n x P_n, where $|H_i| = n^2$, i = 1,2,..., k. Let us denote the successive vertices (in clockwise spiral direction) of graph H_i by $\{u_{i1}, u_{i2}, .., u_{in2}\}$, where u_{i1} is considered as the root vertex of G. Here we define labeling function $f : V(H) \rightarrow \{0,1\}$ as follows.

Case

Case 1: $n \equiv 0, 2(mod4)$

$f(u_{i1}) = 1,$

$f(u_{1j}) = 0;$ if $j \equiv 2, 3(mod4)$

$= 1;$ if $j \equiv 0, 1(mod4), 2 \le j \le n^2$

Subcase I: i is odd

$f(u_{ij}) = 0;$ if $j \equiv 0, 1(mod4)$

$= 1;$ if $j \equiv 2, 3(mod4), 2 \le j \le n^2, 2 \le i \le k$

Subcase II: i is even

$f(u_{ij}) = 0;$ if $j \equiv 0, 3(mod4)$

$= 1;$ if $j \equiv 1, 2(mod4), 2 \le j \le n^2, 1 \le i \le k$

Case 2: $n \equiv 1, 3(mod4)$

$f(u_{i1}) = 1$

Subcase I: i is odd

$f(u_{ij}) = 0;$ if $j \equiv 2, 3(mod4)$

$= 1;$ if $j \equiv 0, 1(mod4), 2 \le j \le n^2, 1 \le i \le k$

Subcase II: i is even

$f(u_{ij}) = 0;$ if $j \equiv 0, 1(mod4)$

$= 1;$ if $j \equiv 2, 3(mod4), 2 \le j \le n^2, 1 \le i \le k$

The labeling pattern defined in above cases satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ in each case which is shown in *Table 1*. Hence the graph under consideration is cordial graph.

Let $n = 4a + b$, $k = 4c + d$, where $n,\ k \in N$.

Table 1: Table for Theorem 3.1

b	d	vertex conditions	edge conditions
0,2	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1,3	0,1,2,3	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$

Illustration 3.1 Cordial labeling of one point union of three copies of grid graph $P_4\ xP_4$ is shown in Fig. 1 as an illustration for the Theorem 3.1. It is the case related to $n = 0\ (mod4)$.

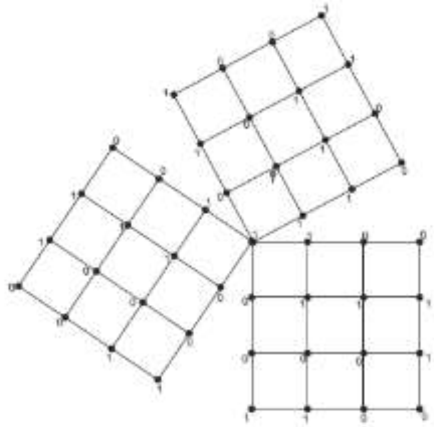

Figure 1: Cordial labeling of one point union of three copies of grid graph $P_4\ x\ P_4$

Theorem 3.2 The one point union of cycle with one chord is cordial.

Proof: Let H be the one point union of k copies $H_1, H_2, \ldots, H_k$ of cycle C_n with one chord. Let $u_{i1}, u_{i2}, \ldots, u_{in}$ denote the vertices of H_i and let $e_i = u_{i2}u_{in}$ be the chord in H_i, i = 1,2..., k.
Here u_{i1} is considered as the root vertex of H, i = 1, 2, ... , k.
To define labeling function f : V (H) → {0, 1} we consider following cases.

Case 1: $n \equiv 0(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 2, 3(mod4)$
$\quad = 1$; if $j \equiv 0, 1(mod4)$, $2 \leq j \leq n$
Subcase II: i is even, $1 \leq i \leq k$
$f(u_{in}) = 1$
$f(u_{ij}) = 0$; if $j \equiv 0, 3(mod4)$
$\quad = 1$; if $j \equiv 1, 2(mod4)$, $2 \leq j \leq n - 1$
Case 2: $n \equiv 1(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$\quad = 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n$
Subcase II: i is even, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$\quad = 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n$
Case 3: $n \equiv 2(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$\quad = 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n$
Subcase II: i is even, $1 \leq i \leq k$
$f(u_{in}) = 0$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$\quad = 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n - 1$
Case 4: $n \equiv 3(mod4)$
$f(u_{i1}) = 1$
$f(u_{1j}) = 0$; if $j \equiv 2, 3(mod4)$
$\quad = 1$; if $j \equiv 0, 1(mod4)$, $2 \leq j \leq n$
For $2 \leq i \leq k$:
$f(u_{in-1}) = 1$,
$f(u_{ij}) = 0$; if $j \equiv 2, 3(mod4)$
$\quad = 1$; if $j \equiv 0, 1(mod4)$, $2 \leq j \leq n, j \neq n - 1$

The labeling pattern defined above satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in *Table 2*. Hence the graph under consideration is cordial graph.
Let $n = 4a + b$, $k = 4c + d$, where n, $k \in N$.

Table 2: Table for the graph G in Theorem 3.2

b	d	vertex conditions	edge conditions
0	0,2	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
1,3	0,1,2,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
2	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Illustration 3.2 Cordial labelling of one point union of three copies of cycle C_5 with one chord is shown in *Fig. 2* as an illustration for the *Theorem 3.2*. It is the case related to $n = 1(mod4)$.

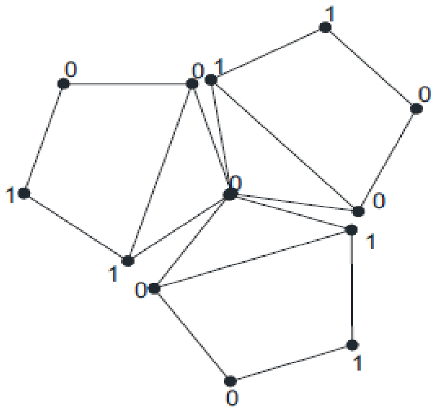

Figure 2: Cordial labeling of one point union of three copies of cycle C_5 with one chord

Theorem 3.3 The one point union of cycle with twin chords is cordial.
Proof: Let H be the one point union of k copies $H_1, H_2, \ldots, H_k$ of cycle C_n with twin chords.
Let $u_{i1}, u_{i2}, \ldots, u_{in}$ denotes the vertices of G_i, $i = 1, 2, \ldots, k$. Let $e_i = u_{i2}u_{in}$ and $e'_i = u_{i3}u_{in}$ be the chords in G_i, $i = 1, 2, \ldots, k$. Here ui_1 is considered as the root vertex of G_i, $i = 1, 2, \ldots, k$. To define labeling function f : V (H) →{0, 1} we consider following cases.

Case 1: $n \equiv 0(mod4)$
$f(u_{i1}) = 1$
$f(u_{1j}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n$
Subcase I: i is odd, $2 \leq i \leq k$
$f(u_{in-1}) = 0$,
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n, i \neq n - 1$
Subcase II: i is even, $2 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n$
Case 2: $n \equiv 1(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n$

Subcase II: i is even, $1 \le i \le k$
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \le j \le n$
Case 3: $n \equiv 2(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \le i \le k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \le j \le n$

Subcase II: i is even, $1 \le i \le k$
$f(u_{in}) = 0$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \le j \le n-1$
Case 4: $n \equiv 3(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \le i \le k$
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \le j \le n$
Subcase II: i is even, $1 \le i \le k$
$f(u_{in-1}) = 0$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \le j \le n, j \ne n-1$

The labeling pattern defined in above cases satisfies the conditions of cordial labeling which is shown in *Table 3*. Hence the graph under consideration is cordial graph.
Let $n = 4a + b$; $k = 4c + d$, where $n, k \in N$.

Table 3: Table for Theorem 3.3

b	d	vertex conditions	edge conditions
0	0,2	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1	0,1,2,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
2	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
3	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1) + 1$	$e_f(0) + 1 = e_f(1)$

Illustration 3.3 Cordial labeling of one point union of three copies of cycle C_5 with twin chords is shown in *Fig. 3* as an illustration for *Theorem 3.3*. It is the case related to $n \equiv 1(mod4)$.

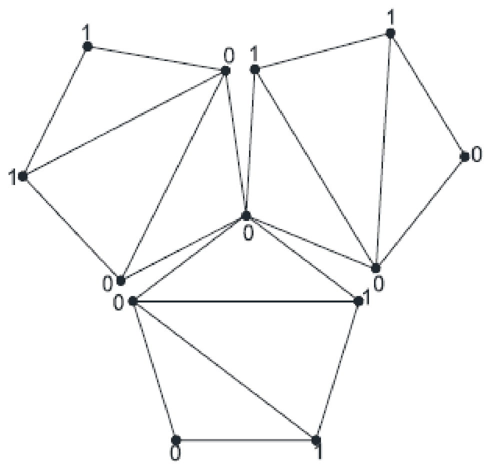

Figure 3: Cordial labeling of one point union of three copies of cycle C_5 with twin chord

III. CONCLUSION

We have discussed cordiality of grid, cycle with one chord, cycle with twin chords in context of one point union of graphs. We contribute three new graph families in the theory of cordial graphs.

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