V-Super Edge Magic Labeling

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***Abstract*: The scope of this paper to suggest that the concept of V-Super Magic Labeling of some new graphs such as Path, Cycle , Sun graph ,are realized .And also we prove that the Spider gaph ,Lollipop graph ,Roman candle graph are also V-super Edge magic Labeling.**

Index Terms: Magic labeling, Super Edge Magic labeling, V-Super Magic labeling, Spider graph, Lollipop graph and Roman candle graph

I. INTRODUCTION

Graph labeling is one of the most important research area and it is used in many real world applications like coding theory , radar, astronomy , X-ray Crystallography ,channel assignment process. A graph contains 1-1 mapping into graph elements carries a non - negative real number is called Labeling. Labeling has many types such that graceful , harmonious, magic , antimagic etc.. J.Sedlack was first introduced a Magic graph. A graph contains a positive real number with sum of the labels around any vertex equals the constant is called Magic labeling. It was introduced by Kotzig and Rosa. Super magic labeling is known as the labeling are consecutive integers beginning from 1 onwards. It was introduced by Stewart . Super edge magic (V-Superedge magic) labeling was introduced by Enomoto , Llado , G.Ringel.

II.PRELIMINARIES

Definition 1.1: A graph whose edges are labeled by non- negative numbers such that sum of the labels of the edges are incident with any vertex is **same** is known as **Magic graph labeling .**

Definition 1.2: A graph of an edge is labeled in distinct elements such that sum of the edge labels at each vertex is same known as **Edge magic graph labeling .**

Definition 1.3: A edge magic labeling g is called **Super Edge Magic Labeling** if g (V) = {1, 2, 3, ….. , v} and g(E)={v+1,v+2,……..v+e}. A graph G is known as super edge magic graph if ∃ super edge magic graph labeling of G.

Definition 1.4: A edge magic total labeling if the property of vertex - labels are smallest possible integers are known as **V-Super edge magic labeling.**

III . V-SUPER EDGE MAGIC LABELING

Definition 2.1: If no vertices repeated more than once in open walk is known as **path.**

Theorem 2.2: If path P_s is V -SEML.

Proof: In a graph H (v, ε) is edge magic the $\sum_{v\in V}\deg(v)\,g(v)+\sum_{\varepsilon\in E} g(\varepsilon)=M\varepsilon$

Let v=4, ε=4 ,The vertex and edge labeling are, $g(v_1)=1$, $g(v_r)$ = s - r +1 , 2 ≤ r ≤ s , $g(e_r)$ = 2s-r

$$\sum_{v\in V}\deg(v)\,g(v)+\sum_{\varepsilon\in E} g(\varepsilon)=M\varepsilon$$

1(1)+3(2)+2(2)+4(1)+7+6+5 = 3M⟺1+6+4+4+6+5 = 3(11)

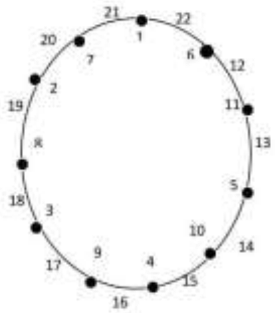
Hence , path P_s is V - SEML.

Definition 2.3: If no vertices (nodes) is repeated more than once in closed walk is known as **cycle.**

Theorem 2.4: If cycle C_s is V- SEML then the magic valance M=$\frac{1}{2}(5s+3)$when s is odd.

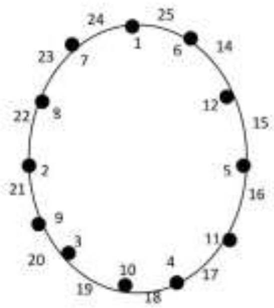
Proof: We assume that s=2r+1. The vertex labeling are {1, r+2,2r+1,…r+2} every labels are calculated by r(mod2r+1) sum of pair of vertices r+2,3r+2,…,r+3 it is totally various when M= 5r+4.The labels of the edges 4r+2, 2r+2,…4r+1 we receives M= 5r+4=$\frac{1}{2}(5s+3)$. Hence, Cycle C_s is V- SEML.

Example : Let r=5 , s=2(5)+1=11 then the magic valance M=5r+4=5(5)+4=29.The vertex and edge labeling are $g(v_1)=1$, $g(v_2)=6$, $g(v_3)=11$, $g(v_4)=5$, $g(v_{11})=7$, $g(v_5)=10$, $g(v_6)=4$, $g(v_7)=9$, $g(v_8)=3$, $g(v_9)=8$, $g(e_2)=12$, $g(e_3)=13$, $g(e_4)=14$, $g(e_5)=15$, $g(e_6)=16$, $g(e_7)=17$, $g(e_8)=18$, $g(e_9)=19$, $g(e_{10})=20$, $g(e_{11})=21$. Hence , the cycle is V-SEML .



Theorem 2.5: If cycle C_s is V- SEML then the magic valance $M=\frac{1}{2}(5s+4)$ when s is even.

Proof: The vertex and edge labeling are $g(a_1)=1$, $g(a_2)=6$, $g(a_3)=12$, $g(a_4)=5$, $g(a_5)=11$, $g(a_6)=4$, $g(a_7)=10$, $g(a_8)=3$, $g(a_9)=9$, $g(a_{10})=2$, $g(a_{11})=8$, $g(a_{12})=7$ and edge labeling are own count.



The magic valance M=32. Hence, Cycle C_{12} is V- SEML.

Definition 2.6 :A lolipop graph is a graph which is obtained by connecting the terminal points of the path P_n to the cycle C_n.

Theorem 2.7: Let L_s be an lollipop graph is V- SEML for all s.

Proof: Let $a_0, a_1, \dots, a_{r-1}, a_0$ be an vertices of a graph and a is adjacent to a_0. b is adjacent to a. Then there exsits the magic valance M=5r+11. The vertex and edge labeling are

$$g(a_l)=\begin{cases}\frac{r+l}{2} & \text{if } l=0,2,\dots,r-2\\ \frac{3r+l+9}{2} & \text{if } l=1,3,\dots,r-1\\ r & \text{if } l=r+1\\ \frac{r+l+2}{2} & \text{if } l=r,r+2,\dots,2r\\ \frac{l-r-1}{2} & \text{if } l=r+3,r+5,\dots,2r-1\\ \frac{3r+6}{2} & \text{if } l=2r+1\end{cases} \quad \text{and}$$

$$g(e_l)=\begin{cases}3r-l+6, & l=0,2,\dots,r-2\\ 2r+6, & l=r-1\\ 4r-l+10, & l=r,r+1\\ 5r-l+10, & l=r+2,r+3,\dots,2r-1\\ 2r+7, & l=2r\end{cases},$$

$g(e_{r-1})=3r+8$, $g(a_0a)=3r+7$, $g(ca)=2r+5$.

Example: Let r=8 and s=2r+2=18, the vertex and edge labeling are $g(a_1)=17$, $g(a_3)=18$, $g(a_5)=19$, $g(a_7)=20$, $g(a_9)=8$, $g(a_{11})=1$, $g(a_{13})=2$, $g(a_{15})=3$, $g(a_{17})=15$, $g(a_2)=5$, $g(a_4)=6$, $g(a_6)=7$, $g(a_8)=9$, $g(a_{10})=10$, $g(a_{12})=11$, $g(a_{14})=12$, $g(a_{16})=13$, $g(c)=16$, $g(b)=14$, $g(e_1)=29$, $g(e_2)=28$, $g(e_3)=27$, $g(e_4)=26$, $g(e_5)=25$, $g(e_6)=24$, $g(e_7)=$

22,$g(e_8)$=34,$g(e_9)$=33,$g(e_{10})$=40,$g(e_{11})$=39,$g(e_{12})$=38,$g(e_{13})$=37, $g(e_{14})$=36,$g(e_{15})$=35,$g(e_{16})$=23,$g(a_{15}a_0)$=32, $g(a_0a)$=31, g(ca)=21.The magic constant M=51. Hence the lollipop graph L_{20} is V-SEML.

Definition 2.8: A **sun graph** is graph which consist of cycle length n where every vertex of the cycle have adjacent leaves.

Theorem 2.9: Let S_s is sun graph is V- SEML when s is odd

Corollary 2.10: If S_s is sun graph is V- SEML when s is even.

Definition 2.11: A tree with one vertex of degree atleast 3 and all other vertices of degree atmost 2 is known as **spider graph** .

Theorem 2.12: The spider graph is V-SEML when s is odd.

Proof: Let s=3r+1 then the vertex and edge labeling are

$$g(y)=\frac{s+2}{3},\ g(a_1)=s,\ g(a_{2t})=2t,\ 1\le t\le \frac{s-4}{6},$$

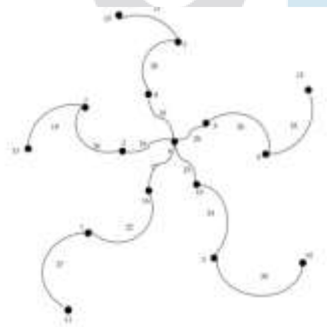
$$g(a_{2t+1})=\frac{s+5}{3}+2t-1,\ 1\le t\le\frac{s-4}{6},\ g(b_{2t})=\frac{s+5}{3}+2t-2,\ 1\le t\le\frac{s-4}{6},$$

$$g(b_{2t+1})=2t-1,\ 0\le t\le\frac{s-4}{6},\ g(c_1)=\frac{2s+1}{3},$$

$$g(c_{2t+1})=s-l,\ 1\le t\le\frac{s-4}{6},\ ,\ g(c_{2t})=\frac{5s-2}{6}-t+1,\ 1\le t\le\frac{s-4}{6}$$

$R=g(x)+g(y):xy\in E(H)$ edge labeling is own count. Then the magic valance $M=v+\varepsilon+R=\frac{7s+5}{3}$.

Example:



Theorem 2.13 :The Roman candle graph is V-SEML for all r.

Proof: The vertices of the graph are denoted as $x_1, x_2,\dots, x_s$ and y_l is the neighbouring vertices,we take $y_{l1},.., y_{lr}$ are pendent vertices neighbor to y_l $1\le t\le s$.

$$g(x_l)=\begin{cases}\frac{t+1}{2}+s & ,t=odd\\ s(r+1)+\frac{s+t-1}{2} & ,t=even\end{cases},\quad g(x_l)=\begin{cases}\frac{t+s}{2} & ,t=odd\\ \frac{t}{2} & ,t=even\end{cases}$$

if $1\le t\le s$ and $1\le l\le r$

$$g(y_{tl})=\begin{cases}s(r+3-l) & ,t=1,l=odd\\ s(r+2-l)+1 & ,t=1,l=even\\ sl+\frac{s+t+1}{2} & ,t=even,l=odd\\ sl+\frac{s+t-1}{2} & ,t=even,l=even\end{cases}$$

$$g(y_1y_{l1})=\begin{cases}s(r+2-t) & ,l=odd,\\ s(r+3+l)-1 & ,l=even\end{cases}$$

$$g(y_ly_{tl})=\begin{cases}s(r+3)+1-t+sl & ,t,l=odd\\ s(r+3)-t+sl & ,t=odd,l=even\\ 2s(r+2)-s(l-1)-t & ,t=even,l=odd\\ 2s(r+2)-s(l-1)-t+1 & ,t=even,l=even\end{cases}$$

$$g(y_{lt})=\begin{cases}\frac{t-1}{2}+s(r+2)-sl & ,t,l=odd, 3\le t\le s\\ s(r+2)+\frac{t+1}{2}-sl, & l=even, t=odd\end{cases}$$

$g(x_t x_{t+1})$=s(r+3)-t, $1\le t\le s-1$,

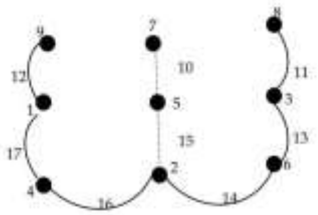
$$g(x_t y_{t+1})=\begin{cases}s(r+2)-t & ,t=odd,\\ s(r+4)+1-t, & t=even\end{cases}$$

Hence ,The magic valance $M=2s(r+2)+s+\frac{s+1}{2}$

Example: Put s=5, r=5 the vertex are $g(x_1)=6$, $g(x_2)=33$, $g(x_3)=7$, $g(x_4)=34$, $g(x_5)=8$,$g(y_1)=3$, $g(y_2)=1$, $g(y_3)=4$, $g(y_4)=2$, $g(y_5)=5$, $g(x_{11})=35$, $g(x_{12})=26$, $g(x_{13})=25$, $g(x_{14})=16$, $g(x_{15})=15$, $g(x_{21})=9$, $g(x_{22})=13$, $g(x_{23})=,19$, $g(x_{24})=23$, $g(x_{25})=29$, $g(x_{31})=31$, $g(x_{32})=27$, $g(x_{33})=,21$, $g(x_{34})=17$,$g(x_{35})=11$,$g(x_{41})=10$, $g(x_{42})=14$, $g(x_{43})=,20$, $g(x_{44})=24$,$g(x_{45})=30$, $g(x_{51})=32$, $g(x_{52})=28$,$g(x_{53})=22$,$g(x_{54})=18$, $(x_{55})=12$,$g(x_1y_1)=69$,$g(x_2y_2)=44$,$g(x_3y_3)=67$,$g(x_4y_4)=42$,$g(x_5y_5)=65$,$g(x_1y_2)=39$,$g(x_2y_3)=38$, $g(x_3y_4)=37$, $g(x_4y_5)=36$, $g(y_1y_{11})=40$, $g(y_1y_{12})=49$, $g(y_1y_{13})=50$,$g(y_1y_{14})=59$,$g(y_1y_{15})=60$,$g(y_2y_{21})=68$, $g(y_2y_{22})=64$, $g(y_2y_{23})=58$,$g(y_2y_{24})=54$,$g(y_2y_{25})=48$,$g(y_3y_{31})=43$,$g(y_3y_{32})=47$,$g(y_3y_{33})=53$,$g(y_3y_{34})=57$,$g(y_3y_{35})=63$,$g(y_4y_{41})=66$,$g(y_4y_{42})$ $=62$,$g(y_4y_{43})=56$,$g(y_4y_{44})=52$,$g(y_4y_5)=46$, $g(y_5y_{51})=41$, $g(y_5y_{52})=45$, $g(y_5y_{53})=51$, $g(y_5y_{54})=55$, $g(y_5y_{55})=61$.Hence ,the

magic valance M=78.The roman candle graph is V-SEML.

Note: The prawn graph is V- SEML.

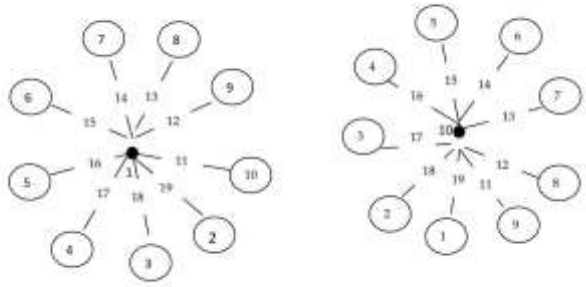


Theorem 2.14: If a star graph $S_{1,s}$ is V-SEML then its middle point get receives the label 1 or s+1.

Proof: Let the vertex and edge labeling are $\{v, v_1, \ldots, v_s\}$ & $\{e_s : 1\le l\le s\}$, c is the middle point. The magic valance $M=2s+3+\frac{(s-1)g(v)+1}{s}$ .Hence, M is an integer if g(v)=1 or g(v)=s+1.

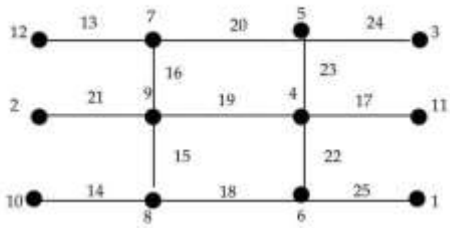
- If g(v)=1 then the magic valance M=2s+4
- If g(v)=s+1 then the magic valance M=3s+3

Example: Put s=9 then the magic valance M= 22 when middle point receives 1. The magic valance M=30 when middle point receives s+ 1.



Hence, the star graph is V-SEML.

Note: The grill graph is V- SEML. The magic valance M=32



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