SELF WEAK COMPLEMENTARY INTERVAL - VALUED FUZZY GRAPH

[1]Punitha. V, [2]Sathya. P

[1]Assistant Professor, [2]M.Phil Scholar
Dept. Of Mathematics, Navarasam Arts And Science College For Women, Arachalur

***Abstract*: Fuzzy graph which have revolutionized the analysis of problematic data to arrive at a better decision making power are of different kinds. Among them, the simplest and generalized form is the interval – valued fuzzy graph. In this paper we introduce the condition for the interval – valued fuzzy graph to be self-weak complementary interval – valued fuzzy graph.**

***Keywords*: Fuzzy graph, Bipolar fuzzy graph, Interval – valued fuzzy graph, self weak complementary.**

1. INTRODUCTION

A fuzzy set was introduced and defined mathematically by L.A. Zadeh in 1965. After that, the concept of fuzzy graph theory was introduced by A. Rosenfeld in 1975 using the fuzzy relation and its operations. The properties of fuzzy graphs dealt with J.N. Mordeson and C.S. Peng.In 1975, the concept of Interval-valued fuzzy set was introduced by Zadeh as an extension of fuzzy sets in which the values of membership degrees are intervals in $[0,1]$ instead of elements in $[0,1]$. Hongmei and Lianhua defined the definition of Interval-valued fuzzy graph. In 2011, Muhammad Akram and Wieslaw A. Dudek investigatd some of the operations on Interval-valued fuzzy graph. This paper is organised as the definition of interval – valued fuzzy graph and the condition for interval – valued fuzzy graph to be self weak complementary.

2. PRELIMINARIES

Definition 1

The **complement** graph$\overline{S^*}$of a simple graphS^* has the vertices as same as$S^*$and two vertices are adjacent in$\overline{S^*}$if and only if they are not adjacent in S^*.

Definition 2

A **fuzzy graph**$F = (V, \sigma, \mu)$ is a triple consisting of a nonempty set Vtogether with a pair of functions$\sigma : V \to [0,1]$ and $\mu : \varepsilon \to [0,1]$such that for all $a,b \in V$, $\mu(ab) \leq \sigma(a) \wedge \sigma(b)$.

Definition 3

A **bipolar fuzzy graph** with an underlying set V is defined to be a pair$F = (Q,R)$ where $Q = (\mu_Q^P, \mu_Q^N)$ is a bipolar fuzzy set in V and$R = (\mu_R^P, \mu_R^N)$ is a bipolar fuzzy set such that,

$\mu_R^P(a,b) \leq \mu_Q^P(a) \wedge \mu_Q^P(b)$,$\mu_R^N(a,b) \geq \mu_Q^N(a) \vee \mu_Q^N(b)$, for every$a,b \in V$.

Definition 3

The **Complement of a bipolar fuzzy graph** $F = (Q, R)$ of a graph$F^* = (V, E)$ is a bipolar fuzzy$\bar{F} = (\bar{Q}, \bar{R})$ of$\overline{F^*} = (V, V^2)$, where$\bar{Q} = Q(\mu_Q^P, \mu_Q^N)and\bar{R} = Q(\overline{\mu_Q^P}, \overline{\mu_Q^N})$ is defined by

$\overline{\mu_Q^P}(ab) = (\mu_Q^P(a) \wedge \mu_Q^P(b)) - \mu_R^P(ab)$,$\overline{\mu_Q^N}(ab) = (\mu_Q^N(a) \vee \mu_Q^N(b)) - \mu_R^N(ab)$,for every $a,b \in V$.

Definition 4

A bipolar fuzzy graph$F$is said to be Self complement$F \cong \bar{F}$.

Definition 5

A bipolar fuzzy graph$F = (Q, R)$ of a graph $F^* = (V,E)$ is said to be **self Weak complement** if F is weak isomorphism with its complement $\bar{F}$, i.e., there exist a bijective homomorphism$f$from F to$\bar{F}$ such that for all$a,b \in V$.

$\mu_Q^P(a) = \overline{\mu_Q^P}(f(a))$, $\mu_Q^N(a) = \overline{\mu_Q^N}(f(a))$and

$\mu_R^P(ab) \leq \overline{\mu_R^P}(f(a)f(b))$, $\mu_R^N(ab) \leq \overline{\mu_R^N}(f(a)f(b))$.

Definition 5

If $F^* = (V, E)$ is a graph, then by **interval-valued fuzzy relation** R on a set E, we mean an interval-valued fuzzy set such that, $\mu_R^-(ab) \le \mu_R^-(a) \wedge \mu_R^-(b)$, $\mu_R^+(ab) \le \mu_R^+(a) \wedge \mu_R^+(b)$, for every $ab \in E$.

Definition 6

By an **interval-valued fuzzy graph** of a graph $F^* = (V, E)$, we mean a pair $F = (Q, R)$, where $Q = [\mu_Q^-, \mu_Q^+]$ is an interval-valued fuzzy set on V and $R = [\mu_R^-, \mu_R^+]$ is an interval-valued fuzzy relation on E.

Definition 7

An interval – valued fuzzy graph $F = (Q, R)$ of a graph $F^* = (V, E)$ is said to be **self Weak complementary interval – valued fuzzy graph** if F is weak isomorphism with its complement $\overline{F}$, i.e., there exist a bijective homomorphism f from F to $\overline{F}$ such that for all $a, b \in V$. $\mu_Q^-(a) = \overline{\mu_Q^-}(f(a))$, $\mu_Q^+(a) = \overline{\mu_Q^+}(f(a))$ and $\mu_R^-(ab)\overline{\mu_R^-}(f(a)f(b))$, $\mu_R^+(ab) \le \overline{\mu_R^+}(f(a)f(b))$.

3. SELF WEAK COMPLEMENTARY OF A BIPOLAR AND INTERVAL – VALUED FUZZY GRAPH

Theorem 1

Let $F = (Q, R)$ be a self weak complementary bipolar fuzzy graph of a graph $F^* = (V, E)$. Then

$\mu_R^P(uv) \le \frac{1}{2}\left(\mu_Q^P(u) \wedge \mu_Q^P(v)\right)$ and

$$\mu_R^N(uv) \ge \frac{1}{2}\left(\mu_Q^N(u) \vee \mu_Q^N(v)\right)$$

Proof:

Let $F = (Q, R)$ be a self weak complementary bipolar fuzzy graph of a graph $F^* = (V, E)$. Then there exist a weak isomorphism $k : F \to \overline{F}$ such that for all $u, v \in V$. Thus

$\mu_Q^P(u) = \overline{\mu_Q^P}(k(u)) = \mu_Q^P(k(u))$ *

$\mu_Q^N(u) = \overline{\mu_Q^N}(k(u)) = \mu_Q^N(k(u))$ **

and $\mu_R^P(u) \le \overline{\mu_R^P}(k(u)k(v))$ 3.1

$\mu_R^N(u) \ge \overline{\mu_R^N}(k(u)k(v))$ 3.2

Using the definition of Complement in the above inequality, for every $u, v \in V$, we have,

$$\mu_R^P(uv) \le \overline{\mu_R^P}(k(u)k(v))$$

$$\overline{\mu_R^P}(k(u)k(v)) = \left(\mu_Q^P(k(u)) \wedge \mu_Q^P(k(v))\right) - \mu_R^P(k(u)k(v))$$

$$\mu_R^N(uv) \ge \overline{\mu_R^N}(k(u)k(v))$$

$$\overline{\mu_R^N}(k(u)k(v)) = \left(\mu_Q^N(k(u)) \vee \mu_Q^N(k(v))\right) - \mu_R^N(k(u)k(v))$$

So $\mu_R^P(uv) + \mu_R^P(k(u)k(v)) \le \left(\mu_Q^P(k(u)) \wedge \mu_Q^P(k(v))\right)$

and $\mu_R^N(uv) + \mu_R^N(k(u)k(v)) \ge \left(\mu_Q^N(k(u)) \vee \mu_Q^N(k(v))\right)$

Hence $\mu_R^P(uv) + \mu_R^P(uv) \le \mu_Q^P(u) \wedge \mu_Q^P(v)$ [∵ by (*)]

$\mu_R^N(uv) + \mu_R^N(uv) \ge \mu_Q^N(u) \vee \mu_Q^N(v)$ [∵by (**)]

$$2\,\mu_R^P(uv) \le \mu_Q^P(u) \wedge \mu_Q^P(v)$$

$$2\,\mu_R^N(uv) \ge \mu_Q^N(u) \vee \mu_Q^N(v)$$

Therefore $\mu_R^P(uv) \le \frac{1}{2}\,\mu_Q^P(u) \wedge \mu_Q^P(v)$ 3.3

$\mu_R^N(uv) \ge \frac{1}{2}\,\mu_Q^N(u) \vee \mu_Q^N(v)$ 3.4

Hence the proof.

Theorem 2

Let $F = (Q, R)$ be an interval – valued fuzzy graph of a graph $F^* = (V, E)$. If

$\mu_R^-(uv) \leq \frac{1}{2}\left(\mu_Q^-(u) \wedge \mu_Q^-(v)\right)$ and

$\mu_R^+(uv) \leq \frac{1}{2}\left(\mu_Q^+(u) \wedge \mu_Q^+(v)\right)$, for every $u, v \in V$. Then $F = (Q, R)$ is a self weak complementary interval – valued fuzzy graph.

Proof:

Let $F = (Q, R)$ be an interval – valued fuzzy graph of a graph $F^* = (V, E)$. Then there exist an identity mapping $f : V \to V$. Thus

$\mu_Q^-(u) = \mu_Q^-(f(u))$ and

$\mu_Q^+(u) = \mu_Q^+(f(u))$, for all $u \in V$.

We have to show that,

$\mu_R^-(uv) \leq \overline{\mu_R^-}(I(u)I(v))$ and

$$\mu_R^+(uv) \leq \overline{\mu_R^+}(I(u)I(v))$$

By the definition of $\overline{\mu_R}$, we have

$$\overline{\mu_R^-(uv)} = \left(\mu_Q^-(u) \wedge \mu_Q^-(v)\right) - \mu_R^-(uv)$$

$\overline{\mu_R^+(uv)} = \left(\mu_Q^+(u) \wedge \mu_Q^+(v)\right) - \mu_R^+(uv)$, for all $u, v \in V$ $\overline{\mu_R^-(uv)} \geq \left(\mu_Q^-(u) \wedge \mu_Q^-(v)\right) - \frac{1}{2}\left(\mu_Q^-(u) \wedge \mu_Q^-(v)\right)$, by 3.3

$$= \frac{1}{2}\left(\mu_Q^-(u) \wedge \mu_Q^-(v)\right)$$

$$\geq \mu_R^-(uv)$$

Thus $\mu_R^-(uv) \leq \overline{\mu_R^-}(I(u)I(v))$, by 3.1

$\overline{\mu_R^+(uv)} \geq \left(\mu_Q^+(u) \wedge \mu_Q^+(v)\right) - \frac{1}{2}\left(\mu_Q^+(u) \wedge \mu_Q^+(v)\right)$, by 3.3

$$= \frac{1}{2}\left(\mu_Q^+(u) \wedge \mu_Q^+(v)\right)$$

$$\geq \mu_R^+(uv)$$

Thus $\mu_R^+(uv) \leq \overline{\mu_R^+}(I(u)I(v))$, by 3.1

Therefore $\mu_R^-(uv) \leq \overline{\mu_R^-}(I(u)I(v))$

$\mu_R^+(uv) \leq \overline{\mu_R^+}(I(u)I(v))$, for all $u, v \in V$.

Hence the proof.

5. Conclusion

In this paper we have investigated and obtain the condition for interval – valued fuzzy graph to be self weak complementary. Further the study of interval – valued fuzzy graph can be extended with projects such as expert systems, neural networks and the shortest path in the networks.

References

1. M. Akram, W.A. Dudek, **"Interval – valued fuzzy graphs"**, Compt. Math.Appl., 2011.
2. P. Bhattacharya
3. , **"Some remarks on fuzzy graphs"**, Pattern Recognit. Lett.6, 1987.
4. Lee K.M, **"Comparision of interval – valued fuzzy sets, Intuitionstic fuzzy sets and bipolar – valued fuzzy sets"**, J. Fuzzy Logic Intell. Systems, 2004.
5. Sunil Mathew, John N. Moderson, Davender S. Malik, **"Fuzzy graph theory"**, volume 363.