GRAPHOIDAL COVERING NUMBER OF A BICYCLIC GRAPH BY GEODESIC PATH

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***Abstract*: In this paper, we introduced the concept of Geodesic graphoidal and its covering number of a graph and prolonged the view of geodesic graphoidal covering number of a bicyclic graph.**

Keywords*: *Geodesic, Bicyclic graph, Graphoidal, Covering number, shortest path.

1. INTRODUCTION

A graph covered by a set of edges or vertices is simply named as covering of a graph. Graph covering is the crucial view in graph theory. In this paper the view of geodesic graphoidal covering number of a graph was disputed. Graphoidal covers of the two cyclic graph is bicyclic graphoidal covering.B. Devadas Acharya and E. Sampath kumar were introduced the view of graphoidal covering. C. Arumugam and Suresh suseela were acquainted the view of acyclic graphoidal covering.

2. PRELIMINARIES

DEFINITION 1 :

A cyclic graph with two cycle is called ***bicyclic graph***.

DEFINITION 2:

The smallest path in a graph G is sometimes called as ***geodesic***. The diameter d(G) of a connected graph G is the length of any longest geodesic.

DEFINITION 3:

A ***graphoidal cover*** of G is a collection χ of paths in G gratifies the succeeding terms:

i) There exist minimum two vertices in each path of χ .
ii) Each vertex of G is an inner vertex which occurs only one path in χ.
iii) An edge in G occurs in one path is not occurs in another path of χ .

DEFINITION 4:

The ***graphoidal covering number*** of a graph G a is defined as the minimal cardinality of a graphoidal cover of G and it is expressed as $\delta(G)$.

$$\text{ie.}, \delta(G) = \min_{\chi \in \alpha(G)} |\chi| .$$

DEFINITION 5:

A Collection χ of shortest paths in G is surely in each path of χ having atleast 2 vertices, Each vertex of G is an inner vertex of almost one path in χ also each edge of G is definitely one path in χ is called as ***Geodesic graphoidal cover of a graph*** G.

DEFINITION 6:

The ***Geodesic graphoidal covering number*** of a graph G is defined as the minimal cardinality of a geodesic graphoidal cover of G. And it is expressed as δ_g.

THEOREM 1:

Suppose that G be the bicyclic graph contains $W(k, l)$ & both the cycles are of even length. Assume m express the counting of vertices of G also assume l express the counting of vertices of degree > 2 on $W(k, l)$. So that,

$$\delta_g = \begin{cases} 3 & \text{when } l = 0 \\ m+2 & \text{when } l = 1 \ \&\ deg(a_l) > 2, a_i = a_j \\ m+3 & \text{when } l > 1 \ \&\ \textit{each } (u,v)\text{segment of B in} \\ & \text{Which each vertices excluding } u \ \&\ v \\ & \text{have degree 2 is shortest path} \\ m+3 & \text{else} \end{cases}$$

PROOF:

Assume $V(W(k,l)) = \{a_0, a_1, \ldots\ldots, a_{k+1}, \ldots\ldots, a_{k+l-2}\}$

$V(B_k) = \{a_0, a_1, \ldots\ldots, a_{k-1}, a_0\}$.

$V(B_l) = \{a_0, a_1, a_k, \ldots\ldots, a_{k+l-2}, a_0\}$.

Here k & l are even.

Case a: When $l = 0$

Thus, $G = W(k,l)$

The geodesic graphoidal path double covering abide by,

$T_1 = \{a_j, a_{j-1}, \ldots\ldots, a_1, a_0, a_k, a_{k+1}, \ldots\ldots, a_0\}$ $\quad \left[j = \frac{k}{2} \ \&\ i = k + \frac{l}{2} - 1\right]$

$T_2 = \{a_j, a_{j+1}, \ldots\ldots, a_0\}$

$T_3 = \{a_0, a_{k+l-2}, \ldots\ldots, a_l\}$ $\quad \Rightarrow \delta_g \le 3$

Subsequently atleast 2 vertices on $W(k,l)$ are outer vertices in all minimal geodesic graphoidal cover. Thus $n \ge 2$. So that,

$\delta_g \ge s - t + 2$ $\quad \Rightarrow \delta_g \ge 3$

$\therefore \ \delta_g = 3$

Case b: When $l > 1$

Assume a_h be the similar vertex of degree > 2 on $W(k,l)$ more than a_0.

Also consider a_h lies on B_k.

Sub case (i): Let $h = \frac{k}{2}$

Assume $\mathcal{G} = G - \{a_1 a_2 \ldots\ldots a_{h-1}\}$ be the monocyclic graph having m vertices & $m = 1$.

Thus, $\delta_g(\mathcal{G}) = m + 1$.

Assume that χ' is the minimal geodesic graphoidal cover of $\mathcal{G}$.Obviously any path in χ' be the shortest path in G & then $\chi = \chi' \cup T$.

Here $T = \{a_0, a_1, \ldots\ldots, a_h\}$ be the geodesic graphoidal cover of G.

$\Rightarrow \ \delta_g(G) \le m + 2$.

For each m vertices & atleast one vertex on $W(k,l)$ be the outer point of any minimal geodesic graphoidal cover χ. Thus, $n \ge m + 1$.

So that, $\delta_g(G) = s - t + n$

$$\geq m + 2$$

$$\delta_g(G) = m + 2.$$

Sub case (ii): Let $a_h \neq a_j$.

Claim that, $h < \frac{k}{2}$

Assume that $\mathcal{G} = G - \{a_{j+1}, a_{j+2}, \ldots.., a_{k-2}\}$ be the monocyclic graph of $m + 1$ vertices & $l = 1$.

we get, $\delta_g(\mathcal{G}) = m + 2$.

Consider χ' is the minimal geodesic graphoidal cover of $\mathcal{G}$. Obviously any path in χ' be the shortest path in G & then $\chi = \chi' \cup T$

Here $T = \{a_j, a_{j+1}, \ldots.., a_{k-1}, a_0\}$ be the geodesic graphoidal cover of G.

$$\delta_g(G) \leq m + 2 + 1$$

$$= m + 3.$$

Also each vertices of G & atleast two vertices on $W(k, l)$ be the outer point of any minimal geodesic graphoidal cover χ. Thus,

$$n \geq m + 2 \quad (\text{Here } a_i \& a_j \text{ are outer points})$$

$$\delta_g(G) = s - t + n$$

$$\geq 1 + m + 2$$

$$= m + 3.$$

Case c:

Let $l > 2$ and here accurately one (u, v) segment of any of the cycles on $W(k, l)$ in all the vertices excluding u and v having degree 2 & this (u, v) segment is not a shortest path.

Assume that (u, v) segment expressed by $\left(u = a_p, a_{p+1}, \ldots \ldots, a_q = v\right)$ here $1 < p, q < \frac{k}{2}$.

Consider $\mathcal{G} = G - \{a_{j+1}, a_{j+2}, \ldots \ldots. a_{k-1}\}$. So that, G' be the monocyclic graph of $m + 1$ vertices & $l = 1$.

By case b,

$$\delta_g(\mathcal{G}) = m + 1 + 1$$

$$= m + 2.$$

Thus, $\delta_g(G) = m + 3$.

The segment (u, v) be expressed by $\left(u = a_p, a_{p+1}, \ldots \ldots, a_n = v\right)$ here $1 < p < \frac{k}{2}, 1 < n < \frac{l}{2}$ and a_p lies on B_k, U_q lies on B_l.

So that, $\mathcal{G} = G - \{a_{j+1}, a_{j+2}, \ldots \ldots. a_{k-1}\}$ be the monocyclic graph of $m + 1$ vertices & $l = 2$.Thus ,

$$\delta_g(\mathcal{G}) = m + 1$$

$$\Rightarrow \delta_g(G) = m + 2.$$

Case d:

Let $l > 2$ & where accurately one (u, v) segment of every cycle on $W(k, l)$ in all the vertices excluding u & v having degree 2 & the segment (u, v) be the shortest path.

By generalization on n,

Let $m = 2$, G holds of $W(k, l)$ & two paths.This two paths lies on some other cycles.

Obviously $T_1 = \{a_j, u_n, u_{n-1}, \dots \dots, u_1\}$ & $T_2 = \{a_i, v_n, v_{n-1}, \dots \dots, v_1\}$ here a_j on B_k and a_i on B_l.

Assume $\mathcal{G} = G - \{a_{j+1}, a_{j+2}, \dots \dots . a_{k-1}\}$ be the monocyclic graph of two vertices & $l = 2$.

Thus ,

$$\delta_g(\mathcal{G}) = 2.$$

Consider χ' is the minimal geodesic graphoidal cover of G. Obviously,any path in G and then $\chi = \chi' \cup \{a_i, a_{j+1}, \dots \dots, a_{k-1}, a_0\}$ be the minimal geodesic cover of G.

i.e., $\chi = \{(u, v)\text{segment} \cup (a_0, a_j)\text{segment} \cup (a_0, a_i)\text{segment}\}$

$$\Rightarrow \delta_g(G) = 3 = m + 1.$$

Now assuming the above result is true for all bicyclic graph containing $W(k, l)$.

Gratifying the statement of case (d) of $m - 1$ vertices with $l > 2$.

Assume G be the bicyclic graph holding $W(k, l)$ gratifying the statement of case (d) with m vertices, Here $m > 2$ with $l > 1$.

Assume $T_1 = \{a_j, u_n, u_{n-1}, \dots \dots, u_1\}$ is the path in G. Obviously, $\deg(u_1) = 1$, $\deg(u_2) = \deg(u_3) =$.......$= \deg(u_n) = 2$ & $\deg(a_j) > 2$ & T is disjoint from $W(k, l)$ if $l = 2$.

Assume $\mathcal{G} = G - \{a_1, a_2, \dots, a_t\}$ be the bicyclic graph holding $W(k, l)$.

Gratifying the statement of case (d) of $m - 1$ vertices having $m > 2$.

If each (u, v) segment of every cycles on $W(k, l)$ in G in which every vertex exclude u & v having degree 2 is the shortest path.

Thus by generalization,

$$\delta_g(G) = m - 1 + 1 = m.$$

Assume χ be the minimal geodesic graphoidal cover of $\mathcal{G}$.

So that $\chi \cup \{T\}$ is the minimal geodesic graphoidal cover of G.

$$\Rightarrow \delta_g(G) \leq m + 1.$$

In case, there is a (u, v) segment of every cycles on $W(k, l)$ claimed (a_k, a_l) segment in $\mathcal{G}$ in which every vertices excluding a_k and a_h having degree 2 & this (a_1, a_h) segment is not shortest path. Thus by case (c),

$$\delta_g(\mathcal{G}) = m + 1.$$

Assume $T = \{a_0, a_1, \dots, a_j\}$ here $1 < j < \frac{k}{2}$ is a shortest path.

Assume χ is the minimal geodesic graphoidal cover of $\mathcal{G}$ & T' is the path in χ, here a_j be the outer vertex. Assume U is the path having each edges of T' & T.

Thus,$(\chi - \{T'\}) \cup \{U\}$ be the geodesic graphoidal cover of G.

$$\Rightarrow \delta_g(G) \leq m + 1.$$

Then each of m vertices on $W(k, l)$ be the outer points of any minimal geodesic graphoidal cover χ.Thus, $n \geq m$

$$\Rightarrow \delta_g(G) = s - t + n$$

$$\therefore \delta_g(G) = m + 1.$$

Hence proved.

CONCLUSION

In this paper, a novel exposition of geodesic graphoidal cover and geodesic graphoidal covering number of a graph G has been acquainted. Furthermore, it has been proved that the declaration of the bicyclic graph for $W(k, l)$ is equal to δ_g and this can be enlarged to tricyclic graph.

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