

RP-94: Formulation of a Special Type of Standard Cubic Congruence of Composite Modulus- an Odd Multiple of Power of Two

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Abstract: In this paper, a special type of standard cubic congruence of composite modulus- an odd multiple of power of two, is considered for formulation. It was not formulated earlier. The author successfully established the formula for the solutions of the said congruence. It is found that the said congruence has exactly four solutions. This paper made it possible to find all the solutions orally. This is the merit of the paper.

Keywords: Binomial expansion, Cubic congruence, Composite modulus, Formulation.

INTRODUCTION

A congruence in an unknown x of the type: $x^3 \equiv a \pmod{m}$, m being a positive composite integer, is called a standard cubic congruence of composite modulus. Much had not been studied about this congruence in the literature of mathematics. A standard cubic congruence of composite modulus is an interesting congruence of study, failed to attract the mathematicians for its discussion. This attracted the author's attention to study the congruence. The author started his study on standard cubic congruence on composite modulus and written many papers on it. Here, the congruence under consideration is of the type: $x^3 \equiv a^3 \pmod{2^m \cdot b}$; b an odd positive integer with one more speciality: a is an even positive integer.

LITERATURE REVIEW

It is found that a standard cubic congruence of composite modulus is seldom discussed in the literature of mathematics. A very brief discussion is found in the book of Thomas Koshy [1]. Also, Zuckerman had defined a cubic congruence and a cubic residue [2]. But no detailed discussion is found. Recently, the author has written some papers on formulations of such cubic congruence and has been published in different international journals:

- 1) $x^3 \equiv a^3 \pmod{2^m 3^n}$ [3],
- 2) $x^3 \equiv a^3 \pmod{3^n \cdot b^r}$ [4],
- 3) $x^3 \equiv a^3 \pmod{2^m \cdot 3^n \cdot b^r}$ [5].

Even the author found a special type of standard cubic congruence of composite modulus yet not formulated. The author wished to formulate the said congruence and his efforts are presented in this paper.

PROBLEM-STATEMENT

Here the problem is-

To establish a suitable formula for the solutions of the special type of cubic congruence of composite modulus- an odd multiple of power of two:

$$x^3 \equiv a^3 \pmod{2^m \cdot b}; b \neq 3^n, b \text{ being an odd positive integer.}$$

ANALYSIS & RESULT

Consider the congruence: $x^3 \equiv a^3 \pmod{2^m \cdot b}; b \neq 3^n, b \text{ being an odd positive integer.}$

If $x \equiv 2^{m-2} \cdot bk + a \pmod{2^m \cdot b}, m \geq 3; k=0, 1, 2, 3, \dots$, then

$$\begin{aligned} x^3 &\equiv (2^{m-2} \cdot bk + a)^3 \\ &\equiv (2^{m-2} \cdot bk)^3 + 3 \cdot (2^{m-2} \cdot bk)^2 \cdot a + 3 \cdot (2^{m-2} \cdot bk) \cdot a^2 + a^3 \pmod{2^m \cdot b} \\ &\equiv 2^{m-2} \cdot bk \{ (2^{m-2} \cdot bk)^2 + 3 \cdot a \cdot 2^{m-2} \cdot bk + 3a^2 \} + a^3 \pmod{2^m \cdot b} \end{aligned}$$

$\equiv 2^{m-2}.bk(4t) + a^3 \pmod{2^m.b}$, if a is an even integer but b an odd positive integer.

$$\equiv a^3 \pmod{2^m.b}.$$

Therefore, it is seen that the said congruence has solutions given by

$$x \equiv 2^{m-2}.bk + a \pmod{2^m.b}; k = 0, 1, 2, 3, 4, 5, 6, \dots \dots \dots$$

But for $k = 4, 5, \dots \dots$

$$x \equiv 2^{m-2}.b.4 + a \equiv 2^m b + a \equiv a \pmod{2^m b},$$

which is same as for $k=0,1, \dots \dots \dots$

Therefore, it can be said that the said congruence has exactly four solutions for $k = 0, 1, 2, 3$.

Sometimes the cubic congruence may be of the type: $x^3 \equiv d \pmod{2^m.b}$.

It can be written as:

$$x^3 \equiv d + k.2^m.b = a^3 \pmod{2^m.b} \text{ for a fixed } k, a \text{ positive integer [3].}$$

ILLUSTRATIONS

Consider the congruence $x^3 \equiv 216 \pmod{5600}$.

It can be written as $x^3 \equiv 6^3 \pmod{2^5.175}$

It is of the type $x^3 \equiv a^3 \pmod{2^m.b}$

with $m = 5, b = 175$, an odd integer; $a = 6$, an even positive integer.

By author's formulation, it has exactly four solutions given by

$$\begin{aligned} x &\equiv 2^{m-2}.bk + a \pmod{2^m.b} \\ &\equiv 2^{5-2}.175k + 6 \pmod{2^5.175} \\ &\equiv 2^3.175k + 6 \pmod{32.175} \\ &\equiv 8.175k + 6 \pmod{5600} \end{aligned}$$

$$\equiv 1400k + 6 \pmod{5600}; k = 0, 1, 2, 3.$$

$$\equiv 6, 1406, 2806, 2406 \pmod{5600}.$$

These are the required solutions.

Consider one more example $x^3 \equiv 64 \pmod{1960}$.

It can be written as $x^3 \equiv 4^3 \pmod{2^3.245}$

It is of the type $x^3 \equiv a^3 \pmod{2^m.b}$

with $m = 3, b = 245$, an odd integer; $a = 4$, an even positive integer.

By author's formulation, it has exactly four solutions given by

$$\begin{aligned} x &\equiv 2^{m-2}.bk + a \pmod{2^m.b} \\ &\equiv 2^{3-2}.245k + 4 \pmod{2^3.245} \\ &\equiv 2^2.245k + 4 \pmod{8.245} \\ &\equiv 4.245k + 4 \pmod{1960} \end{aligned}$$

$$\equiv 490k + 4 \pmod{1960}; k = 0, 1, 2, 3.$$

$$\equiv 4, 494, 984, 1474 \pmod{1960}.$$

These are the required solutions.

Also consider the congruence $x^3 \equiv 104 \pmod{112}$.

It can be written as $x^3 \equiv 104 + 112 = 216 = 6^3 \pmod{112}$.

It can also be written as $x^3 \equiv 6^3 \pmod{2^4 \cdot 7}$.

It is of the type $x^3 \equiv a^3 \pmod{2^m \cdot b}$ with $a = 6, b = 7, m = 4$.

The solutions are $x \equiv 2^{m-2} \cdot bk + a \pmod{2^m \cdot b}$ with $k = 0, 1, 2, 3$.

$$i. e. x \equiv 2^{4-2} \cdot 7k + 6 \pmod{2^4 \cdot 7}$$

$$i. e. x \equiv 4 \cdot 7k + 6 \pmod{112}$$

$$i. e. x \equiv 28k + 6 \pmod{112}; k = 0, 1, 2, 3.$$

$$\equiv 6, 34, 62, 90 \pmod{112}.$$

These are the required solutions.

CONCLUSION

Thus, it can be concluded that the standard cubic congruence of the type

$$x^3 \equiv a^3 \pmod{2^m \cdot b}; b \neq 3^n, b \text{ odd positive integer, and}$$

a even positive integer, has exactly four solutions given by

$$x \equiv 2^{m-2} \cdot bk + a \pmod{2^m \cdot b}; m \geq 3; k = 0, 1, 2, 3.$$

MERIT OF THE PAPER

In this paper, a special type of cubic congruence of the type:

$$x^3 \equiv a^3 \pmod{2^m \cdot b}; b \text{ odd positive integer, } a \text{ even positive integer,}$$

is formulated. A formula for all the solutions is established. The solutions can also be obtained orally. This is the merit of the paper.

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