RP-98: Formulation of Solutions of a Special Standard Cubic Congruence of Composite Modulus

Prof. B M Roy

Head, Department of Mathematics
Jagat Arts, Commerce & I H P Science College, Goregaon
Dist: Gondia, M. S., INDIA, Pin: 441801
(Affiliated to R T M Nagpur University, Nagpur)

***Abstract*: In this paper, the author considered a standard cubic congruence of composite modulus for formulation of its solutions. The author's efforts has provided a formulation for solutions of the said congruence. The discovered formulae are tested true by solving different examples and by its verifications. Such formulation was not considered by earlier mathematicians and no effective method is found to find its solutions. Formulation is the merit of the paper. It is proved time-saving, easy and simple. Sometimes the solutions can be obtained orally. This is one more merit of the paper. This made the study of cubic congruence interested.**

***Keywords*: Binomial cubic expansion, Cubic Congruence, Composite Modulus, Formulation.**

INTRODUCTION

A standard cubic congruence of composite modulus is a congruence of the type: $x^3 \equiv a^3 \ (mod\ m)$; m being a composite positive integer. Its solutions are the values of x that satisfies the congruence. Some standard cubic congruence has unique solutions; some has three solutions; some has four solutions and some has $p\ or\ p^2$ solutions; p being a positive prime integer. Also, some of the congruence have infinite number of solutions.

.LITERATURE REVIEW

The author has gone through many books of Number Theory but failed to find any discussion on it. Earlier mathematicians showed a very little interest to study the congruence. Hence no formulation or any discussion is found. The author intentionally started his research on standard cubic congruence and successfully discovered formulations of standard cubic congruence of prime and composite modulus [1], [2], [3], [4], [5].

Now he wishes to consider one special type of standard cubic congruence of composite modulus for formulation.

NEED OF RESEARCH

Finding no discussion and any formulation for the said congruence in the literature of mathematics, the author showed his eagerness to make it an interested topic of study and tried his best to formulate the congruence. His efforts of establishment of the formulation are presented in this paper. This is the need of the research.

PROBLEM- STATEMENT

Here the problem is-

"To formulate two standard cubic congruence of composite modulus of the types

(1) $\boldsymbol{x^3 \equiv a^3\ (mod\ a^m); m \geq 4}$;

(2) $\boldsymbol{x^3 \equiv a^3\ (mod\ a^m); m \leq 3}$ ".

ANALYSIS & RESULTS

Case-I:

For the congruence: $\boldsymbol{x^3 \equiv a^3\ (mod\ a^m); m \geq 4.}$

If $x \equiv a^{m-2}k + a\ (mod\ a^m)$, then

$$x^3 \equiv (a^{m-2}k + a)^3$$

$$\equiv (a^{m-2}k\ )^3 + 3.(a^{m-2}k)^2.a + 3.a^{m-2}k.a^2 + a^3\ \ (mod\ a^m)$$

$$\equiv\ a^{m-2}k\{\ (a^{m-2}k)^2 + 3a.a^{m-2}k + 3a^2\} + a^3\ \ (mod\ a^m)$$

$$\equiv\ a^{m-2}k\{\ a^2t\} +\ a^3\ \ (mod\ a^m),$$

$$\equiv a^m kt + a^3 \ (mod \ a^m)$$

$$\equiv a^3 \ (mod \ a^m).$$

Thus, it can be said that $x \equiv a^{m-2}k + a \ \ (mod \ a^m)$ is a solution of it for some

$k = 0, 1, 2, \dots \dots . a^2 - 1, a^2, a^2 + 1, \dots \dots \dots$

It can also be seen that if $k = a^2, a^2 + 1, \dots \dots$, $x \equiv a^{m-2}. a^2 + a = a^m + a \equiv a \ (mod \ a^m)$

It is the same solution as for $k = 0,1, \dots \dots \dots \dots, a^2 - 1.$

Thus, the said congruence has a^2 incongruent solutions.

Case-II

For the congruence $x^3 \equiv a^3 \ (mod \ a^m); m \leq 3; \ a \ any \ Integer.$

It is equivalent to $x^3 \equiv 0 \ (mod \ a^m); m \leq 3.$

For its solutions, let us consider $x = at, for \ a \ positive \ integer \ t.$

Then, $x^3 = (at)^3 = a^3. t^3 \equiv 0 \ (mod \ a^3).$

Thus, it can be seen that $x^3 \equiv 0 \ (mod \ a^3)$ or $x^3 \equiv 0 \ (mod \ a^2)$ or $x^3 \equiv 0 \ (mod \ a).$

So, $x \equiv at \ (mod \ a^m) \ with \ m = 1, 2, 3$ are the solutions of the said congruence. It is also seen that the above congruence have the same solutions $x = at$ for $m = 1,2,3; t \ a \ positive$

$integer.$

ILLUSTRATIONS OF CASE-I

Consider the congruence $x^3 \equiv 2^3 \ (mod \ 2^5)$

It is of the type $x^3 \equiv a^3 \ (mod \ a^m) \ with \ a = 2, m = 5.$ It has $2^2 = 4$ solutions.

These solutions are given by $x \equiv a^{m-2}k + a \ (mod \ a^m)$

$$\equiv 2^{5-2}k + 2 \ (mod \ 2^5)$$

$$\equiv 2^3 k + 2 \ (mod \ 2^5)$$

$$\equiv 8k + 2 \ (mod \ 32)$$

$$\equiv 2, 10, 18, 26 \ (mod \ 32).$$

These solutions are also verified true.

Consider one more example $x^3 \equiv 4^3 \ (mod \ 4^4).$

It is of the type $x^3 \equiv a^3 \ (mod \ a^m) \ with \ a = 4, m = 4.$ It has $4^2 = 16$ solutions.

These solutions are given by $x \equiv a^{m-2}k + a \ (mod \ a^m)$

$$\equiv 4^{4-2}k + 4 \ (mod \ 4^4)$$

$$\equiv 4^2 k + 4 \ (mod \ 4^4)$$

$$\equiv 16k + 4 \ (mod \ 256)$$

$$\equiv 4, 20, 36, 52, 68, 84, 100, 116, 132, 148, 164, 180,$$

$$196, 212; 228, 244 \ (mod \ 256).$$

These solutions are also verified true.

Consider one more congruence $x^3 \equiv 5^3 \ (mod \ 5^6).$

It is of the type $x^3 \equiv a^3 \ (mod \ a^m) \ with \ a = 5, m = 6.$ It has $5^2 = 25$ solutions.

These solutions are given by $x \equiv a^{m-2}k + a \ (mod \ a^m)$

$$\equiv 45^{6-2}k + 5 \ (mod\ 5^6)$$

$$\equiv 5^4 k + 5 \ (mod\ 5^6)$$

$$\equiv 625k + 5 \ (mod\ 15625)$$

$$\equiv 5, 630, 1255, 1880, 2505, 3130, 3755, 4380, 5005,$$

$$5630, 6255, 6880, 7505, 8130, 8755, 9380, 10005,$$

$$10630, 11255, 11880, 12505, 13130, 13755, 14380,$$

$$15005 \ (mod\ 15625).$$

These solutions are also verified true.

ILLUSTRATIONS OF CASE-II

Consider the congruence $x^3 \equiv 4^3 \ (mod\ 4^3)$. It is equivalent to $x^3 \equiv 0 \ (mod\ 4^3)$.

It is of the type $x^3 \equiv 0 \ (mod\ a^3)\ with\ a = 4$.

Its solutions are given by $x \equiv at = 4t \ (mod\ 4^3)$, for a positive integer t.

Therefore, the required solutions are $x \equiv 4, 8, 12, 16, \dots \dots (mod\ 64)$.

If one consider the congruence $x^3 \equiv 4^3 \ (mod\ 4^2)$. It is equivalent to $x^3 \equiv 0 \ (mod\ 4^2)$.

It is of the type $x^3 \equiv 0 \ (mod\ a^2)$. Its solutions are given by $x \equiv at = 4t \ (mod\ 4^2)$, for a positive integer t.

Therefore, the required solutions are $x \equiv 4, 8, 12, 16, \dots \dots (mod\ 16)$.

CONCLUSION

Thus it can be concluded that the said congruence $x^3 \equiv a^3 \ (mod\ a^m); m \geq 4$, a positive integer has a^2incongruent solutions given by

$x \equiv a^{m-2}k + a \ (mod\ a^m); k = 0, 1, 2, \dots \dots, a^2 - 1$.

But the congruence $x^3 \equiv 0 \ (mod\ a^m); m \leq 3$ has $solutions$

$x \equiv at \ (mod\ a^m)\ with\ m = 1, 2, 3\ ; t\ any\ positive\ integer$.

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