Performance Measures of a Repairable System with Multiple Units using Regenerative Point Technique and Semi-Markov Process

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***Abstract*- The primary objective of this paper is to evaluate the performance measures of a repairable system having two non-identical units (main and duplicate units) with the concept of change in environmental conditions. Initially, the main unit is operative whereas the duplicate unit is at cold standby. Single server visits the failed unit immediately for conducting repair. The expressions for some important reliability measures such as MTSF, Availability and Profit of the system model have been derived.**

***Index Terms*- Performance measures, Repairable system, Environmental conditions.**

I. INTRODUCTION

In view of the frequent and vital use in management and industrial sectors, the repairable systems of two or more identical units have been investigated stochastically in detail by the scholars including Gopalan and Naidu (1984), Goel and Sharma (1989) and Singh (1989) under strict control of environment conditions such as pollution, moisture, voltage, climate, temperature and other natural catastrophic. But in case of high cost of identical units, the non-identical unit (may be a substandard unit) might be kept as spare in cold standby not only to improve the reliability of the system but also to maintain performance of the system in emergency, as referred by Mokkadis et. al. (1989). Each unit is capable of performing the same kind of functions but their degree of reliability and desirability may differ from unit to unit. Deswal et al. (2013) discussed standby systems of non-identical units with different failure and repair policies. Also, sometimes it becomes very difficult to keep the environmental conditions under control which may fluctuate due changing climate and other natural catastrophic. While considering this fact in mind, Rajni (2013) and Barak (2019) and Kumar et. al. (2019), obtained reliability analysis of a cold standby system under different weather conditions. Further, the performance measures of cold standby repairable systems of non-identical units under different weather conditions have not been studied so far by the researchers in the field of reliability.

Hence, in the present Paper, a repairable system of two non-identical units – one is original (called main unit) and other is a substandard unit (called duplicate unit) has been analyzed stochastically in detail under two weather conditions – normal and abnormal. The environmental conditions when satisfied to the system correspond to normal weather; otherwise, it is supposed that the system is in abnormal weather. Initially, the system is operative with main unit and duplicate unit is kept a spare in cold standby. Both units have direct complete failure from normal mode. Each unit is capable of performing the same set of functions with different degree of reliability and desirability. There is a single server who visits the system immediately whenever needed to do repair of the failed unit in normal weather only. The operation and repair of the units are not allowed in abnormal weather as a precautionary measure to avoid excessive damage to the system. However, operation and repair of the units are as usual in normal weather. The units work as new after repair.

The distributions of failure time of the units and change of weather conditions follow negative exponential while that of repair times of the units are taken as arbitrary. All random variables are statistically independent. The switch devices and repairs are perfect. The expressions for various measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server and profit function in steady state are derived using semi-Markov process and regenerative point technique. The numerical results giving particular values to the parameters and various costs are obtained for mean time to system failure (MTSF), availability and profit to depict their tabular behavior. The application of the present work can be visualized in a software industry where application software is run through two different databases-one is initially operative and other is kept in cold standby.

II. NOTATIONS

E	The set of regenerative states

MO/DO	Main/Duplicate unit is good and operative
$\overline{MWO}$ / $\overline{DWO}$	Main/Duplicate unit is good and waiting for operation in abnormal weather
MCs/DCs	Main/Duplicate unit is in cold standby mode
$\overline{MCs}$ / $\overline{DCs}$	Main/Duplicate unit is in cold standby mode in abnormal weather
λ/λ_1	Constant failure rate of Main /Duplicate unit
β/β_1	Constant rate of change of weather from normal to abnormal/abnormal to normal weather
MFur/DFur	Main/duplicate unit failed and under repair
MFUR/DFUR	Main/duplicate unit failed and under repair continuously from previous state
MFwr/DFwr	Main/duplicate unit failed and waiting for repair
MFWR/DFWR	Main/duplicate unit failed and waiting for repair continuously from previous state
$\overline{MFwr}$ / $\overline{DFwr}$	Main/Duplicate unit failed and waiting for repair due to abnormal weather
$\overline{MFWR}$ / $\overline{DFWR}$	Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather
g(t)/G(t)	pdf/cdf of repair time of Main unit
$g_1(t)/G_1(t)$	pdf/cdf of repair time of Duplicate unit
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]
$q_{ij.kr}(t)/Q_{ij.kr}(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]
$q_{ij.k,(r,s)}{}^{n}(t)/Q_{ij.k,(r,s)}{}^{n}(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.
$M_i(t)$	Probability that the system is up initially in regenerative state Si at time t without visiting to any other regenerative state
$W_i(t)$	Probability that the server is busy in state Siupto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
m_{ij}	The conditional mean sojourn time in regenerative state Si when system is to make transition in to regenerative state Sj. Mathematically, it can be written as $$m_{ij} = E(T_{ij}) = \int_0^\infty t\,d[Q_{ij}(t)] = -q_{ij}^{*\prime}(0),$$ where T_{ij} is the transition time from state Si to Sj; Si, Sj ε E.
μ_i	The mean Sojourn time in state Si this is given by $$\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij},$$ where T_i is the sojourn time in state Si.
Ⓢ/©/$©^n$	Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution n times
** / *	Symbol for Laplace Steiltjes Transform (L.S.T.)/ Laplace transform (L.T.)
´(desh)	Used to represent alternative result

The following are the possible transition states of the system

S_0 = (MO, DCs), S_1 = (MFur, DO), S_2 = ($\overline{MWO}$, $\overline{DCs}$), S_3= ($\overline{MFwr}$, $\overline{DWO}$),

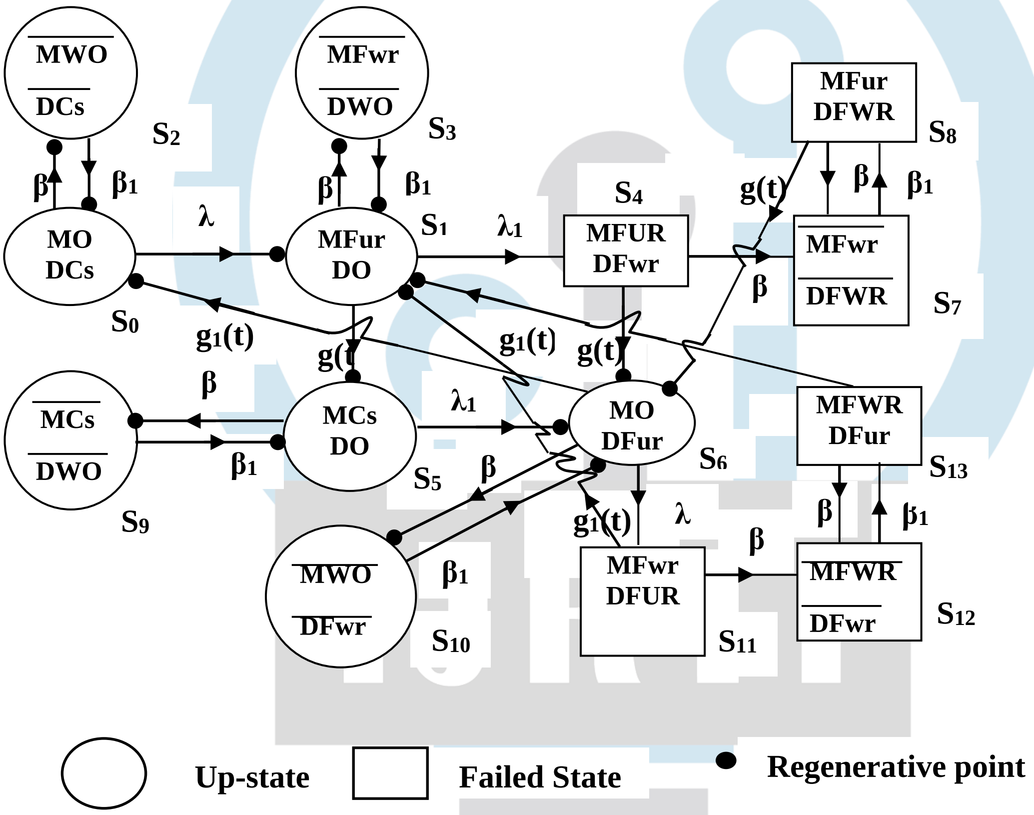
S_4= (MFUR, DFwr), S_5= (MCs, DO), S_6= (MO, DFur), S_7= ($\overline{MFwr}$, $\overline{DFWR}$),

S_8=(MFur,DFWR),S_9=($\overline{MCs}$, $\overline{DWO}$),S_{10}=($\overline{MWO}$, $\overline{DFwr}$),

S_{11}=(MFwr,DFUR) , S_{12}=($\overline{\text{MFWR}}$, $\overline{\text{DFwr}}$) ,S_{13}=(MFWR,DFur)

The states S_0, S_1, S_2, S_3, S_5, S_6, S_9 and S_{10} are regenerative while the states S_4, S_7, S_8, S_{11}, S_{12} and S_{13} are non-regenerative as shown in below figure

Transition Diagram of the Proposed Model



III. TRANSITION PROBABLILTIES AND MEAN SOJOURN TIMES

The differential transition probabilities are:

$dQ_{01}(t)=\lambda e^{-(\lambda+\beta)t}dt$, $dQ_{02}(t)=\beta e^{-(\lambda+\beta)t}dt$, $dQ_{13}(t)=\beta e^{-(\beta+\lambda_1)t}\,\overline{G(t)}\,dt$,

$dQ_{14}(t)=\lambda_1 e^{-(\beta+\lambda_1)t}\,\overline{G(t)}\,dt$, $dQ_{15}(t)=g(t)\,e^{-(\beta+\lambda_1)t}\,dt$, $dQ_{20}(t)=\beta_1 e^{-\beta_1 t}\,dt$,

$dQ_{31}(t)=\beta_1 e^{-\beta_1 t}\,dt$, $dQ_{46}(t)=g(t)e^{-\beta t}dt$, $dQ_{47}(t)=\beta e^{-\beta t}\,\overline{G(t)}\,dt$, $dQ_{56}(t)=\lambda_1 e^{-(\beta+\lambda_1)t}\,dt$, $dQ_{59}(t)=\beta e^{-(\beta+\lambda_1)t}\,dt$, $dQ_{60}(t)=g_1(t)e^{-(\beta+\lambda)t}dt$, $dQ_{6,10}(t)=\beta e^{-(\beta+\lambda)t}\,\overline{G_1(t)}\,dt$, $dQ_{6,11}(t)=\lambda e^{-(\beta+\lambda)t}\,\overline{G_1(t)}\,dt$, $dQ_{78}(t)=\beta_1 e^{-\beta_1 t}\,dt$, $dQ_{86}(t)=g(t)\,e^{-\beta_1 t}\,dt$,

$dQ_{87}(t)=\beta_1 e^{-\beta_1 t}\,\overline{G(t)}\,dt$, $dQ_{95}(t)=\beta_1 e^{-\beta_1 t}\,dt$, $dQ_{10,6}(t)=\beta_1 e^{-\beta_1 t}\,{}^{t}dt$,

$dQ_{11,1}(t)=g_1(t)\, e^{-\beta t}dt$, $dQ_{11,12}(t)= \beta e^{-\beta t}\, \overline{G_1(t)}\, dt$, $dQ_{12,13}(t)= \beta_1 e^{-\beta_1 t}\, dt$,

$dQ_{13,1}(t)=g_1(t)\, e^{-\beta t}dt$, $dQ_{13,12}(t)= \beta e^{-\beta t}\, \overline{G_1(t)}\, dt$

…(1)

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$, we have

$p_{01}=\frac{\lambda}{\beta+\lambda}$, $p_{02}=\frac{\beta}{\beta+\lambda}$, $p_{13}=\frac{\beta}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{14}=\frac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{15}=g^*(\beta+\lambda_1)$,

$p_{20}=1$, $p_{31}=1$, $p_{46}=g^*(\beta)$, $p_{47}=1-g^*(\beta)$, $p_{56}=\frac{\lambda_1}{\beta+\lambda_1}$ $p_{59}=\frac{\beta}{\beta+\lambda_1}$, $p_{60}=g^*_1(\beta+\lambda)$,

$p_{6,10} = \frac{\beta}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{6,11}=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{78}=1$ $p_{86}=g^*(\beta)$,

$p_{87}=1-g^*(\beta)$, $p_{95}=1$, $p_{10,6}=1$, $p_{11,1}=g_1^*(\beta)$, $p_{11,12}=g_1^*(\beta)$ $p_{12,13}=1$, $p_{13,1}=g^*_1(\beta)$,

$p_{13,12}=1-g^*_1(\beta)$, $p_{16.4}=\frac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))g^*(\beta)$, $p_{16.4(7,8)}{}^{n}=\frac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))(1-g^*(\beta))$,

$p_{61.11}=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))g_1^*(\beta)$, $p_{61.11,(12,13)}{}^{n}=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))(1-g_1^*(\beta))$

…(2)

It can be easily verified that

$p_{01}+p_{02}=p_{13}+p_{16.4}+p_{16.4(7,8)}{}^{n}+p_{15}=p_{20}=p_{31}=p_{46}+p_{47}=p_{56}+p_{59}=1$

$p_{60}+p_{6,10}+p_{61.11}+p_{61.11(12,13)}{}^{n}=p_{78}=p_{86}+p_{87}=p_{95}=p_{10,6}=p_{11,1}+p_{11,12}=p_{12,13}=p_{13,1}+p_{13,12}=1$

…(3)

The mean sojourn times (μ_i) in the state S_i are

$\mu_0= m_{01}+m_{02}=\frac{1}{\beta+\lambda}$, $\mu_1= m_{13}+m_{14}+m_{15}=\frac{1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $\mu_2 = m_{20}=\frac{1}{\beta_1}$,

$\mu_3= m_{31}=\frac{1}{\beta_1}$, $\mu_4= m_{46}+m_{47}=\frac{1}{\beta}(1-g^*(\beta)$, $\mu_5= m_{56}+m_{59}=\frac{1}{\beta+\lambda_1}$,

$\mu_6= m_{60}+m_{6,10}+m_{6,11}=\frac{1}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$,

$\mu_7= m_{78}=\frac{1}{\beta_1}$ $\mu_8= m_{86}+m_{87}=\frac{1}{\beta}(1-g^*(\beta))$, $\mu_9= m_{95}=\frac{1}{\beta_1}$, $\mu_{10}= m_{10,6}=\frac{1}{\beta_1}$,

$\mu_{11}= m_{11,1}+m_{11,12}=\frac{1}{\beta}(1-g_1^*(\beta))$, $\mu_{12}= m_{12,13}=\frac{1}{\beta_1}$, $\mu_{13}= m_{13,1}+m_{13,12}=\frac{1}{\beta}(1-g^*_1(\beta))$,

$\mu'_1=m_{13}+m_{15}+m_{16.4}+m_{16.4,(7,8)}{}^{n}=\frac{(1-g^*(\beta+\lambda_1))(\beta\beta_1 g^*(\beta)+\lambda_1(\beta+\beta_1)(1-g^*(\beta)))}{\beta\beta_1(\beta+\lambda_1)g^*(\beta)}$,

$\mu'_6=m_{60}+m_{61.11}+m_{61.11,(12,13)}{}^{n}+m_{6,10}=\frac{(1-g_1^*(\beta+\lambda))(\beta\beta_1 g_1^*(\beta)+\lambda(\beta+\beta_1)(1-g_1^*(\beta)))}{\beta\beta_1(\beta+\lambda)g_1^*(\beta)}$

…(4)

IV. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\emptyset_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$\emptyset_0(t) = Q_{01}(t)Ⓢ\emptyset_1(t) + Q_{02}(t)Ⓢ\emptyset_2(t)$

$\emptyset_1(t) = Q_{13}(t)Ⓢ\emptyset_3(t) + Q_{15}(t)Ⓢ\emptyset_5(t) + Q_{14}(t)$

$\emptyset_2(t) = Q_{20}(t)Ⓢ\emptyset_0(t)$

$\emptyset_3(t) = Q_{31}(t)Ⓢ\emptyset_1(t)$

$\emptyset_5(t) = Q_{56}(t)Ⓢ\emptyset_6(t) + Q_{59}(t)Ⓢ\emptyset_9(t)$

$\emptyset_6(t) = Q_{60}(t)Ⓢ\emptyset_0(t) + Q_{6,10}(t)Ⓢ\emptyset_{10}(t) + Q_{6,11}(t)$
$\emptyset_9(t) = Q_{95}(t)Ⓢ\emptyset_5(t)$
$\emptyset_{10}(t) = Q_{10,6}(t)Ⓢ\emptyset_6(t)$
…(5)

Taking L.S.T. of above relations (5) and solving for $\emptyset_0^{**}(s)$, we get

$$\phi_0^{**}(s) = \frac{Q_{01}^{**}(s)Q_{13}^{**}(s)(1-Q_{6,10}^{**}(s)Q_{10,6}^{**}(s))(1-Q_{59}^{**}(s)Q_{95}^{**}(s)) + Q_{01}^{**}(s)Q_{15}^{**}(s)Q_{56}^{**}(s)Q_{6,11}^{**}(s)}{(1-Q_{02}^{**}(s)Q_{20}^{**}(s))(1-Q_{13}^{**}(s)Q_{31}^{**}(s))(1-Q_{6,10}^{**}(s)Q_{10,6}^{**}(s))(1-Q_{59}^{**}(s)Q_{95}^{**}(s)) - Q_{01}^{**}(s)Q_{15}^{**}(s)Q_{56}^{**}(s)Q_{60}^{**}(s)}$$

…(6)
we have

$$R^*(s) = \frac{1-\phi_0^{**}(s)}{s}$$

…(7)
The reliability of the system can be obtained by taking inverse Laplace transform of (7).
The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N_1}{D_1}$$

…(8)
where
$N_1=p_{56}(1-p_{6,10})((1-p_{13})(\mu_0+p_{02}\mu_2)+p_{01}(\mu_1+p_{13}\mu_3))+p_{01}p_{15}((1-p_{6,10})(\mu_5+p_{59}\mu_9)+p_{56}(\mu_6+p_{6,10}\mu_{10}))$
$D_1=p_{01}p_{56}((1-p_{6,10})(1-p_{13})-p_{15}p_{60})$
…(9)

V. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are given as:

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$
$A_1(t) = M_1(t) + q_{13}(t)©A_3(t) + q_{15}(t) © A_5(t) + (q_{16.4}(t)+q_{16.4,(7,8)}^{n}(t)) ©A_6(t)$
$A_2(t) = q_{20}(t)©A_0(t)$
$A_3(t)=q_{31}(t)©A_1(t)$
$A_5(t)=M_5(t)+q_{56}(t)©A_6(t)+q_{59}(t)©A_9(t)$
$A_6(t) = M_6(t)+q_{60}(t)©A_0(t)+(q_{61.11}(t)+q_{61.11(12,13)}^{n}(t))©A_1(t)+q_{6,10}(t)©A_{10}(t)$
$A_9(t)=q_{95}(t)©A_5(t)$
$A_{10}(t)=q_{10,6}(t)©A_6(t)$
…(10)

where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$M_0(t)=e^{-(\beta+\lambda)t}$, $M_1(t)=e^{-(\beta+\lambda_1)t}\overline{G(t)}$, $M_5(t)= e^{-(\beta+\lambda_1)t}$, $M_6(t)= e^{-(\beta+\lambda)t}\overline{G_1(t)}$

…(11)
Taking L.T. of above relations (10) and (11) and solving for $A_0^*(s)$,we have

$$A_0^*(s)=\frac{\begin{gathered}(M_0^*(s)(1-q_{13}^*(s)q_{31}^*(s))+q_{01}^*(s)M_1^*(s)(1-q_{6,10}^*(s)q_{10,6}^*(s))(1-q_{59}^*(s)q_{95}^*(s))\\ -(q_{61.11}^*(s)+q_{61.11(12,13)^n}^*(s))M_0^*(s)(q_{15}^*(s)q_{56}^*(s)+(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))\\ (1-q_{59}^*(s)q_{95}^*(s)))+q_{01}^*(s)q_{15}^*(s)(1-q_{6,10}^*(s)q_{10,6}^*(s))M_5^*(s)+\\ q_{56}^*(s)M_6^*(s))(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))q_{01}^*(s)(1-q_{59}^*(s)q_{95}^*(s))M_6^*(s)\end{gathered}}{\begin{gathered}(1-q_{02}^*(s)q_{20}^*(s))(1-q_{13}^*(s)q_{31}^*(s))(1-q_{59}^*(s)q_{95}^*(s))(1-q_{6,10}^*(s)q_{10,6}^*(s))\\ -(q_{61.11}^*(s)+q_{61.11(12,13)^n}^*(s))(1-q_{02}^*(s)q_{20}^*(s))(q_{15}^*(s)q_{56}^*(s)+(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))\\ (1-q_{59}^*(s)q_{95}^*(s))-q_{01}^*(s)q_{15}^*(s)q_{56}^*(s)q_{60}^*(s)-q_{01}^*(s)q_{60}^*(s))\\ (1-q_{59}^*(s)q_{95}^*(s))(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))\end{gathered}} \quad \ldots(12)$$

The steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s)=\frac{N_2}{D_2}$$

…(13)

$N_2=p_{56}(1-p_{13})(p_{60}\mu_0+p_{01}\mu_6)+p_{01}(1-p_{6,10})(p_{56}\mu_1+p_{15}\mu_5)$

$D_2=p_{56}(1-p_{13})(p_{60}\mu_0+p_{01}\mu_6'))+p_{01}(1-p_{6,10})(p_{56}\mu_1'+p_{15}\mu_5)$

…(14)

VI. BUSY PERIOD ANALYSIS OF THE SERVER

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state S_i at t=0.The recursive relations for $B_i(t)$ are as follows:

$B_0(t)=q_{01}(t)$ © $B_1(t)+q_{02}(t)$ © $B_2(t)$

$B_1(t)=W_1(t)+q_{13}(t)$©$B_3(t)+q_{15}(t)$ © $B_5(t)+(q_{16.4}(t)+q_{16.4,(7,8)}^n(t))$ ©$B_6(t)$

$B_2(t)=q_{20}(t)$©$B_0(t)$

$B_3(t)=q_{31}(t)$©$B_1(t)$

$B_5(t)=q_{56}(t)$©$B_6(t)+q_{59}(t)$©$B_9(t)$

$B_6(t)=W_6(t)+q_{60}(t)$©$B_0(t)+(q_{61.11}(t)+q_{61.11,(12,13)}^n(t))$©$B_1(t)+q_{6,10}(t)$©$B_{10}(t)$

$B_9(t)=q_{95}(t)$©$B_5(t)$

$B_{10}(t)=q_{10,6}(t)$©$B_6(t)$

…(15)

where $W_i(t)$ be the probability that the server is busy in state S_i due to failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative states so,

$W_1(t)=e^{-(\beta+\lambda_1)t}\,\overline{G(t)}\ +(\lambda_1 e^{-(\beta+\lambda_1)t}$ ©1) $\overline{G(t)}$,$W_6(t)=e^{-(\beta+\lambda)t}\,\overline{G_1(t)}\ +(\lambda e^{-(\beta+\lambda)t}$©1) $\overline{G_1(t)}$

…(16)

Taking L.T. of above relations (15) and (16) and solving for $B_0^*(s)$.We obtain

$$B_0^*(s)=\frac{\begin{gathered}W_1^*(s)q_{10}^*(s)(1-q_{6,10}^*(s)q_{10,6}^*(s))(1-q_{59}^*(s)q_{95}^*(s))+\\ q_{01}^*(s)W_6^*(s)(q_{15}^*(s)q_{56}^*(s)+(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))(1-q_{59}^*(s)q_{95}^*(s)))\end{gathered}}{\begin{gathered}(1-q_{02}^*(s)q_{20}^*(s))(1-q_{13}^*(s)q_{31}^*(s))(1-q_{59}^*(s)q_{95}^*(s))(1-q_{6,10}^*(s)q_{10,6}^*(s))\\ -(q_{61.11}^*(s)+q_{61.11(12,13)^n}^*(s))(1-q_{02}^*(s)q_{20}^*(s))(q_{15}^*(s)q_{56}^*(s)+(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))\\ (1-q_{59}^*(s)q_{95}^*(s))-q_{01}^*(s)q_{15}^*(s)q_{56}^*(s)q_{60}^*(s)-q_{01}^*(s)q_{60}^*(s))\\ (1-q_{59}^*(s)q_{95}^*(s))(q_{16.4}^*(s)+q_{16.4(7,8)^n}^*(s))\end{gathered}} \quad \ldots(17)$$

The time for which server is busy due to repair is given by

$$B_0^*(\infty)=\lim_{s\to 0} sB_0^*(s)=\frac{N_3}{D_2}$$

…(18)

where
$N_3=p_{01}p_{56}(W_1{*}(0)(1-p_{6,10})+W_6{*}(0)(1-p_{13}))$ and D_2 is already defined
…(19)

VII. EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $N_i(t)$ are given as

$N_0(t) =Q_{01}(t)Ⓢ(1+N_1(t)) + Q_{02}(t) ⓈN_2(t)$
$N_1(t) =Q_{13}(t)ⓈN_3(t)+ Q_{15}(t) ⓈN_5(t) +Q_{16.4}(t) ⓈN_6(t)+Q_{16.4,(7,8)}{}^n(t))Ⓢ(1+N_6(t))$
$N_2(t)=Q_{20}(t)ⓈN_0(t)$
$N_3(t)=Q_{31}(t)Ⓢ(1+N_1(t))$
$N_5(t)=Q_{56}(t)Ⓢ(1+N_6(t))+Q_{59}(t)ⓈN_9(t)$
$N_6(t) =Q_{60}(t)ⓈN_0(t)+Q_{61.11}(t)ⓈN_1(t)+Q_{61.11,(12,13)}{}^n(t)Ⓢ(1+N_1(t))+Q_{6,10}(t)ⓈN_{10}(t)$
$N_9(t)=Q_{95}(t)ⓈN_5(t)$
$N_{10}(t)=Q_{10,6}(t)Ⓢ(1+N_6(t))$
…(20)

Taking L.S.T. of relations (20) and solving for $N_0^{**}(s)$,we get

$$N_0^{**}(s)=\frac{\begin{array}{l}Q_{01}^{**}(s)((1+Q_{16.4(7,8)^n}^{**}(s))(1-Q_{6,10}^{**}(s)Q_{10,6}^{**}(s))(1-Q_{59}^{**}(s)Q_{95}^{**}(s))\\-(Q_{61.11}^{**}(s)+Q_{61.11(12,13)^n}^{**}(s))(Q_{15}^{**}(s)Q_{56}^{**}(s)+(Q_{16.4}^{**}(s)+Q_{16.4(7,8)^n}^{**}(s))\\(1-Q_{59}^{**}(s)Q_{95}^{**}(s))Q_{15}^{**}(s)Q_{56}^{**}(s)(1+Q_{61.11(12,13)^n}^{**}(s))+(1-Q_{59}^{**}(s)Q_{95}^{**}(s))\\(Q_{16.4}^{**}(s)+Q_{16.4(7,8)^n}^{**}(s))(Q_{6,10}^{**}(s)Q_{10,6}^{**}(s)+Q_{61.11(12,13)^n}^{**}(s))\end{array}}{\begin{array}{l}(1-Q_{02}^{**}(s)Q_{20}^{**}(s))(1-Q_{13}^{**}(s)Q_{31}^{**}(s))(1-Q_{6,10}^{**}(s)Q_{10,6}^{**}(s))(1-Q_{59}^{**}(s)Q_{95}^{**}(s))\\-(Q_{61.11}^{**}(s)+Q_{61.11(12,13)^n}^{**}(s))(1-Q_{02}^{**}(s)Q_{20}^{**}(s))(Q_{15}^{**}(s)Q_{56}^{**}(s)+\\(Q_{16.4}^{**}(s)+Q_{16.4(7,8)^n}^{**}(s))(1-Q_{59}^{**}(s)Q_{95}^{**}(s))-Q_{01}^{**}(s)Q_{15}^{**}(s)Q_{56}^{**}(s)Q_{60}^{**}(s)\\-Q_{01}^{**}(s)Q_{60}^{**}(s)(1-Q_{59}^{**}(s)Q_{95}^{**}(s))(Q_{16.4}^{**}(s)+Q_{16.4(7,8)^n}^{**}(s))\end{array}} \quad \text{…(21)}$$

The expected numbers of visits per unit time by the server are given by

$$N_0(\infty)=\lim_{s\to 0} sN_0^{**}(s)=\frac{N_4}{D_2}$$

…(22)
where
$N_4=p_{01}p_{56}((1-p_{6,10})(1+p_{15}-p_{14}p_{47})-(1-p_{13})p_{6,11}p_{11,1})$ and D_2 is already specified.
…(23)

VIII. PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as
$P=K_0A_0-K_1B_0-K_2N_0$
where
K_0=Revenue per unit up-time of the system
K_1=Cost per unit for which server is busy
K_2= Cost per unit visit by the server and A_0, B_0, N_0 are already defined.

IX. PARTICULAR CASE

Suppose $g(t)=\alpha e^{-\alpha t}$,$g_1(t)=\alpha_1\, e^{-\alpha_1 t}$

By using the non-zero elements pij, we can obtain the following results:

$p_{01}=\dfrac{\lambda}{\beta+\lambda}$, $p_{02}=\dfrac{\beta}{\beta+\lambda}$, $p_{13}\dfrac{\beta}{\alpha+\beta+\lambda_1}$, $p_{14}\dfrac{\lambda_1}{\alpha+\beta+\lambda_1}$, $p_{15}=\dfrac{\alpha}{\alpha+\beta+\lambda_1}$, $p_{20}=1$, $p_{31}=1$,

$p_{46}=\dfrac{\alpha}{\alpha+\beta}$, $p_{47}=\dfrac{\beta}{\alpha+\beta}$, $p_{56}=\dfrac{\lambda_1}{\beta+\lambda_1}$, $p_{59}=\dfrac{\beta}{\beta+\lambda_1}$, $p_{60}=\dfrac{\alpha_1}{\alpha_1+\beta+\lambda}$, $p_{6,10}=\dfrac{\beta}{\alpha_1+\beta+\lambda}$,

$p_{6,11}=\dfrac{\lambda}{\alpha_1+\beta+\lambda}$, $p_{78}=1$, $p_{86}=\dfrac{\alpha}{\alpha+\beta}$, $p_{87}=\dfrac{\beta}{\alpha+\beta}$, $p_{95}=1$, $p_{10,6}=1$, $p_{11,1}=\dfrac{\alpha_1}{\alpha_1+\beta}$,

$p_{11,12}=\dfrac{\beta}{\alpha_1+\beta}$, $p_{12,13}=1$, $p_{13,1}=\dfrac{\alpha_1}{\alpha_1+\beta}$, $p_{13,12}=\dfrac{\beta}{\alpha_1+\beta}$, $p_{16.4}=\dfrac{\lambda_1}{\alpha+\beta+\lambda_1}\;\dfrac{\alpha}{\alpha+\beta}$

$p_{16.4(7,8)}{}^{n}=\dfrac{\lambda_1}{\alpha+\beta+\lambda_1}\;\dfrac{\beta}{\alpha+\beta}$, $p_{61.11}=\dfrac{\lambda}{\alpha_1+\beta+\lambda}\;\dfrac{\alpha_1}{\alpha_1+\beta}$,

$p_{61.11,(12,13)}{}^{n}=\dfrac{\lambda}{\alpha_1+\beta+\lambda}\;\dfrac{\beta}{\alpha_1+\beta}$

$\mu_0=\dfrac{1}{\beta+\lambda}$, $\mu_1=\dfrac{1}{\alpha+\beta+\lambda_1}$, $\mu_2=\dfrac{1}{\beta_1}$, $\mu_3=\dfrac{1}{\beta_1}$, $\mu_4=\dfrac{1}{\alpha+\beta}$, $\mu_5=\dfrac{1}{\beta+\lambda_1}$, $\mu_6=\dfrac{1}{\alpha_1+\beta+\lambda}$,

$\mu_7=\dfrac{1}{\beta_1}$, $\mu_8=\dfrac{1}{\alpha+\beta}$, $\mu_9=\dfrac{1}{\beta_1}$, $\mu_{10}=\dfrac{1}{\beta_1}$, $\mu_{11}=\dfrac{1}{\alpha_1+\beta}$, $\mu_{12}=\dfrac{1}{\beta_1}$, $\mu_{13}=\dfrac{1}{\alpha_1+\beta}$

$\mu_1'=\dfrac{\alpha\beta_1+\beta\lambda_1+\beta_1\lambda_1}{\alpha\beta_1(\alpha+\beta+\lambda_1)}$, $\mu_6'=\dfrac{\alpha_1\beta_1+\beta\lambda+\beta_1\lambda}{\alpha_1\beta_1(\alpha_1+\beta+\lambda)}$

$W^*_1(0)=\dfrac{(\alpha+\lambda_1)}{\alpha(\alpha+\beta+\lambda_1)}$, $W^*_6(0)=\dfrac{(\alpha_1+\lambda)}{\alpha_1(\alpha_1+\beta+\lambda)}$

MTSF (T_0) = $\dfrac{N_1}{D_1}$, Steady state availability (A_0) = $\dfrac{N_2}{D_2}$,

Busy period of the server (B_0) = $\dfrac{N_3}{D_2}$,

Expected number of visits by the server (N_0) = $\dfrac{N_4}{D_2}$

where

$N_1=(\beta+\beta_1)(\lambda_1(\alpha_1+\lambda)(\alpha+\lambda+\lambda_1)+\alpha\lambda(\alpha_1+\lambda+\lambda_1))$

$D_1=\lambda\lambda_1\beta_1(\alpha\lambda+\lambda_1(\alpha_1+\lambda))$

$N_2=\alpha\alpha_1\beta_1(\alpha+\lambda_1)(\lambda_1+\lambda)(\alpha_1+\lambda))$

$D_2=\alpha\lambda_1(\alpha+\lambda_1)(\alpha_1^2\beta_1+\lambda(\alpha_1\beta_1+\beta\lambda+\beta_1\lambda))+\alpha_1\lambda(\alpha_1+\lambda)(\lambda_1(\alpha\beta_1+\beta\lambda_1+\beta_1\lambda_1)+\alpha^2\beta_1)$

$N_3=\lambda\lambda_1\beta_1(\alpha+\lambda_1)(\alpha_1+\lambda)(\alpha+\alpha_1)$

$$N_4=\frac{\alpha\alpha_1\beta_1\lambda\lambda_1((\alpha_1+\lambda)(\alpha_1+\beta)((\alpha+\beta)(2\alpha+\beta+\lambda_1)-\beta\lambda_1)-\lambda\alpha_1(\alpha+\lambda_1)(\alpha+\beta))}{(\alpha+\beta)(\alpha_1+\beta)}$$

X. TABLES

Table 1: MTSF vs. Normal Weather Rate (β_1)

Normal Weather Rate(β_1)	α=2,α_1=2.5, β=0.01, λ=0.5,λ_1=0.6	α=1.5	α_1=2	β=0.05	λ=0.3	λ_1=0.4
1.1	11.02792208	9.734759	10.42727	11.42532	16.99522	15.77851
1.2	11.01964286	9.727451	10.41944	11.38393	16.98246	15.76667
1.3	11.01263736	9.721267	10.41282	11.3489	16.97166	15.75664
1.4	11.00663265	9.715966	10.40714	11.31888	16.96241	15.74805
1.5	11.00142857	9.711373	10.40222	11.29286	16.95439	15.74061

1.6	10.996875	9.707353	10.39792	11.27009	16.94737	15.73409
1.7	10.99285714	9.703806	10.39412	11.25	16.94118	15.72834
1.8	10.98928571	9.700654	10.39074	11.23214	16.93567	15.72323
1.9	10.98609023	9.697833	10.38772	11.21617	16.93075	15.71866
2.0	10.98321429	9.695294	10.385	11.20179	16.92632	15.71455

Table 2: Availability vs. Normal Weather Rate (β_1)

Normal Weather Rate (β_1)	$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$	$\alpha=1.5$	$\alpha_1=2$	$\beta=0.05$	$\lambda=0.3$	$\lambda_1=0.4$
1.1	0.952288701	0.93391	0.944042	0.950654	0.969052	0.967458
1.2	0.952322813	0.933956	0.944081	0.950824	0.969075	0.967482
1.3	0.952351679	0.933995	0.944115	0.950968	0.969094	0.967502
1.4	0.952376422	0.934029	0.944144	0.951091	0.96911	0.967519
1.5	0.952397867	0.934058	0.944169	0.951198	0.969124	0.967534
1.6	0.952416632	0.934083	0.94419	0.951292	0.969137	0.967547
1.7	0.952433191	0.934106	0.94421	0.951375	0.969148	0.967558
1.8	0.95244791	0.934126	0.944227	0.951448	0.969157	0.967569
1.9	0.95246108	0.934144	0.944242	0.951514	0.969166	0.967578
2	0.952472933	0.93416	0.944256	0.951573	0.969174	0.967586

Table 3: Profit vs. Normal weather rate (β_1)

Normal Weather Rate (β_1)	$K_0=5000, K_1=350, K_2=300$ $\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$	$\alpha=1.5$	$\alpha_1=2$	$\beta=0.05$	$\lambda=0.3$	$\lambda_1=0.4$
1.1	4577.52164	4476.754344	4531.372	4564.752	4704.575	4693.31
1.2	4577.68561	4476.97648	4531.562	4565.568	4704.684	4693.425
1.3	4577.824362	4477.164458	4531.724	4566.259	4704.776	4693.522
1.4	4577.9433	4477.325595	4531.862	4566.851	4704.856	4693.605
1.5	4578.046384	4477.465256	4531.981	4567.365	4704.924	4693.677
1.6	4578.136586	4477.587467	4532.086	4567.814	4704.985	4693.74
1.7	4578.21618	4477.695306	4532.178	4568.211	4705.038	4693.796
1.8	4578.286932	4477.791167	4532.261	4568.563	4705.085	4693.845
1.9	4578.350238	4477.876941	4532.334	4568.879	4705.127	4693.89
2	4578.407215	4477.95414	4532.4	4569.163	4705.165	4693.929

XI. CONCLUSION

To make the study more concrete, the numerical results giving particular values to the various parameters and cost are obtained to depict the behavior of Mean Time to System Failure (MTSF), availability and profit functions as shown respectively in Table 1, 2 and 3. It is revealed that MTSF decreases with the increase in normal weather rate (β_1) and failure rates (λ, λ_1) of the units. And, it increases with increase of abnormal weather rate (β) and repair rates (α, α_1) of the units. The results show that availability and profit of the system model keep on increasing as normal weather rate (β_1) and repair rates (α, α_1) increase while their values decline with the increase of abnormal weather rate (β) and failure rates (λ, λ_1) of the units. Thus, on the basis of the results obtained for a particular case, it is interpreted that a system of non-identical units which is not allowed to operate in abnormal weather conditions can be made more available and profitable to use either by providing normal weather for operation or by providing better repair facilities like calling server of high repair rates (may be called as expert server).

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