Influence of Cohesive Soil on Stability of Vertical Cuts

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***Abstract*: Tension crack development is one of the major causes of failure, especially for vertical cuts. The field of stability analysis is at its peak but still the effect of tension cracks are still needs to be explored and examined. To understand the behavior of earth in such cases, a set of analytical results for the stability of homogeneous vertical cuts has been presented. In this paper, Using limit equilibrium analysis of slices, it is tried to provide general solution for the true collapse values considering general shear stress parameter and drained condition of soil . For the purpose of this research Morgenstern-Price method of slip surface analysis based on SLOPE/W model simulation is used and comparison is made between the decreases in safe depth of vertical cuts with tension crack with respect to that of without them.**

***Keywords*: Vertical cut, Stability, Earth pressure, Limit Equilibrium, Depth, Width, and Regression Equation.**

Introduction

The presence of tension crack is always increase chances of failure of a vertical cut. Also the safe depth that cut is reduced. Hence it is necessary to estimate the depth of tension crack and the depth to which is vertical cut is made. The importance of tension crack and their effect on stability is always duly emphasis in soil mechanics and in vertical these may extend to considerable depth and can be predicted and understood by using C and Φ values.

Earlier researchers tries to investigate the effect of cracks on stability of slopes used of limit equilibrium methods in their original form (e.g. Spencer, 1968; Kaniraj & Abdullah, 1993), or based on variation in computation (e.g. Baker, 1981, 2003). Others considered tension cracks in undrained conditions (e.g. Baker & Leshchinsky, 2003). Latest works involves finite-element upper-bound limit analyses, where the presence of a crack is at well defined location. (Antao et al., 2008).

Formulation of problem

The field of stability analysis is at its peak but still the effect of tension cracks are still needs to be explored and examined. This paper is concerned only with vertical tension cracks, since this is an initial step in the understanding of the behavior, and with all the relevant literature regarding this area is confined to the case of vertical cracks.

In reference to some researchers about lower-bound analyses (e.g. Terzaghi, 1943; Baker, 2003; Antao et al., 2008), that considered a limited tensile strength for the soil, the maximum crack depth is fixed. Whereas, tension is one of the possible reasons of cracks, and there are experimental evidence that shows that cracks can be caused and/or deepened by many factors , such as continuous drying and wetting (Konrad & Ayad, 1997), removal of water (Dyer et al., 2009; Peron et al., 2009) and weathering (Hales & Roering, 2007).

In the following, the vertical slice method of limit analysis is employed to investigate the stability of uniform Cohesion(C) and Angle of internal friction (Φ), vertical cuts are subject to cracks of any possible depth and location. Three different types of problems will be dealt with

1. Determination of the critical failure depth of cuts for vertical cut with a crack which occurs for cohesive soils using SLOPE/W
2. Determination depth of tension crack
3. Determination of the safe top width in case of cracks.

Theory and Approach to Simulation

For sophisticated analyses, soil, and loading conditions, limit equilibrium computer programs are generally used to perform the computations. Computer programs are available that can handle a wide variety of slip surface geometry, soil stratigraphy, soil shear strength and internal soil reinforcement. Computing software such as SLOPE/W are tools to simulate the situation that is to be analyzed which uses certain pre-defined methods that should be judged before presenting it as a final solution.

In order, to understand the behavior of vertical cut of C – Φ soil using computing software, it is important to understand way to address for a general solution Slip Surface Stability Concept.

General Limit Equilibrium Method

The method of general limit equilibrium (GLE) formulation was developed by Fredlund at the University of Saskatchewan in the 1970's (Fredlund and Krahn 1977). This method comprises of the key elements of all the methods. In GLE formulation technique there are two factors of safety equations and allows for a range of interslice shear-normal force conditions.

1. The GLE factor of safety (FOS) equation with respect to moment equilibrium (F_m) is:

$$FOS = \frac{\sum \quad (C'\beta R + (N - u\,\beta)R \;\; tan\, tan\, \Phi')}{\sum \quad Wx - \sum \quad Nf \pm \sum \quad Dd}$$

2. The factor of safety (FOS) equation with respect to horizontal force equilibrium (F_f) is:

$$FOS = \frac{\sum \quad (C'\beta \cos\alpha + (N - u\beta) \tan \tan \Phi' \cos\alpha)}{\sum \quad N \sin\alpha - \sum \quad D \cos\omega}$$

The terms in the equations are:
C' = effective cohesion
Φ' = effective angle of friction
u = pore-water pressure
N = slice base normal force
W = slice weight
D = concentrated point load
β, R, x, f, d, ω = geometric parameters
α = inclination of slice base

One of the basic variables in the equations is N, the normal at the base of each slice. The following equation is obtained by the considering equilibrium of vertical forces.

$$N = \frac{W + (Xr - Xl) - \frac{\sum \quad (C'\beta \sin\alpha + u\beta \tan \tan \Phi' \sin\alpha)}{FOS}}{\cos \cos \alpha + \frac{\sin\alpha \tan\Phi'}{FOS}}$$

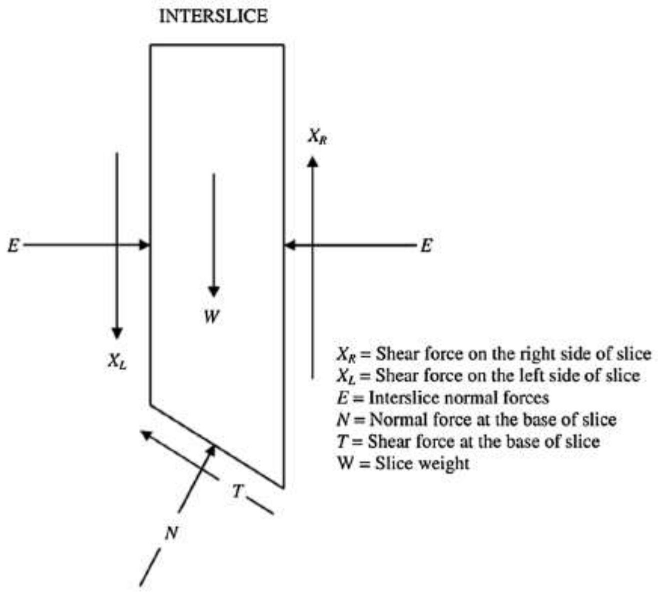

Figure 1: General force diagram of slice in SLOPE/W.

The concept of factor of safety for moment and forces was initially introduced by Spencer (1967). The interslice shear forces in the GLE technique are handled with an equation developed by Morgenstern and Price (1965).
The equation is:
X = E λ F(x)
Where, F(x) = a interslice force function,
λ = the percentage (in decimal form) of the function used,
E = the interslice normal force, and
X = the interslice shear force.

F is F_m when N is substituted into the moment factor of safety equation and F is F_f when N is substituted into the force factor of safety equation. The literature on slope stability analysis often refers to the denominator of this equation as m. A basic point is that the normal at the base of slice is dependent on the interslice shear forces Xr and Xl on each side of a slice.

Interslice Force Function and λ in SLOPE/W
The GLE method is beneficial for observing and for finding the interslice force functions and their influence in computed factor of safety, also after completion of analysis there is only one factor of safety. Hence in SLOPE/W, F_m and F_f are the same that is both moment and force equilibrium are satisfied and same value for in the equation for the normal at the slice base. This means the factor of safety is the equal for each and every slice. This has an important effect on the computed stress distributions within the sliding mass and along the slip surface. Another important point about the GLE method is that it is not restricted by the shape of the slip surface.

Morgenstern-Price Method

Morgenstern-Price (M-P) factors of safety are obtained at the point where the factor of safety satisfies both moment and force equilibrium. The M-P method can utilize any general appropriate function and SLOPE/W can accommodate a wide range of different interslice force functions, but for this research work half sine function is assumed.

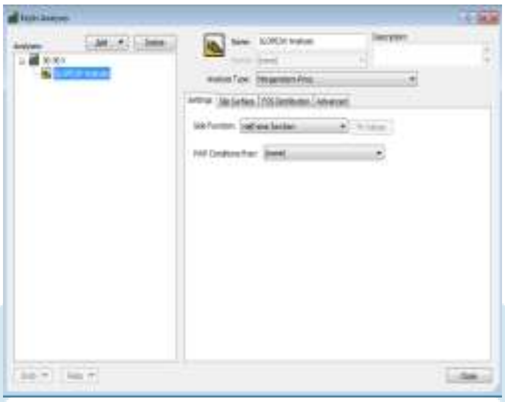

Figure 2: Key In dialog box and half sine function used in SLOPE/W.

Shear Stress Parameter

In concern with this research work following Mohr-Coulomb concept is utilized by SLOPE/W and considering various values of C and Φ modeling and research has been conducted. Before the numerical simulation of vertical cuts, maximum and minimum values of cohesion (C) and angle of friction (Φ) that can give realistic solution from the software are researched so as to reach a general solution for field and these properties are given in table 1

Table 1: Realistic values of C and Φ of soil available in nature. (Geotechdata, 2013)

Properties	Maximum	USCS Classification	Minimum	USCS Classification
C	50 kPa	SM,SC	10 kPa	MH
Φ	50˚	GS	10˚	Pt

SLOPE/W is very simple programming tool that can be used to model vertical slip surfaces. Therefore to understand the concept of Slip surfaces in vertical cuts to generate a relation between depth and shear stress parameter, SLOPE/W is found to work well. The results obtained from SLOPE/W are in good agreement with the results form Bell's Relation of earth Pressure. To study the depth and top width (as geometrical parameter) and C-Φ relation Drained condition with no tension cracks have been considered for the present study

Tension Crack Search

To analyze models with consideration tension crack, SLOPE/W is allowed for automatic search for cracks in tension at the slice base in the crest area. At the end of analysis, SLOPE/W checks all the slices and examines that if any negative slice base normals in the crest area. Initial slice with a negative normal is exempted from the simulation, and then iterations for analysis are done until there are no further negative base normals.

Simulation Results of Vertical Cuts with Tension Cracks using SLOPE/W

The detailed model of vertical cut setup for tension cracks in GLE software SLOPE/W is generated And here shear stress parameter i.e. Cohesion (kPa) and angle of internal friction (˚) is taken as dependent variable and depth of vertical cut is taken as dependent variable. For the purpose to understand the true behavior of slip surface under tension crack , factor of safety is kept close to and greater than 1 and such that its range is always 1 to 1.1

Values of Safe Depth of Cuts (FOS > 1) for Various Values of C (kPa) and Φ

Table 2: Variation of safe depth (m) for various values of C (kPa) and Φ considering tension cracks.

Φ / **C**	10˚	20˚	30˚	40˚	50˚
10	**1.8**	**2.4**	**2.9**	**3.5**	**4.3**
20	**3.2**	**4.3**	**5.7**	**6.4**	**9.5**
30	**5.2**	**6.5**	**7.9**	**9.2**	**12.3**
40	**6.5**	**8**	**9.9**	**12.8**	**16.1**
50	**9.7**	**11.5**	**13.3**	**17.2**	**22.1**

The variation of the depth with tension cut with reference to C and Φ is plotted in fig 3 and 4

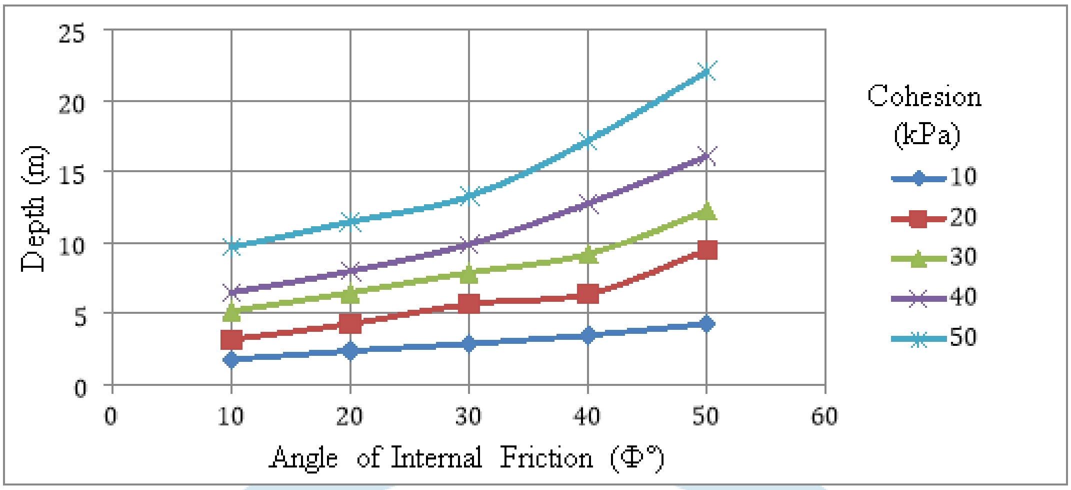

Figure 3: Variation of depth Vs angle of internal friction with respect to cohesion with tension crack by SLOPE/W.

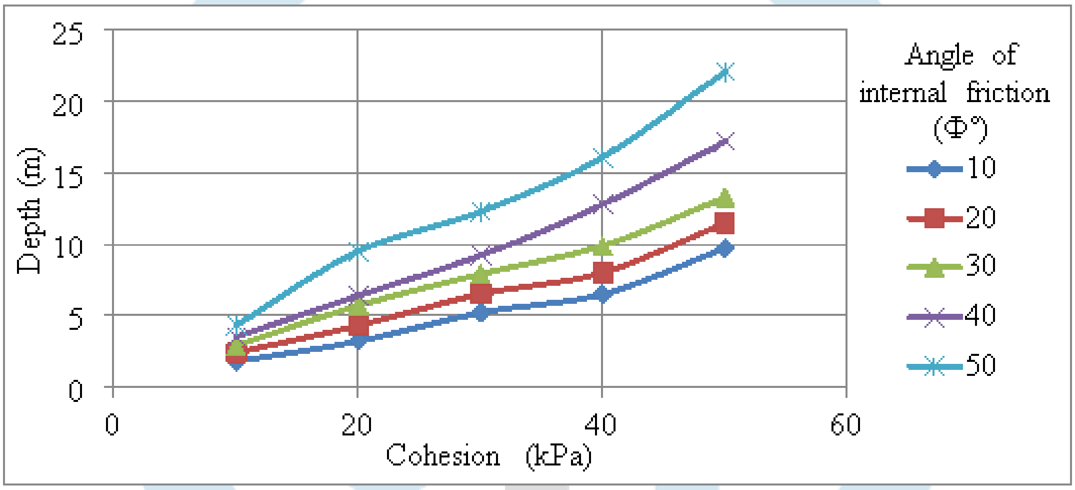

Figure 4: Variation of depth Vs cohesion with respect to angle of internal friction with tension crack By SLOPE/W.

Change in Safe Depths between With and Without Tension Crack

The depth of tension crack is well established and the variation is as follows

Table 3: Safe depth (m) for various values of C (kPa) and Φ (˚) without considering tension cracks

C \ **Φ**	10˚	20˚	30˚	40˚	50˚
10	**2.7**	**3.2**	**3.9**	**5.5**	**7.3**
20	**5.4**	**6.3**	**8.1**	**9.6**	**12.6**
30	**8.1**	**9.3**	**11.2**	**14.8**	**18.3**
40	**10.4**	**12.7**	**15.3**	**18.9**	**24.5**
50	**13.5**	**16.5**	**19**	**23.7**	**29.8**

Table 4: Change in safe depth (m) for various values of C (kPa) and Φ considering with and without tension cracks.

C \ **Φ**	10˚	20˚	30˚	40˚	50˚
10	**0.9**	**0.8**	**1**	**2**	**3**
20	**2.2**	**2**	**2.4**	**3.2**	**3.1**
30	**2.9**	**2.8**	**3.3**	**5.6**	**6**
40	**3.9**	**4.7**	**5.4**	**6.1**	**8.4**
50	**3.8**	**5**	**5.7**	**6.5**	**7.7**

The variation of the change in depths between with and without tension cut with reference to C and Φ is plotted in figure 5 and 6.

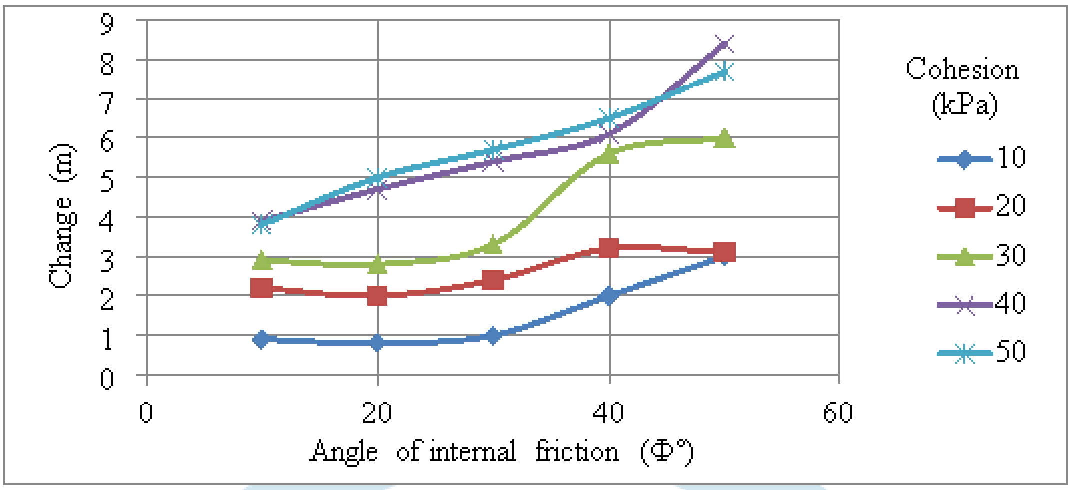

Figure 5: Variation of change in depths between with and without tension cut Vs angle of internal friction with respect to cohesion in the both the condition.

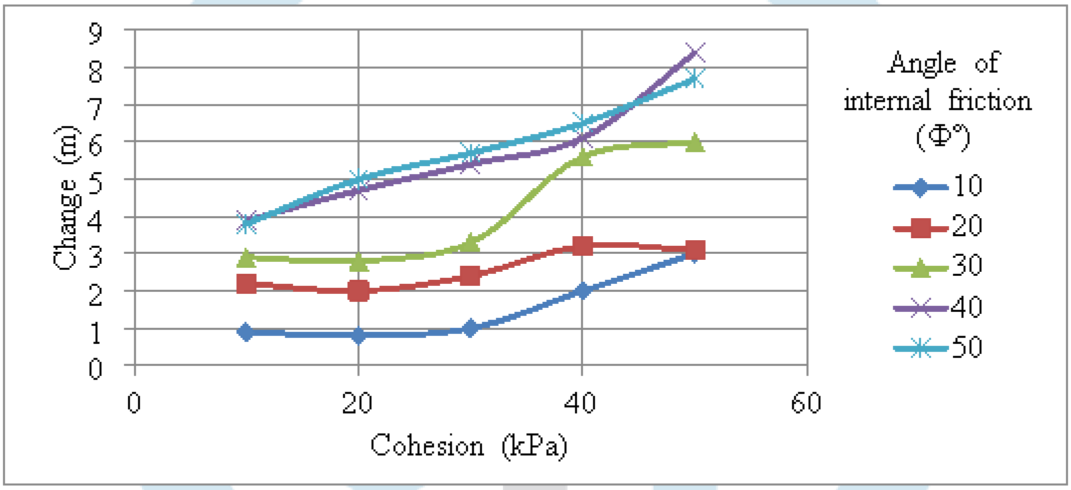

Figure 6: Variation of change in depths between with and without tension cut Vs cohesion with respect to angle of internal friction in the condition.

Values of Depth of Tension Crack for Various Values of C and Φ.

Table 5: Values of depth (m) of tension crack for various values of C (kPa) and Φ (˚).

Φ / **C**	10˚	20˚	30˚	40˚	50˚
10	**0.6**	**0.8**	**1**	**1.3**	**1.9**
20	**2**	**2.2**	**2.4**	**3.3**	**4.5**
30	**3**	**3.6**	**3.8**	**4.8**	**6**
40	**4**	**4.7**	**5**	**6**	**7.1**
50	**4.6**	**5.1**	**5.4**	**6.6**	**7.7**

The variation in depth of tension crack with reference to C and Φ is shown in figure 7 and 8.

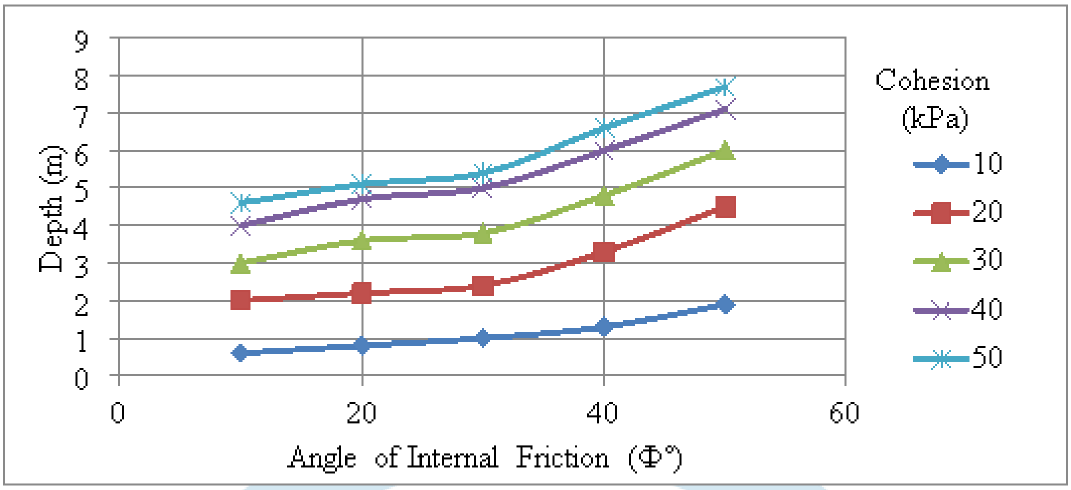

Figure 7: Variation of depth of tension crack Vs angle of internal friction with respect to cohesion by SLOPE/W.

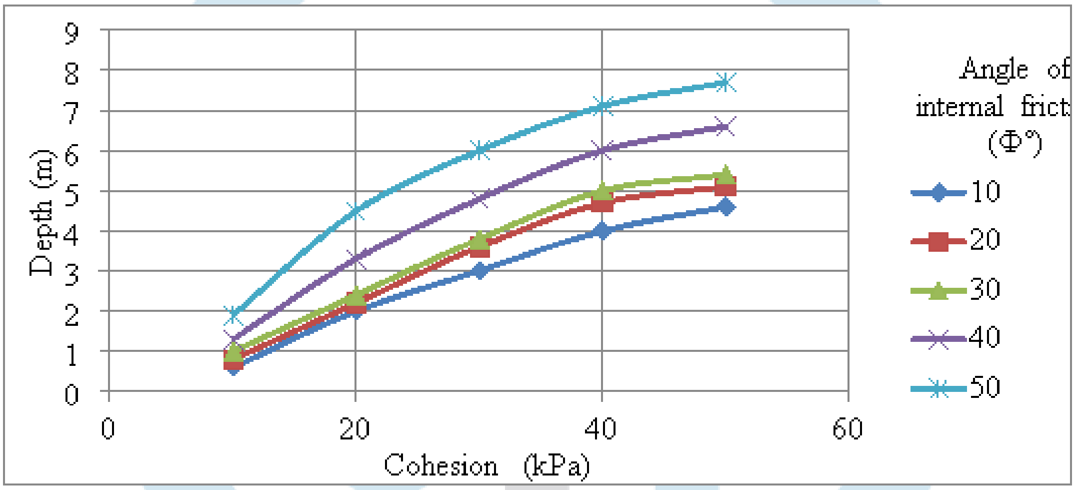

Figure 8: Variation of depth of tension Crack Vs cohesion with respect to angle of internal friction By SLOPE/W.

Similar graphical variation of depth with respect to C and Φ is observed as that in case of with tension cracks, but change in depths are very significant, also it becomes evident that with increase of Φ depth varies parabolically and similarly with increase in C depth varies linearly. Hence, it can be concluded that Morgenstern-Price method also supports the fact that tension crack makes still unstable and these should be considered while analyzing.

Values of Safe Width at Top (FOS > 1) For Various Values of C and Φ

Table 6: Values of safe width (m) with respect to C (kPa) and Φ (˚) by SLOPE/W.

C \ **Φ**	10˚	20˚	30˚	40˚	50˚
10	**1**	**1.6**	**1.3**	**1.2**	**0.9**
20	**1.1**	**1.7**	**2.2**	**1.6**	**2.5**
30	**2.2**	**1.6**	**2**	**1.6**	**2.7**
40	**3**	**2.1**	**2.7**	**3.2**	**3.6**
50	**6.4**	**5.9**	**5.6**	**6.2**	**7.8**

The variation of the safe width at top with reference to C and Φ in case of tension crack is plotted under in figure 9 and 10.

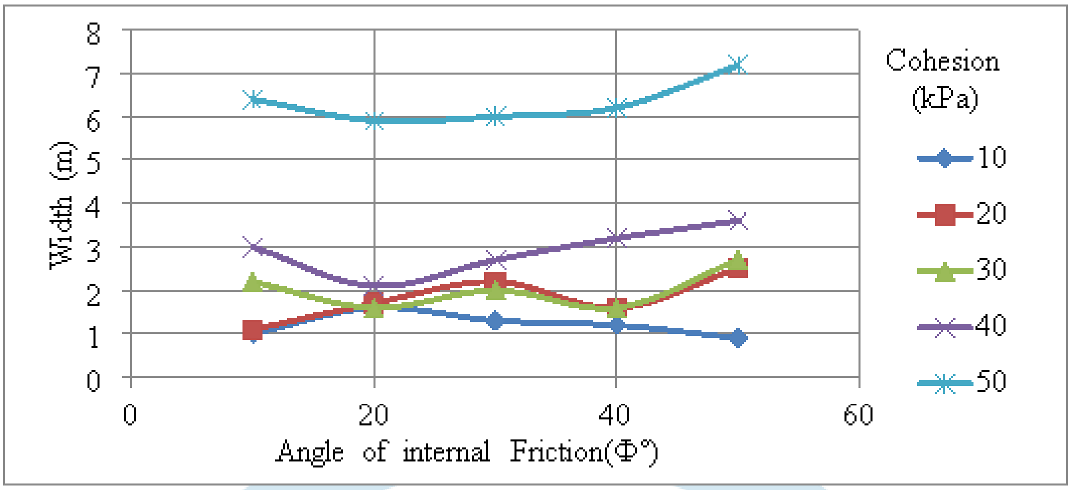

Figure 9: Variation of safe width at top Vs angle of internal friction with respect to cohesion by SLOPE/W in case of tension Crack.

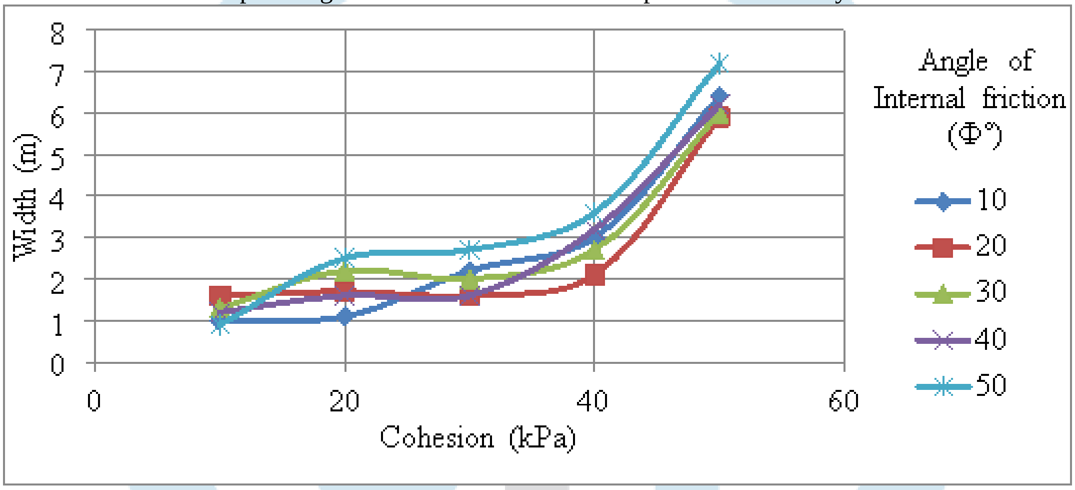

Figure 10: Variation of safe width at top Vs cohesion with respect to angle of internal friction by SLOPE/W in case of tension Crack.

Graphical variation of safe width at the top is similar to that in case of without tension cracks. This is due to the fact that soil after tension crack behaves like that of without them.

Vertical Cuts with Tension Cracks

After analyzing regression results, it has been observed that for depths of vertical cuts with tension crack and height of tension crack with respect to internal angle of friction and cohesion single curvilinear regression analysis gives better results, where as for safe width multiple linear regression show conservative results for most of the cases .

Correlation between Depths of Cuts with Angle of Internal Friction

The resultant regression analysis after correlating of height of cuts with cohesion and angle of internal friction is expressed by following equations with its corresponding correlation coefficient

Table 7: Various equation for depth (m) considering tension cracks with varying cohesion (kPa) and respective R^2.

C (kPa)	Equation	R^2
10	$H_t = 0.039\ \Phi + 1.4$	0.997
20	$H_t = 2.513\ e^{0.025\Phi}$	0.979
30	$H_t = 4.231\ e^{0.020\Phi}$	0.991
40	$H_t = 0.003\ \Phi^2 + 0.039\ \Phi + 1.4$	0.999
50	$H_t = 0.003\ \Phi^2 – 0.035\ \Phi + 9.76$	0.997

Correlation between Depths of Cuts with Cohesion.

The resultant regression analysis after correlating of height of cuts with cohesion is expressed by following equations with its corresponding correlation coefficient

$H_t = X e^{YxC}$

Table 8: Various equation for depth (m) of tension cracks with varying angle of internal friction (˚) and respective R^2.

Φ (˚)	**X**	**Y**	**R^2**
10	0.163	1.019	0.985
20	0.264	0.943	0.990
30	0.353	0.917	0.996
40	0.358	0.972	0.992
50	0.470	0.973	0.989

Correlation between Depth of Tension Cracks with Angle of Internal Friction

The resultant regression analysis after correlating of Depth of tension crack with angle of internal friction is expressed by following multiple linear equations with its corresponding correlation coefficient:

$h\,(m) = X\Phi^2 + Y\Phi + Z$

Table 8: Various equation for depth (m) of tension cracks with varying cohesion (kPa) and respective R^2

C(kPa)	**X**	**Y**	**Z**	**R^2**
10	0.000	– 0.007	0.64	0.991
20	0.001	– 0.054	2.4	0.993
30	0.001	– 0.013	3.08	0.985
40	0.001	– 0.010	3.86	0.988
50	0.001	– 0.013	4.62	0.989

Correlation between Depth of Tension Crack with Cohesion.

The resultant regression analysis after correlating of depth of tension crack with cohesion is expressed by following multiple linear equations with its corresponding correlation coefficient:

Table 9: various equation for depth (m) of tension cracks with varying angle of internal friction (˚) and respective R^2

Φ(˚)	**Equation**	R^2
10	$h = -0.001C^2 + 0.168C - 0.96$	0.999
20	$h = -0.001C^2 + 0.209C - 1.2$	0.996
30	$h = -0.001C^2 + 0.208C - 1$	0.995
40	$h = -0.002C^2 + 0.265C - 1.14$	0.999
50	$h = 3.653\ln(C) - 6.470$	0.998

Correlation between Top Width with Cohesion and Angle of Internal Friction.

The resultant regression analysis after correlating of top width of cuts with cohesion and angle of internal friction is expressed by following multiple linear equations with its corresponding correlation coefficient:

$B_t = 0.1138C + 0.0146\Phi - 0.92$ with $R^2=0.75$, n=25.

Similarly, for this value does not consider every field aspect and are obtained for FOS just above 1, it is reliable to apply a FOS accordingly to decrease depth of vertical cuts and to increase safe width while using this data in field.

Conclusion

It is now possible to deal with complex stratigraphy, highly irregular tension crack conditions, a variety of linear and nonlinear shear strength models, In case of vertical cuts with tension cracks, change in depths are very significant compared to that of without them, also it becomes evident that with increase of internal friction depth varies parabolically and similarly with increase in cohesion depth varies linearly. And variation of safe width at the top is similar to that in case of without tension cracks. This is due to the fact that soil after tension crack behaves like that of without them.

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