Metro Domination Number of Diamond Snake Graph

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***Abstract*: A subset D of the vertex set V of the graph $G(V, E)$ is said to be a dominating set if every vertex in $V - D$ is adjacent to at least one vertex in D. The minimum cardinality of the dominating set is called the domination number. The metro domination number is the order of a minimum dominating set which resolves as a metric as a metric set. It is denoted by$\gamma_\beta(G)$.In this paper we determine the metro domination number of diamond snake graphs.**

***Keywords*: fan graph, fire cracker graph, dominating set, domination number, metric dimension, metro domination.**

1 Introduction

Every graph considered here are simple, finite, undirected and connected. A graph $G = (V, E)$ and $u, v \in V$. $d_G(u, v)$is denoted as distance between u and v in G. We refer [5,6,7,8,9,11] for the works on metro domination.

2 Preliminary Results

Definition 2.1: A set D of vertices in a graph $G(V, E)$ is said to be a dominating set G, if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of the dominating set.

Definition 2.2: A subset $S \subseteq V$ is called resolving set if every pair of $u, v \in V$, there exists a vertex $w \in V$ such that the distance between vertices $u, v \in V$ is represented as $d(u, w) \neq d(v, w)$. A set of vertices $S \subseteq V(G)$ resolves G, then S is a resolving set of G and its minimum cardinality is a metric basis of G and its cardinality is the metric dimension of G and its $\beta(G)$.

Definition 2.3: Metro domination number is introduced by B.Sooryanarayana and Raghunath[4]. A dominating set D of V(G)which is both dominating set as well as resolving set is called the metro dominating set of G. The minimum cardinality of metro dominating set of G is called metro domination number of G, denoted by $\gamma_\beta(G)$.

Definition 2.4: The graph G consists of collection of n cycles C_4, these cycles are connected in such way that two adjacent cycles sharing a common vertex, the resulting graph is called the diamond snake graph and it is denoted by D_n. A diamond snake graph has $3n+1$ vertices and $4n$ edges, where n is the number of blocks in the diamond snake. A snake is an Eulerian path that has no chords [3].

Definition 2.4: A fan graph $F_{m,n} = \overline{K_m} + P_n$, where $\overline{K_m}$ is the empty graph (consists of m isolated vertices with no edges) and P_n is the path graph on n vertices. [3]

Definition 2.5: An fire cracker $F(m, n)$ is a graph obtained by the series of interconnected m copies of n stars by linking one leaf from each. [3]

Corollary 2.6: For any integer n, $\beta(D_n) = \begin{cases} 3n, & n \text{ is even} \\ 3n-1, & n \text{ is odd} \end{cases}$

3 Main Results

Theorem 3.2: For any integer n, $\gamma_\beta(D_n) = \begin{cases} 3n, & n \text{ is even} \\ 3n-1, & n \text{ is odd} \end{cases}$

Proof: Let $G \cong D$ be a diamond graph on $3n+1$ vertices with $4n$ edges and let D be a dominating set of graph G. By the definition of diamond snake graph G, consists of collection of n cycles C_4, these cycles are connected in such a way that any two adjacent cycles sharing a common vertex, where n is the number of blocks in the diamond snake. But we know that $\gamma(D_n) = n+1$ and corollary 2.6, since a metro dominating set D is also dominating set.

Thus $\gamma_\beta(D_n) \geq 3n$ (1)

We define a set D as follows,

$$D = \begin{cases} w_{2l-1} : l \geq 1 \\ u_{2l} : l \geq 1 \\ v_{2l} : l \geq 1 \end{cases}$$

Choose D in the abovecases then D is a dominating setand $|D| = 3n$. By using 2.6, the dominating set also serves as metric set.

Thus $\gamma_\beta(D_n) \leq 3n$ (2)

From (1) and (2),

$\gamma_\beta(D_n) = 3n$

Example: Metro domination number of diamond snake graph (D_3) is 5.

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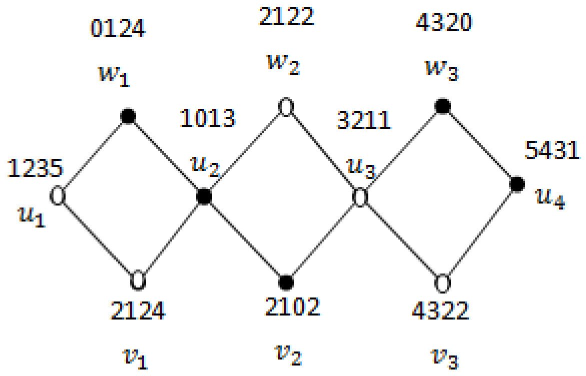

Figure 1: Diamond snake graph $\gamma_\beta(D_3) = 5$

REFERENCES

1. C. Berge, Graphs and hypergraphs, North-Holland, Amsterdam,1973
2. B. Zmazek and J. Zerovnik, “On domination number of graph bundles”, Institute of mathematics, physics and mechanics, 43(2005),pp.1-10.
3. A. Sugumaran and E. Jayachandran, “Domination number of some graphs”, International Journal of Scentific Development and Reserch, 3(2018),no.11,pp.386-391.
4. Raghunath P and Sooryanarayana B, “Metro domination number of a graph”, Twentieth annual conference of Ramanujan Mathematics Society, July 25-30(2005) university of Calicut, Calicut.
5. G C Basavaraju, M Vihukumar and Raghunath, “Metro domination of square of path”, Annals of Pure and Applied mathematics, 14(2017), no.3,pp.539-545.
6. G CBasavaraju, M Vishukumar and Raghunath, “Metro domination of powers of path”, International Journal of Emerging Technology and Advance Engineering, 8(2018), no.1.
7. G CBasavaraju, M Vishukumar and Raghunath, “On the metro domination number of Cartesian product of $P_2 X P_n$”,International Journal of mathematical archieve,9(2018),no.3,pp.1-2
8. G CBasavaraju, M Vishukumar and Raghunath, “On the metro domination number of Cartesian product of $P_2 X P_n$ and $C_m X C_n$”, Journal of Engineering and applied sciences, 14(2019), no.1, pp.114-119
9. G CBasavaraju, M Vishukumar and Raghunath, “On the K-metrodomination number of Cartesian product of $P_2 X P_n$”, Journal of Advanced Reserch in Dynamical and Control Systems, 11(2019), no.1, pp.425-431.
10. LalithaLamani and Yogalaksmi S, “Neighborhood, Locating Dominating and Almost Locating Dominating Set of a Ladder Graph”, International Journal of Mathematics Trends and Technology, 67(2021), no.1, pp.121-128.
11. G CBasavaraju, M Vishukumar and Raghunath, “Metro Domination of Square Cycle”, International Journal of Mathematics and its applications, 5(2017), no.1-E, pp.641-645.