Metro Domination Number of Diamond Snake Graph

Rajeshwari Shibaraya¹,a), Basavaraju G C²,b), Anant Kumar Kulkarni³,c) Vishukumar M ⁴,d)

¹Department of Mathematics
Srinivas Institute of engineering and technology, Karnataka, India
²Department of Mathematics
Brindavan College of engineering, Bengaluru, Karnataka, India
³Department of Mathematics
Srinivas institute of technology, Karnataka, India
⁴Department of Mathematics
Reva University, Bengaluru, Karnataka, India

Abstract: A subset D of the vertex set V of the graph G(V, E) is said to be a dominating set if every vertex in V – D is adjacent to at least one vertex in D. The minimum cardinality of the dominating set is called the domination number. The metro domination number is the order of a minimum dominating set which resolves as a metric as a metric set. It is denoted by \( \gamma_D(G) \). In this paper we determine the metro domination number of diamond snake graphs.

Keywords: fan graph, fire cracker graph, dominating set, domination number, metric dimension, metro domination.

1 Introduction

Every graph considered here are simple, finite, undirected and connected. A graph \( G = (V, E) \) and \( u, v \in V, d_G(u, v) \) is denoted as distance between \( u \) and \( v \) in \( G \). We refer [3,6,7,8,9,11] for the works on metro domination.

2 Preliminary Results

Definition 2.1: A set \( D \) of vertices in a graph \( G(V, E) \) is said to be a dominating set \( G \), if every vertex in \( V – D \) is adjacent to some vertex in \( D \). The domination number \( \gamma(G) \) of a graph \( G \) is the minimum cardinality of the dominating set.

Definition 2.2: A subset \( S \subseteq V \) is called resolving set if every pair of \( u, v \in V \), there exists a vertex \( w \in V \) such that the distance between vertices \( u, v \in V \) is represented as \( d(u, w) \neq d(v, w) \). A set of vertices \( S \subseteq V(G) \) resolves \( G \), then \( S \) is a resolving set of \( G \) and its minimum cardinality is a metric basis of \( G \) and its cardinality is the metric dimension of \( G \) and its \( \beta(G) \).

Definition 2.3: Metro domination number is introduced by B. Sooryanarayana and Raghunath[4]. A dominating set \( D \) of \( V(G) \) is both dominating set as well as resolving set is called the metro dominating set of \( G \) and G. The minimum cardinality of metro dominating set of \( G \) is called metro domination number of \( G \), denoted by \( \gamma_D(G) \).

Definition 2.4: The graph \( G \) consists of collection of \( n \) cycles \( C_4 \), these cycles are connected in such a way that two adjacent cycles sharing a common vertex, the resulting graph is called the diamond snake graph and it is denoted by \( D_n \). A diamond snake graph has \( 3n + 1 \) vertices and \( 4n \) edges, where \( n \) is the number of blocks in the diamond snake. A snake is an Eulerian path that has no chords [3].

Definition 2.5: A fan graph \( F_{m,n} = \overline{K_m} + P_n \), where \( \overline{K_m} \) is the empty graph (consists of \( m \) isolated vertices with no edges) and \( P_n \) is the path graph on \( n \) vertices. [3]

Definition 2.6: An fire cracker \( F(m, n) \) is a graph obtained by the series of interconnected \( m \) copies of \( n \) stars by linking one leaf from each. [3]

Corollary 2.6: For any integer \( n \), \( \beta(D_n) = \begin{cases} 3n, & n \text{ is even} \\ 3n - 1, & n \text{ is odd} \end{cases} \)

3 Main Results

Theorem 3.1: For any integer \( n \), \( \gamma_D(D_n) = \begin{cases} 3n, & n \text{ is even} \\ 3n - 1, & n \text{ is odd} \end{cases} \)

Proof: Let \( G = D \) be a diamond graph on \( 3n + 1 \) vertices with \( 4n \) edges and let \( D \) be a dominating set of graph \( G \). By the definition of diamond snake graph \( G \), consists of collection of \( n \) cycles \( C_4 \), these cycles are connected in such a way that any two adjacent cycles sharing a common vertex, where \( n \) is the number of blocks in the diamond snake. But we know that \( \gamma(D_n) = n + 1 \) and corollary 2.6, since a metro dominating set \( D \) is also dominating set.

Thus \( \gamma_D(D_n) \geq 3n \) \hspace{1cm} (1)

We define a set \( D \) as follows,
Choose $D$ in the above cases then $D$ is a dominating set and $|D| = 3n$. By using 2.6, the dominating set also serves as metric set.

Thus $\gamma_{\beta}(D_n) \leq 3n$ (2)

From (1) and (2),

$\gamma_{\beta}(D_n) = 3n$

Example: Metro domination number of diamond snake graph $(D_3)$ is 5.

![Diamond snake graph](image)

Figure 1: Diamond snake graph $\gamma_{\beta}(D_3) = 5$

REFERENCES