Influence of Non-Newtonian Rheology on Gas-Liquid and Liquid-Liquid Flow in Closed Conduits - A Review

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***Abstract*: This article presents a brief review on the multiphase flow of non-Newtonian fluids with the primary focus on the hydrodynamic parameters of gas-liquid and liquid-liquid two phase flow in closed conduits. The review includes experimental studies, analytical models and numerical simulations. The nature of the hydrodynamics of gas/non-Newtonian liquid flow is complex as compared to gas/Newtonian liquid because of high apparent viscosity and the associated elastic behavior. Several studies are available in literature for the prediction of flow pattern, holdup and pressure drop for two-phase flow regimes for gas-Newtonian two phase flow. Hence research work on the hydrodynamics of Newtonian /non-Newtonian fluid mixtures require a concentrated effort. Most of the research gaps are found regarding the study of Taylor bubbles, liquid/liquid two phase flow in channels, gas/liquid and liquid/liquid flow of non-Newtonian fluids in microchannels/millichannels.**

***Keywords*: non-Newtonian liquid, gas-liquid two phase flow, biphasic liquid flow, Taylor bubbles, bubble interactions.**

CONTENTS

1. INTRODUCTION

Multiphase flow is one of the important areas of fluid mechanics. It is characterized by the presence of a deformable interface,which is influenced by the relative motion of the individual phases and their physical properties. Due to their widespread occurrence in practical applications and everyday life, several studies have been reported on the flow hydrodynamics of different two phase systems namely gas-liquid, liquid-liquid, solid-liquid and gas-solid systems. Some studies are also reported on three and four phase systems. However, the majority of the studies involving liquid flow are reported for Newtonian liquids. Non-newtonian liquids although gaining increasing importance in polymer and food processing, reacting systems, slurry transportation etc. has not received

much attention till date and the studies often report contradictory results. A thorough understanding of the hydrodynamics of multiphase flow involving non-Newtonian liquids is essential for design of processes and equipment and also for intensification of reacting as well as non-reacting processes. Therefore, the motivation of this article is to increase future research prospects in multiphase flow of non-Newtonian liquids.

The paper presents a bird's eye view of the relevant studies on gas-liquid and liquid-liquid flow in pipelines. Both these flows are characterized by a deformable interface coupled with the compressibility of the gas phase in the former case. An extensive survey has revealed that the interest on non Newtonian liquids in two phase flow systems is quite recent and the earliest studies reported for gas-liquid flows are in the early 1970's. Henceforth, there had been a gamut of literature but still several aspects remain unexplored. We have attempted to include the relevant literature on pipeline flow of gas-liquid and liquid-liquid mixtures from 1970 to 2020. Since two phase hydrodynamics is significantly influenced by flow rheology, we begin with an overview of the types and characteristics of non-Newtonian liquids (Section 2). The literature on the simplest type of gas-liquid flow viz rising bubbles in quiescent and moving liquid columns are dicuussed in Section 3. The influence of flow rheology on bubble characteristics and interactions between different bubble types are presented in Section 4. Subsequently, we focus on simultaneous gas-liquid flow through pipes and pipe fittings in Section 5 and touch upon the few reported studies on liquid-liquid flow in Section 6. The paper concludes by discussing the lacunae as noted from the survey and identifies avenues for further research. In each section, the literature is arranged in chronological order and the studies classified according to the major topics are compiled in tabular form for the convenience of the reader.

2. OVERVIEW OF NON-NEWTONIAN FLOW

A non-Newtonian liquid is characterised by a flow curve (shear stress, τ_{yx} versus shear rate, $\dot{\gamma}_{yx} = \frac{du_x}{dy}$, specified in the y-direction for flow in the x direction) which is nonlinear and/or does not pass through the origin. Broadly non Newtonian liquids are classified as (i) time independent, purely viscous or inelastic liquids (pseudoplastic, dilatant and viscoplastic), (ii) time dependent liquids (thixotropic and rheopectic) and (iii) viscoelastic liquids. In general, most real liquids exhibit a combination of two or even three types of non-Newtonian features and in process calculations, these are identified on the basis of the dominant non Newtonian characteristic they exhibit under specified flow conditions.

Time independent viscous behaviour is usually modelled by curve fitting, giving empirical relationships for shear stress (τ_{yx}) or apparent viscosity (μ_{app}) versus shear rate ($\dot{\gamma}_{yx}$). The simplest and most common equation describing these liquids is a power law or Ostwald de Waele model, viz

$$\tau_{yx} = k(\dot{\gamma}_{yx})^n \qquad (1)$$

and $$\mu_{app} = k(\dot{\gamma}_{yx})^{n-1} \qquad (2)$$

where the fitting parameters *k* and *n* are termed as the fluid consistency coefficient and the flow behaviour index respectively. The most common time independent non Newtonian flow behaviour is psuedoplasticity or shear thinning behaviour characterized by $n<1$ while dilatant liquids exhibit shear thickening behavior with $n>1$. Although the power law model offers the simplest representation of shear thinning behaviour and is most widely used in process engineering applications, it suffers from several shortcomings as the model is valid for a limited range of shear rates. Due to this, the values of *k* and *n* depend on the range of shear rate considered and the dimension of *k* depends on the numerical value of *n*. The model also does not predict the zero shear (at low shear rates) viscosity, μ_0 and infinite shear viscosity (under high shear conditions), μ_∞. So for significant deviations from the power law model at very high and very low shear rates, models incorporating μ_0 and μ_∞ are used. Two typical models are the Carreau model

$$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = \left\{1+(\lambda\dot{\gamma}_{yx})^2\right\}^{(n-1)/2} \qquad (3)$$

and the Cross viscosity model

$$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = \frac{1}{1+k(\dot{\gamma}_{yx})^n} \qquad (4)$$

fitted with curve-fitting parameters, *n* (<1), λ and *k*. When deviations from power law are significant only at low shear rates, the three constant Ellis model is preferred.

$$\mu = \frac{\mu_0}{1 + \left[\tau_{yx} / \tau_{1/2} \right]^{\alpha - 1}} \qquad (5)$$

with the adjustable parameters, α (>1) and $\tau_{1/2}$ denoting the degree of shear thinning behaviour (higher α referring to more shear thinning) and value of shear stress at $\mu_{app} = \mu_0/2$.

The other type of inelastic time independent liquid, i.e. the viscoplastic liquid flows or deforms only beyond a critical yield stress (τ_0) and exhibits either linear (Bingham plastic) or nonlinear (pseudo plastic) flow curve which does not pass through the origin for $(\tau_{yx} > \tau_0)$. The relevant model equations for viscoplastic liquids are -

for $|\tau_{yx}| < |\tau_0|$, $\dot{\gamma}_{yx} = 0$

and for $|\tau_{yx}| > |\tau_0|$, $\tau_{yx} = \tau_0 + \mu_B \dot{\gamma}_{yx}$ in the Bingham plastic model (6)

$\tau_{yx} = \tau_0 + k(\dot{\gamma}_{yx})^n$ in the Herschel Buckley model (7)

And $(\tau_{yx})^{1/2} = (\tau_0)^{1/2} + \mu_{app}(\dot{\gamma}_{yx})^{1/2}$ in the Casson fluid model (8)

For time dependent liquids, the relation between shear stress and shear rate additionally depends on the duration of shearing and kinematic history. Thixotropic liquids when sheared at a constant rate exhibits a decrease in apparent viscosity with time of shearing and the reverse behavior is exhibited by rheopectic liquids.

The qualitative flow curves for time independent and time dependent liquids are shown on a linear scale in Figs.1 and 2 respectively. The typical relation for a Newtonian liquid is included in Figure 1. In Fig. 2, a larger area enclosed by the hysteresis loop signifies a stronger time dependent behaviour.

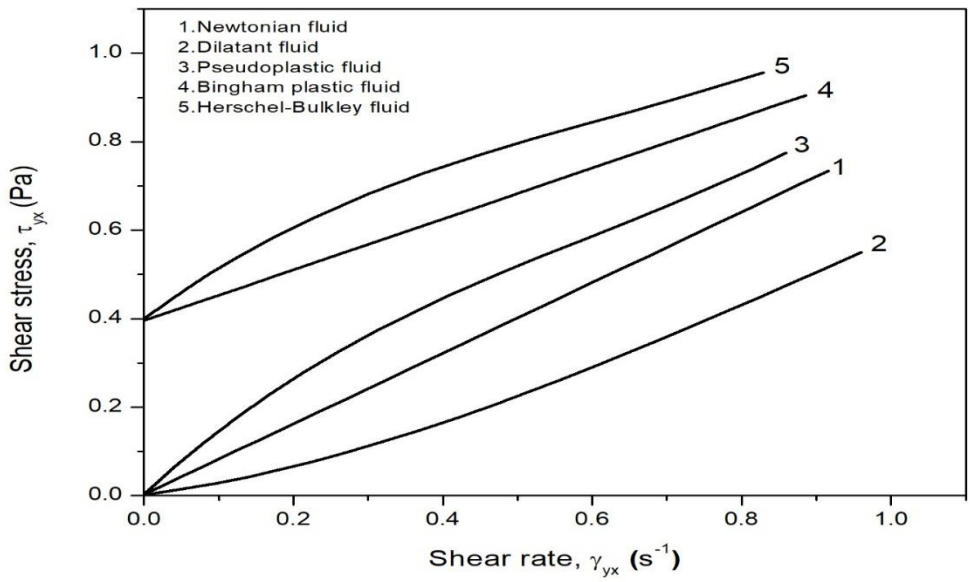

Fig. 1: Schematic flow curve for time independent liquids

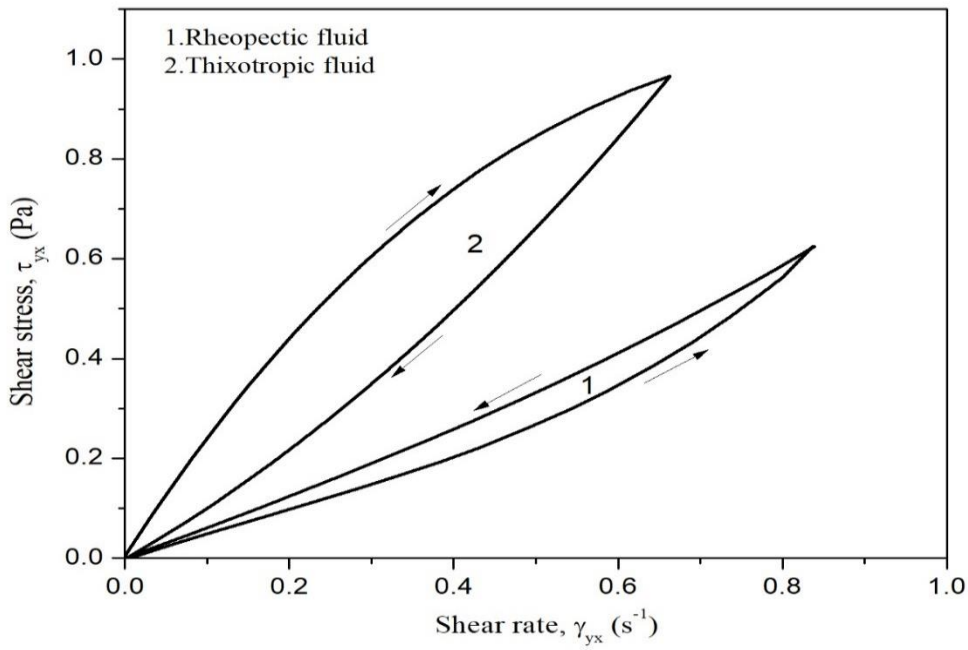

Fig. 2: Schematic flow curve for time dependent liquids

Viscoelastic liquids exhibit characteristics of ideal liquids and elastic solids and exhibit partial recovery after deformation. It is common practice to describe viscoelastic fluid behaviour in steady shear in terms of τ_{yx} and the first normal stress difference, N_1 ($= p_{xx} - p_{yy}$), both expressed as function of shear rate where p_{xx} and p_{yy} are the normal stresses within the sheared liquid in the x and y directions respectively. Fig. 3 shows typical curves depicting the variation of τ_{yx} and N_1 as function of $\dot{\gamma}_{yx}$ for 1.5% CMC and 0.7% PAA solutions as obtained from our experiments.

In order to identify the extent of elasticity of the liquid, τ_{yx} and N_1 are combined in several ways to define a fluid relaxation or characteristic time (t_c). The ratio $\left(\frac{N_1}{2\tau_{yx}}\right)$ termed as the recoverable shear indicates highly elastic behaviour for values > 0.5 [1]. The Maxwellian relaxation time is given by

$$\lambda = \frac{N_1}{2\tau_{yx}\dot{\gamma}_{yx}} \text{ which reduces to } \lambda = \left(\frac{k_1}{2k}\right)^{1/(p-n)}.$$

when both τ_{yx} and N_1 are approximated as power law functions of shear rate, i.e.

$$N_1 = k_1(\dot{\gamma}_{yx})^p \text{ and } \tau_{yx} = k(\dot{\gamma}_{yx})^n$$

The loss tangent, which is the ratio of the loss modulus (G″) and storage modulus (G′) at a particular shear rate can also be used to quantify the relative importance of the elastic and viscous component.

Thus while the state of flow for a Newtonian liquid can be described by two dimensionless numbers, usually the Reynolds (ratio of inertial to viscous forces) and Froude (ratio of inertial to gravitational force) number, a viscoelastic liquid requires at least one additional parameter involving elastic force. This is often taken as the Deborah number (*De*) which is the ratio of fluid response time and the process characteristic time or the Weissenberg number (*Wi*) which is the shear rate times the relaxation time . While *Wi* describes flow with a constant stretch history such as simple shear, *De* describes flows with a non-constant stretch history and a higher *De* signifies elasticity to be of more practical significance. From the definition of Deborah number, it is evident that the same

liquid may exhibit strongly elastic response or an essentially inelastic response depending on the process time. Accordingly for two phase flow, the interfacial interactions are expected to influence the viscoelastic behaviour and the relative importance of elastic and viscous component needs to be estimated for specific flow situations. In addition, an accurate quantification of flow hydrodynamics requires a knowledge of the shear rate characterising the flow in question and this presents a challenge, not much explored, in two phase flows.

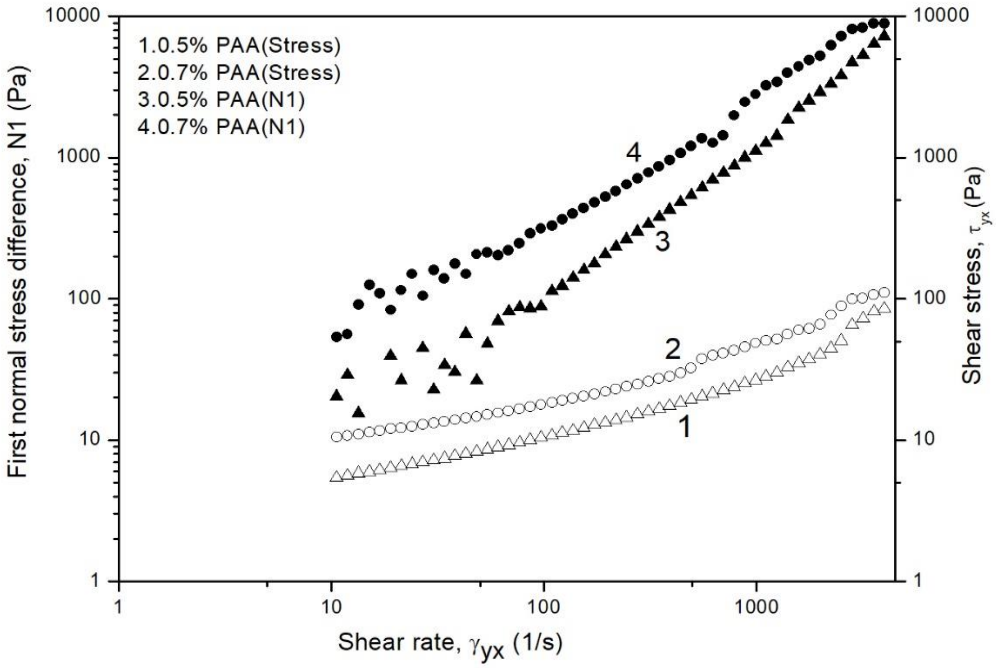

Fig.3: τ_{yx} and N_1 vs. $\dot{\gamma}_{yx}$ for viscoelastic liquids (data collected from experiments)

3. BUBBLE BEHAVIOR IN NON-NEWTONIAN MEDIA

A suitable design and operation of gas-liquid processes requires knowledge of bubble behavior in stationary and moving liquid columns. Therefore, researchers have focussed on rise of single and multiple bubbles of different sizes and shapes as well as their interactions during rise. The studies comprise of experimental investigations, numerical simulations and theoretical analysis with the majority of the studies focussed on rise velocity, bubble trajectory and shape. Similiar to Newtonian liquids, the bubbles above a critical volume assume an elongated axisymmetric shape which remains constant with furthur increase in gas volume. These bubbles, termed as Taylor bubbles (Fig. 4), govern the hydrodynamics of gas-liquid slug flow and are also observed during liquid draining from closed top tubes. While the promotion of slug flow in reduced dimensions leads to intensification of biphasic flow processes, gas slugs often lead to choking and pulsing in multiphase pipelines. Owing to the importance of Taylor bubbles in gas-liquid two phase flow, we focus on Taylor bubble behaviour in a separate subsection 3.2 and discuss the general literature on rise of bubbles in subsection 3.1.

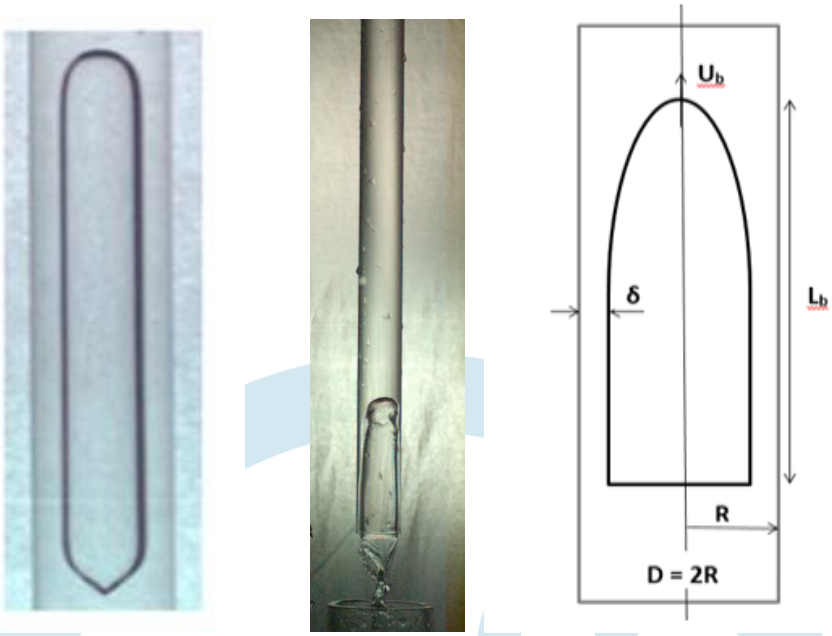

Fig. 4: Snapshot of (a) Taylor bubble rising in 1.5% CMC in circular pipe (b) Taylor finger during liquid draining from circular conduit (c) Schematic of a Taylor bubble inside a pipe

3.1 BUBBLE RISE

One of the earliest studies on rise of small bubbles dates back to Kee and Chhabra [2] who noted the shape and coalescence of bubbles in different concentrations of Newtonian (glycerine) and non-Newtonian solutions (carboxy methyl cellulose, polyacrylamide with surface active agents).Their photographic studies revealed that the shape of the bubbles was independent of gas type and volume and could broadly be classified into spherical, prolate teardrop, oblate cusped and spherical cap. With increase in viscoelasticity, larger bubbles became spherical and the transition to non-spherical bubbles was shifted to higher bubble volumes.

Ichihara et al. [3] reported experiments in viscoelastic liquids like silicone oil and syrup to investigate the effect of liquid rheology on the acoustic properties of a bubbly mixture. They found the velocity of pressure waves was not affected by bubbles when both rigidity and the liquid viscosity are high. The results were relevant for developing mathematical and experimental methods to investigate seismoacoustic phenomena in volcanoes.

Sikorski et al. [4] studied the shape and velocity of air bubbles rising through 1.2 and 1.8 wt% carbopol suspensions/dispersions having yield stress of 24 and 34 Pa respectively in a Plexiglas container of 0.8 m height and square cross-sectional sides of 0.129 m. They noted the rise velocity to increase almost linearly with bubble radius above a critical value. The bubbles exihibited a rounded nose and a tapered tail due to elasticity of the liquid and in liquids with a larger yield stress, bubbles exhibited a larger length-to-radius aspect ratio, a flatter nose and a longer tail.

Zhang et al. [5] performed experimental and numerical investigations on the motion of a single bubble rising in a quiescent non-Newtonian viscous liquid. They selected xanthan gum and carboxy methyl cellulose for the experiments and obtained satisfactory agreement of terminal bubble shape and velocity with the computational results. They showed that the bubble shape and the zero-shear viscosity of shear thinning liquids significantly affects the local viscosity around the bubble.

Shaobi et al. [6] used PIV technique to understand the turbulent characteristics induced by a chain of nitrogen bubbles rising in stationary glycerol and CMC solutions in a Plexiglas column(0.15 m × 0.15 m × 1.70 m). They discussed the influence of polymer concentration, gas flow rate and rheological property with the objective of understanding the mass transfer mechanism of rising bubbles in non-Newtonian liquids. The results depicted that the turbulent kinetic energy profiles are almost symmetrical along the column center and the values are higher in the central region of the column.

Cunha and Albernaz [7] investigated the motion of a spherical bubble in a nonlinear viscoelastic media subjected to an acoustic pressure field. The influence of Deborah number, pressure amplitude and volume fraction of additives were explored to determine the extensional viscosity.Their results showeded that the nonlinear response of a single bubble is strongly affected by both orientation and elasticity of the additives. They proposed the need for additional studies to understand the combined effect of additive orientation and bubble oscillation in a nonlinear viscoelastic fluid.

Amirnia et al. [8] studied the rise of small air bubbles in aqueous solutions of xanthan gum and carboxy methyl cellulose (CMC) in a rectangular column of dimensions 0.27 m × 0.3 m × 2.4 m.They reported the rise velocity to increase as a power law in bubble volume for small bubbles in the Stokes flow regime while for larger bubbles, the rise velocity was constant, similar to that observed in Newtonian liquids. They also noted the bubble shape to change from spherical to cusped and then to spherical cap with increase

in volume. The smaller bubbles rose along a straight path and the larger bubbles exihibited a zig-zag path through the polymer solutions.

Bohm et al. [9] reported an experimental investigation of single bubbles ascending in a rectangular geometry of 0.16 m wide, 0.007 m long and 1.5 m high connected with a bubble separator at the top. They analysed the rising path, shape, absolute and relative terminal rise velocity, friction factor and oscillation frequency of the bubbles. Their experiments showed that the larger channel depth influences the terminal rise velocity in shear-thinning liquids and higher values obtained at larger depths. They also noted higher rise velocity and enhanced oscillations for larger bubbles.

Battistella et al. [10] used a Front-tracking CFD model to investigate the behaviour of a single bubble rising in a power-law fluid where the power-law exponent (n) was varied between 0.5 to 1.5.

3.2 RISE OF TAYLOR BUBBLE

Due to the unique flow physics and important applications, the rise of Taylor bubbles has been extensively studied since the 1940s. However, the studies are primarily confined to Newtonian liquids. Rise of Taylor bubbles in non-Newtonian liquids has not received much attention till date and the reported studies are fairly recent.

Carew et al. [11] experimentally studied the motion of long bubbles through different concentrations of Carbopol 981 and polyacrylamide solutions in three different circular pipes of diameters 0.025, 0.045 and 0.07 m. They derived a theoretical expression for rise velocity of air bubbles in water taking into account the momentum exchange of the bubble nose shape.With increase in angle of inclination from the horizontal, the rise velocity increased from zero to a maximum value between 30º to 70º and then decreased in the vertical orientation. They attributed the decrease in rise velocity to the increasing viscosity or surface tension which resulted in a bubble with a blunter nose.
Shosho and Ryan [12] studied the motion of long bubbles in inclined tubes of 1.22 m length and five different inside diameters of 0.0127, 0.0191, 0.0254, 0.0318 and 0.0381 m. They used water, glycerin and different concentrations of corn syrup, CMC, PAA, PVP and HEC as the liquid medium and expressed Froude number (Fr), Eotvos number (EÖ) and Morton number (M) as function of conduit inclination. Their experiments showed that in both Newtonian and non-Newtonian liquids, the Froude number increased with angle of inclination, till it attained a maximum value and then decreased with further increase in inclination . The maximum Froude number was noted between 30° to 45° for Newtonian liquids and 30° to 45° and 60° to 75° for non-Newtonian liquids with low M ($< 10^{-6}$) and high M ($> 10^{-2}$) respectively.

Nogueira et al. [13] used Particle Image Velocimetry and Shadowgraphy simultaneously for estimating Taylor bubble shape and the flow field around it.

Sousa et al. [14,15] reported flow around single Taylor bubbles rising in CMC and PAA solutions respectively. They used particle image velocimetry (PIV) and shadowgraphy to study bubble rise in an acrylic cylindrical column of 6 m height and 0.032 m internal diameter. Sousa et al. [14] experimented with 0.1 to 1.0 wt% CMC solutions and the test liquids used by Sousa et. al. [15] were 0.1 to 0.8% PAA solutions. The experiments in both the studies revealed that with increasing solution viscosity, the wake flow pattern changes from turbulent to laminar and a negative wake is observed at higher polymer concentrations. In PAA solutions with higher viscoelasticity, the wakes were longer with a recirculation region. They also reported a flow of stretched liquid below the wakes for 0.1 and 0.2 wt.% PAA solutions.

Sousa et al. [16] further reported the effect of gas expansion on the velocity of a Taylor bubble rising in 0.01 and 0.1 wt% CMC solutions confined in a vertical tube 7.0 m long and 0.032 m internal diameter. They used PIV to measure the velocity field in the liquid ahead of the Taylor bubble. Their experiemtns revealed expansion of gas slug to induce an increase in bubble velocity and a corresponding displacement of liquid ahead of the bubble. Furthur, the increase in bubble velocity was equal to the maximum velocity in the displaced liquid ahead of it.

Rabenjafimanantsoa et al. [17] used the PIV technique to investigate the motion of expanding slug bubbles in vertical flow of non-Newtonian liquids. The liquids selected viz 200 and 400 ppm of polyanionic cellulose (PAC) dissolved in water resembled drilling fluids and the experiments were performed in a pipe of 5 m height and 0.08 m ID. Their focus was to study various sections of the Taylor bubble particularly the nose and the tail region. The results revealed highly viscous liquids to form a more a stable tail with a flat edge.

Abhishek et al. [18] carried out a computational analysis to determine the effects of steady and pulsatile co-current flow on the shape and rise velocity of a Taylor bubble rising through water/CMC solution in a vertical tube of 0.01 m diameter. They reported the length of Taylor bubble to be greater in Newtonian than in shear thinning liquid for a given zero-shear dynamic viscosity. An increase in the CMC mass fraction increased the amplitude of oscillation in the bubble velocity. The bigger bubble disintegrated into smaller bubbles for high amplitude and mean velocity of the pulsatile flow.

Jalaal and Balmforth [19] examined the effect of yield stress on the thin viscoplastic film at the tube wall as a long bubble rises in the conduit. They used lubrication theory to analyse the limit of low Capillary number and the effect of a yield stress. The mismatch between theoretical and numerical simulation results was due to the increasing impact of yield stress on the flow pattern at the ends of the bubble.

Araujo et al. [20] reported a numerical study on the rise of Taylor bubbles through vertical columns of stagnant and flowing inelastic liquids exihibiting both shear thinning and shear thickening behavior. They used commercial CFD with VOF method and analysed the influence of rheological properties on the nose, liquid film and wake of the bubble. The simulation revealed the development of higher shear rates on the liquid film surrounding the Taylor bubbles. For a shear thickening liquid, an increase of viscosity was observed near the bubble while an opposite effect was noted for purely shear thinning liquids.

Liu et al. [21] experimentally investigated the rise of single Taylor bubbles in stagnant and downward flowing non-Newtonian liquids in a 2.44 m long inclined pipe of 0.1524 m inner diameter. Their results showed the bubble velocity to increase with increase in inclination angle and decrease of liquid viscosity. Based on their experimental results, they proposed an empirical correlation of Froude number in inclined pipes (Fr_θ) as function of pipe inclination relative to vertical (θ) and apparent viscosity (μ_{app}), defined as

$$Fr_\theta = (a + b\mu_{app}{}^{c})\left[0.35(\sin\theta)^{d} + 0.45(\cos\theta)^{e}\right] \quad (9)$$

Where a = 0.99416, b = −0.18458, c = 0.21337, d = 0.48996, and e = 0.02661 respectively.

4. BUBBLE INTERACTIONS IN STATIONARY AND MOVING LIQUIDS

Both experimental and numerical studies are reported on bubble interactions in stationary non-Newtonian liquid columns.

4.1 NUMERICAL STUDIES

An extensive numerical study has been reported by Liu et al. [22-24] in three papers published in consecutive years. Liu et. al [22] used CFD software-fluent 6.3 to simulate the motion and interaction of three equal-interval bubbles in 0.5 wt% CMC solution. Their volume of fluid (VOF) simulations showed the effect of initial bubble diameter, initial horizontal distance between bubbles and non-Newtonian rheology on lateral coalescence and rise of bubbles. They noted that the bubbles coalesce when the initial horizontal bubble interval is less than a critical horizontal interval and repulsive interactions are experienced for larger than critical interval between the bubbles. Subsequently, Liu et al. [23] extended the simulations to the coalescence and breakup of multiple parallel bubbles rising in power-law liquids. They also carried out experiments by injecting air through 0.4% CMC solution in a bubble column of 0.15 m × 0.15 m × 1.5 m. The effect of bubble diameter, number and interval as well as flow index of power-law on bubble coalescence and breakup was investigated. A large bubble size and low flow index resulted in breakup of the coalescing bubble. Liu et al. [24] also reported numerical simulations of multiple bubbles rising from a multi orifice system arranged horizontally in (shear-thinning) 0.5% CMC solution. They used three-dimensional Volume of Fluid method to analyse the effect of bubble size, bubble interval, bubble arrangement, rheological property and bubble rise velocity on interaction and coalescence of bubbles as well as the mechanism of bubble coalescence and breakup. An increase in bubble volume and shear-thinning effect of the liquid enhanced the breakup process. A sudden decrease of bubble rise velocity occurred during the coalescence process due to the loss of kinetic energy.

Recently,Sun et al. [25] used the VOF method to simulate the interaction and drag coefficient of three horizontal bubbles namely a large bubble and two symmetrical small bubbles rising by its side in pseudoplastic liquids. They studied the effect of initial diameter of large bubble, initial bubble interval and liquid property on the rise velocity and drag coefficient of the bubbles. Their simulations showed that an increase in bubble diameter decreased the drag coefficient of both large and small bubbles. They proposed two modified correlations for the drag coefficient of the larger and smaller bubles, viz:

$$C_{D1} = \frac{4g\,d_{e1}{}^{3}}{3u_{b1}{}^{2}d_{h1}{}^{2}} \quad (10)$$

and $$C_{D2} = \frac{4(\rho_1-\rho_g)g\,d_{e2}{}^{3} - 6C_L(\rho_1-\rho_g)(u_{b2}-u_w)d_{e2}{}^{2}}{3(\rho_1-\rho_g)(u_{b2}-u_w)^{2}d_{h2}{}^{2}} \quad (11)$$

where ρ_l , ρ_g denote the density of the liquid and gas phases and d_h and d_e denote the maximum horizontal projective bubble diameter and the equivalent bubble diameter respectively. u_b refers to the rise velocity with subscripts 1 and 2 denoting the large and the small bubble respectively. C_L denotes the lift coefficient and u_w is the average velocity in the wake region of the leading bubble and ahead of the trailing bubble.

Kamat et al. [26] numerically simulated the Newtonian thin-film theory of Munro et al. [27] for power-law liquids and defined the regimes as well as the points of transition that arise during the coalescence of two identical bubbles in a low viscosity power-law liquid. The simulations were performed to predict the interface shape and the flow field. The analysis focused separately on the tip and the thin film to predict the scaling transition in each domain with and without the influence of the conditions in other domain.

Vahabi et al. [28] numerically simulated the interaction of a pair of unequal in-line bubbles ascending in an Oldroyd-B liquid described by weakly compressible smoothed particle hydrodynamics (WC-SPH). They investigated the effects of Deborah number,

polymer concentration, density and viscosity ratio, Reynolds number and Bond number on rising bubbles. In their experiment, when the bigger bubble is placed below the smaller bubble, coalescence occurred due to the higher rise velocity of the larger bubble and bubble deformation increased with increase in Bond number.

4.2 EXPERIEMENTAL STUDIES

One of the earliest experimental investigations was reported by Li [29] who studied the formation, interaction and coalescence of air bubbles in different concentration of carboxymethylcellulose and polyacrylamide solution. The experiments in a Plexiglas cylindrical tank of 0.3 m diameter and 0.5 m height revealed bubble rise velocity to depend on the injection period and a short period of injection to result in coalescence. They also proposed a theoretical model for non-spherical bubbles formed at an orifice. The theoretical results agreed well with their experimental data.

Lin and Lin [30] performed a systematic study on the coalescing mechanism of two in-line bubbles rising through 1.5% PAA solution in a two-dimensional bubble column of 30 cm width, 0.012 m depth and 1.20 m height. Their experiments using particle image analyzer showed that both shear-thinning and viscoelastic effects play important role in bubble coalescence. The trailing bubble is accelerated towards the leading bubble due to drag force generated by the shear-thinning effect resulting from a reduction in viscosity and also a pushing force generated from viscoelastic effect due to the circulating upward flow from the leading bubble. The same authors [31] reported experiments on the coalescence mechanism of two unequal air bubbles in similar experimental set up with same polymer solution. They revealed that the coalescence of two unequal bubbles had led to higher acceleration in comparison to the coalescence of two equal bubbles.

Sousa et al. [32] studied the interaction between consecutive Taylor bubbles rising in stagnant non-Newtonian solutions using Particle Image Velocimetry (Fig. 5). The experiments were carried out in a vertical acrylic column of 0.032 m internal diameter and 7 m height using different concentrations of CMC (0.1 wt% to 0.8 wt%) and PAA (0.1 wt% & 0.2 wt%) solutions. They reported the interactions to be similar in Newtonian and non-Newtonian liquids for CMC solutions of lower concentration while a negative wake behind the Taylor bubbles inhibited coalescence at higher concentrations. For PAA solutions, the wake length was longer.

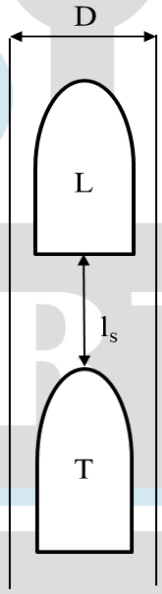

Fig. 5: Leading (L) and trailing (T) Taylor bubbles having distance (l_s) between them

Wenyuan et al. [33] used PIV to investigate the flow field surrounding two parallel moving bubbles released from a pair of orifice submerged in 0.35%, 0.50% and 0.80% (weight/weight) carboxy methyl cellulose (CMC) solution. They analysed the velocity vector, velocity contours and individual velocity components to understand the influence of gas flow rate, solution concentration, distance between injection points, angle between the bubble center lines and the vertical distance between the bubbles. The study showed that the liquid velocity both in front and behind the bubble pair increased with gas flow rate due to the shear-thinning caused by the previous bubble and decreased with increase of CMC concentration due to the increased drag force acting on the bubbles.They also estimated the inter bubble distance at which the mutual repulsion shifted to mutual attraction.

Velez-Cordero et al. [34] reported experimental and numerical simulations of a pair of interacting bubbles rising in shear-thinning inelastic liquids. They used the Lagrangian-Eulerian technique for their simulations and performed experiments in a rectangular acrylic column of inner cross section 5×10^{-3} m^2. The test liquids were different concentrations of xanthan gum and glycerine solutions (83% glycerine/water, 0.02% Xanthan gum in 75% glycerine/water, 0.1% Xanthan gum in 60% glycerine/water). Their results showed that the thinning wake formed behind the bubbles play an important role in the speed of the bubble pair and the formation of clusters in thinning fluids.

Sun et al. [35] carried out experiments in a bubble column (0.15 m × 0.15 m × 1.4 m) using 0.5%, 0.7% and 0.9% (w/w) of sodium carboxy methyl cellulose (SCMC) aqueous solution. Carbon dioxide gas at constant flow rate was introduced through a nozzle of fixed diameter. They noted that an increase in shear thinning behavior decreased the minimum in-line coalescence distance of bubbles while increasing the terminal rise velocity and average acceleration of the trailing bubble.

5. GAS-LIQUID FLOW IN CLOSED CONDUITS

Among the different two phase flow systems involving non Newtonian liquids, the simultaneous flow of gas and liquid has been most extensively studied with the earliest papers in the 1970's. Both experimental as well as numerical investigations are performed through conduits of different geometry and orientation. Some studies, albeit few, focus on flow through pipe fittings and in microchannels. The studies has mostly estimated flow pattern, pressure drop and in situ gas voidage (or liquid holdup). A few studies have attempted to identify interfacial interactions such as interfacial waviness, effect of pipe roughness and entrainment, droplet detachment and redeposition etc. during two phase seaparated flow primarily in the stratified flow distribution. Figure 6 shows various possible flow patterns which are generally governed by the physical properties of the liquids, input fluxes of the two phases, size and orientation of the pipes.

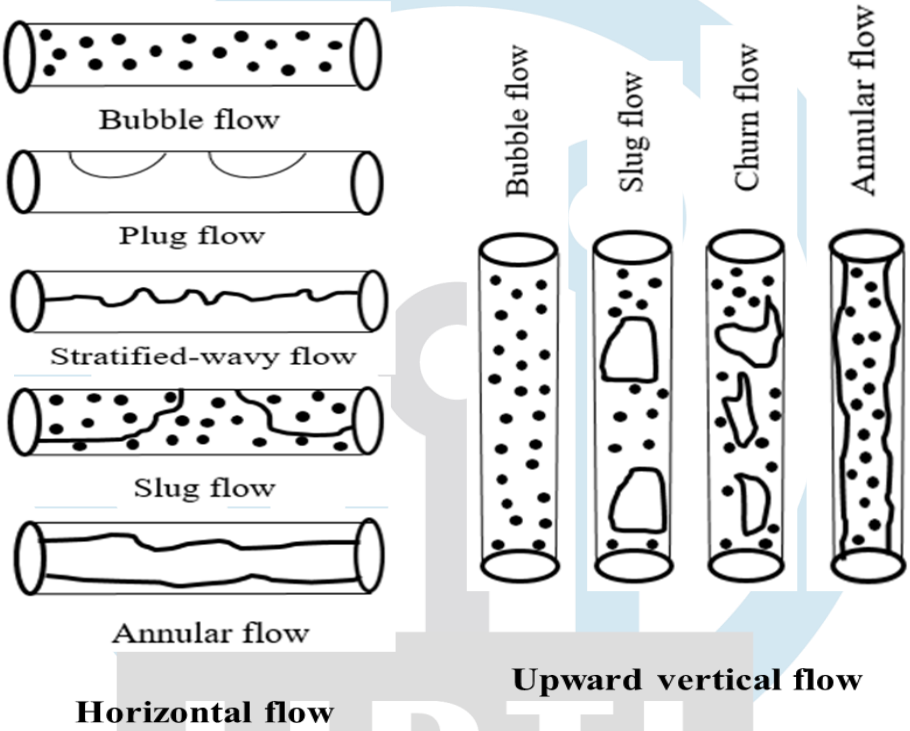

Fig. 6: Flow patterns in horizontal and vertical two phase flow

The survey in this section is presented in three subsections. In subsection 5.1, we present the experimental and analytical studies arranged in chronological order and in 5.2, the numerical studies on the hydrodynamics of gas-liquid flow are discussed. Subsection 5.3 presents a brief discussion on the empirical correlations proposed to predict pressure gradient and in-situ gas voidage during gas- non Newtonian liquid flow.

5.1 EXPERIEMENTAL AND ANALYTICAL STUDIES

Mahalingam and Valle [36] described the momentum transfer during isothermal flow of gas-pseudoplastic liquid mixture in horizontal pipes. They noted strong interfacial interactions by comparing the experimental values of pressure drop with those predicted from a proposed model for the annular flow regime.They observed a sharp decrease in pressure drop in the slug flow regime and an increase in liquid holdup with increasing pseudoplasticity.

Srivastava and Narasimhamurty [37] carried out experimental studies on pressure drop, role of turbulence promoters and effect of pipe geometry on laminar flow of 0.5% SCMC solution. The experiments were performed in circular pipes of 0.0217 and 0.0365 m ID. The researchers noted stream line behavior in non-Newtonian flow and much lower energy loss due to gas slug in non Newtonian as compared to Newtonian flow.

Rosehart et al. [38] reported void fraction,slug velocity and slug frequency for cocurrent slug flow of air and highly viscous non-Newtonian solutions namely CMC 7H3S, Polyhall 295 and Carbopol 941 in a horizontal tube of 0.0254 m ID and 10.67 m length. They proposed different correlations to predict slug parameters as function of liquid rheology and apparent viscosity.

Otten and Fayed [39] estimated pressure drop and drag reduction during slug flow in 0.0254 m ID acrylic tube using carbopol 941. They found that the data on two-phase drag reduction were well correlated by the empirical Lockhart-Martinelli approach at

lower values and the single and two-phase data agreed with Arunachalam's curve. However, the Virk's universal drag reduction curve underpredicted the experiemtnal data.

Tyagi and Srivastava [40] developed equations to predict liquid film thickness during annular flow of air- non Newtonian liquid. The equations used the von-Karman velocity distribution and expressed liquid film thickness as function of phase flow rates, their physical properties and energy losses.

Heywood and Charles [41] predicted pressure drop and holdup for stratified flow of gas and a power law liquid. They reported that drag reduction occurred over the largest range of gas and liquid flow rates at the lowest flow behavior index for laminar flow and the maximum drag reduction occurred for shear-thickening liquids with flow behavior index ≥ 2.

Srivastava et al. [42] estimated void fraction and flow patterns during two-phase flow of air and pseudoplastic liquids (SCMC and Sodium alginate solutions) in circular pipes of 3.048 m length and 0.0217 and 0.0365 m diameter. Their primary focus was to understand the effect of turbulence promotors and pipe diameter. The experiments revealed void fraction to be unaffected by pipe diameter while being a significant fuction of flow pattern and gas flow rate. Turbulence promoters also did not influence the void fraction but the flow patterns were a function of turbulator pitch.

Chhabra and Richardson [43] outlined the intermittent flow pattern comprising of the bubbly, plug and slug distribution during co-current air-non Newtonian liquid flow in a 0.042 m diameter horizontal pipe. The liquids selected were CMC solution, Natrosol and Separan AP-30 and the superficial gas and liquid velocity was varied from 0.15 to 7.0 m/s and 0.25 to 2.0 m/s respctively. They reported that the viscosity of the shear-thinning liquids and viscoelasticity have almost no effect on flow pattern transition.

Bishop and Deshpande [44] investigated stratified flow of gas and pseudoplastic liquid in 0.052 and 0.025m diameter horizontal circular ducts. They compared the behavior of uniform and non-uniform stratified flow. Non-uniform stratified flow with an interfacial level gradient was observed at low liquid velocity. The Heywood-Charles model was used to predict pressure drop and liquid hold up in uniform stratified flow.

Das et al. [45] carried out experiments to evaluate two-phase pressure drop across horizontal bends namely U-bend and bends with 45°, 90° and 135° included angle in a conduit of 0.019 m ID. They used different concentrations of SCMC solution and noted plug and slug flow to be the predominant flow patterns. Based on experiments, they proposed correlations to predict two-phase friction factor as function of tube radius, radius of curvature of the bend and the the physical properties and flow rates of the test fluids.

The same authors Das et al. [46] also proposed correlations to predict liquid holdup in slug flow based on their experiments with air and different concentration of aqueous CMC solution. The experiments were performed in a 8 m long horizontal and a 4 m long vertical Perspex tube of 19 mm ID. Their correlations expressed gas holdup as a function of tube diameter fluid properties and phase flow rates.

Johnson and White [47] experimentally determined air migration velocity in aqueous xanthan gum solution in a pipe of 0.2 m ID and 12 m long.The main parameters governing the gas velocity were well bore geometry, well inclination, volumetric flow rates, void fraction and fluid rheology namely the yield stress and shear thinning bahaviour.The gas migration velocity was larger in more viscous liquid.

Das and Biswas [48] analysed their experimental data on pressure drop for two-phase air/ pseudoplastic liquid slug flow in a vertical tube of 0.019 m ID and 4 m length.They used CMC solution of four different concentrations - 0.5, 0.67, 0.83 and 1.0 kg/m^3 and proposed correlations to predict two-phase friction factor as function of the physical and dynamic variables of the system.

Kaminsky [49] incorporated the effect of pipe wall roughness in the prediction of pressure drop for single-phase and two-phase flow of viscous non-Newtonian liquids namely 0.063 to 0.5% separan AP30, 18.3 to 36.5% Kaolin Clay, 0.5 to 1% CMC, 48.9 to 54% Crushed anthracite. The improved prediction methods provided satisfactory results of pressure drop, liquid holdup and two-phase flow regime.

Banerjee and Das [50] experimentally investigated two-phase pressure drop across 0.0127 m globe and gate valves for different concentrations of SCMC solutions (0.2, 0.4, 0.6 and 0.8 kg/m^3) and proposed correlations of acceptable accuracy.

Samanta et al. [51] evaluated two-phase pressure drop for gas/non-Newtonian liquid flow across orifices and found it to be same as the frictional pressure drop. The experiments were performed in a perspex tube of 4 m length and 0.019 m ID using 0.2, 0.4, 0.6 and 0.8 kg/m^3 of CMC solutions. The pressure drop increased with increase in air flow rate for a constant liquid flow rate and orifice opening. As the flow behavior index increased, the pressure drop decreased.

Dziubinski et al. [52] presented a flow pattern map for gas/non-Newtonian liquid flow in a vertical pipe of 5 m length and diameters of 0.0505, 0.0406 and 0.0253 m. Their experiments with 5% CMC solution revealed negligible effect of non-Newtonian features on the flow regimes.

Ruiz-viera et al. [53] predicted pressure gradient and drag reduction during two phase flow of a lubricating grease/air mixture. They used different pipe geometries with smooth and rough walls and predicted pressure drop gradient from the Sisko model. The accuracy of the proposed model was tested using Fanning friction factor for a non-Newtonian liquid. At high superficial air velocity, drag reduction was found to increase with pipe diameter.

Biswas and Das [54, 55] evaluated frictional pressure drop for air/non-Newtonian liquid flow through helical coils in horizontal and vertical orientation.The experiments by Biswas and Das [54] were carried out in the intermittent flow regime with different concentrations of SCMC solutions. The test rig comprised of 36 different helical coils made from 15 m long PVC pipes of diameter ranging from 0.00904 to 0.00118 m. The authors noted the effect of gas flow rate, liquid flow rate, concentration of SCMC solution and coil diameter during horizontal two phase flow and adopted the Lockhart-Martinelli correlation to predict the two-phase frictional pressure drop. Biswas and Das [55] developed an empirical correlation to predict two phase frictional pressure drop in vertical helical coils based on experimental data in a cylindrical vessel of 0.45 m^3 capacity using nine coils and four different concentrations of SCMC solutions.

Xu et al. [56] performed experimental and theoretical studies on co-current flow of air and CMC solution in inclined smooth pipes. Their experiments in transparent tubes of 0.02, 0.04 and 0.06 m diameters revealed bubbly, stratified, plug , slug , churn and annular patterns. The effect of liquid phase properties,conduit inclination and pipe diameter on two-phase flow characteristics were analysed. The Heywood-Charles model for horizontal flow was modified to accommodate stratified flow in inclined pipes, taking into account the average void fraction and pressure drop of the mixture. The model predicted experimental data within ± 20% of average void fraction and corresponding pressure drop.

Xu and Wu [57] proposed a new correlation to predict average void fraction in the intermittent flow regime during gas/non-Newtonian liquid flow in upward inclined pipes.They carried out experiments for different concentration of CMC solutions in 10 m long and 0.06 m diameter Perspex tubes connected with a U-bend and obtained good agreement between predicted and experimental results.

Xu et al. [58] investigated drag reduction by gas injection in stratified and slug flow regimes for a power-law fluid. The experiments were performed with CMC solutions in a horizontal Plexiglas pipe of 0.05 m diameter and 30 m length. The proposed analysis was successfully validated against experimental data over a wide range of operating conditions, fluid properties and pipe diameters.

Levitsky et al. [59] theoretically analysed the role of fluid rheology on sound wave propagation in thin-walled elastic tubes during flow of a viscoelastic liquid containing gas bubbles. The solution was a polymer of high molecular weight dissolved in a low molecular weight solvent (2.5% solution of polystyrene in toluene). Their analysis could explain the influence of non-Newtonian behavior of the liquid on wave propagation during gas-liquid flow in an elastic tube.

Naraigh and Spelt [60] derived a flat-interface model to describe horizontal, stable stratified flow, where the bottom layer exhibited non-Newtonian rheology. They developed a linear-stability analysis to predict the conditions for which the flat-interface becomes unstable under the influence of non-Newtonian rheology. They noted that an increase in yield stress up to the point of unyielded region in the bottom layer destabilizes flow while an increase in the flow index is stabilizing.

Bar et al. [61] used ANN to predict frictional pressure drop for gas/non-Newtonian liquid flow through piping components such as orifices, gate and globe valves, elbows and a horizontal pipe of 0.0127 m diameter. They used SCMC solutions with concentration of 0.2 to 0.8 kg/m^3 for their experiments and achieved excellent accuracy.

One of the earliest studies on gas/non-Newtonian two-phase flow in microchannels was reported by Yang et al. [62]. The test fluids were used nitrogen gas and three aqueous solutions of 0.4% carboxy methyl cellulose (CMC), 0.2% polyacrylamide (PAA) and 0.2% xanthan gum (XG). Their experiments in square and triangular microchannles of hydraulic diameter 2.5×10^{-3}, 2.886×10^{-3}, and 0.866×10^{-3} m revealed slug, churn and annular flow. The flow regime maps displayed significant influence of non-Newtonian rheology on flow pattern transitions. Geometric factors like the cross-sectional shape and hydraulic diameter of the channel also affected the flow regime map.

Xu [63] estimated the average void fraction during gas/non-Newtonian liquid down flow in pipes inclined at different angles. Perspex pipes of 0.06 m diameter and 10 m length were used for the experiments. They proposed a new correlation for stratified flow which gave good agreement with experimental results.

Majumder et al. [64, 65] used dimensional analysis to develop correlations for gas holdup and two phase pressure drop for both Newtonian and non-Newtonian liquids. The experiments were carried out with water, amyl alcohol, glycerine and CMC solutions in a vertical pipe of 0.01905 m diameter and 3.4 m height. The range of air and liquid flowrates were 0.5×10^{-4} to 1.92×10^{-4} m^{-3}/s and 1.6×10^{-4} to 6.7×10^{-4} m^{-3}/s respectively. Majumder et al. [64] adopted slip velocity, drift-flux and momentum exchange models to analyse gas-holdup data and Majumder et al. [65] reported higher values of frictional pressure drop at higher gas and lower liquid flow rate.

Suleimanov [66] analysed experimental data to propose a mechanism of slip effect during gas/non-Newtonian liquid flow in a capillary at a pressure higher than the saturation pressure.

Wang et al. [67] investigated the gas-liquid phase split at a 0.5×10^{-3} m horizontal T-junction. They used nitrogen gas and CMC solution of 0.1, 0.2 and 0.3 wt% for the experiments and observed slug, slug-annular and annular flow patterns. Their experiments revealed that increasing pseudoplasticity of the liquid phase increased the liquid take off at the side arm and the effect was much more pronounced for the annular as compared to the other flow pattern.

Velez-Cordero et al. [68] studied bubbly flow in an elastic liquid such as Boger-type fluid having nearly constant viscosity. The experiments were carried out in a rectangular transparent acrylic column of inner cross-section of 5×10^{-3} m^2 and the liquid was ionic polyacrylamide in glycerine-water mixture. Their experiments showed that the dispersion of bubble swarm changed dramatically depending on bubble size. Furthur, unlike Newtonian liquids, larger bubbles resulted in higher mass transfer in Boger-type liquids.

Yang et al. [69] analysed the instability of a non-Newtonian liquid sheet subjected to two gas streams of unequal velocity resulting in para-sinuous and para-varicose distubances. They also examined the influence of density ratio, surface tension and liquid viscosity on the instability of the planar sheet.

Xu [70] formulated a correlation to predict liquid slug holdup during gas/non-Newtonian liquid flow in horizontal and inclined pipes. They validated it against experimental results using water and different concentrations of CMC solutions in a 0.06 m diameter perspex tube**.**

Li et al. [71] developed a model for horizontal annular flow of gas and shear thinning liquid. They included the influence of entrainment and aeration and predicted pressure drop, film thickness, void fraction, frictional multiplier and Lockhart-Martinelli parameter. The model predicted results agreed within 10% of published experimental data.

Jing-yu xu et al. [72] developed a correlation for slug flow in horizontal pipes. The correlation incorporated liquid slug holdup and ignored the pressure drop across the gas slug in the homogeneous flow model. It was noted that the correlation had a better predictive ability compared to other empirical correlations.

Hajmohammadi and Nourazar [73] analytically studied the fluid flow and heat transfer consequences on the insertion of a thin gas layer in micro cylindrical Couette flows involving lubricant as power-law liquid. They concluded that the thin gas layer contributes to certain factors such as torque change to set liquid motion, maximum temperature change at shaft, and initiation of Couette–Taylor instability. The thin gas layer stabilizes or destabilizes the flow, depending on the magnitude of the power index of liquid.

A few analytical models have been proposed to predict liquid holdup, pressure gradient and frictional multiplier during non-uniform stratified flow. This flow distribution is one of the important but least understood flow patterns and is generally encountered in petroleum and chemical industries. The non-uniformity of flow influences the transition from stratified to annular distribution. Very limited studies are available in literature and in particular, there is a lack of experimental data. Li et al. [74] adopted the power-law model for non-uniform two-phase flow and the model of Heywood and Charles for uniform flow. The analysis predicted the axial distribution of liquid level and interfacial level gradient for non-uniform gas/ SCMC flow in horizontal tubes. The predictions were within 10% of published experimental data and showed that low liquid and high gas velocity as well as high liquid viscosity were primarily responsible for non-uniformity of flow with an interface level gradient. Subsequently, Li et al. [75] incorporated the effects of interfacial tension and different interface shapes in the model. The interface shape was predicted by solving the Young-Laplace equation and the effect of interfacial tension on holdup and pressure difference between the two phases was noted to be significant, especially for small liquid holdup.

Picchi et al. [76] proposed two different theoretical approaches for stratified flow of gas and power law liquid. The steady fully developed two-fluid model (NTF) used correlations for friction factor and the pre-integrated model (PTF) applied a mechanical approach. In the latter case, the friction factors were obtained from a pre-integrated cross sectional velocity distribution, which was based on channel flow geometry and transformed into pipe flow.

Brujan et al. [77] experimentally analysed shock wave emission from a hemispherical cloud of bubbles rising in elastic (PAM solutions) and inelastic liquids (CMC solutions). Their results showed a substantial reduction of the shock wave pressure in elastic PAM solution. They suggested that numerical simulations can provide a better understanding of the bubble cloud behavior as compared to experiments.

Thandalam et al. [78] estimated flow pattern, pressure drop and liquid holdup during co-current upflow of air, water and different concentrations of SCMC solution through vertical helical coils made from tube of diameters 0.009, 0.015 and 0.02 m. They observed three flow regimes namely plug, slug and stratified flow and noted the effect of various geometrical parameters like tube diameter, coil diameter, pitch difference and liquid physical properties on the flow patterns. The flow pattern transitions were predicted by probability neural network and Lockhart-Martinelli model. Correlations were also developed to interpret pressure drop based on flow patterns. The results showed that both gas holdup and frictional pressure drop depended on liquid concentration and geometrical variables.

Picchi et al. [79] experimentally investigated the flow characteristics of air/shear-thinning liquid systems in horizontal and slightly inclined pipes, 9 m long and 22.8×10^{-3} m ID. They used air and different concentrations of CMC solutions (1%, 3% and 6%) as test fluids and reported that the dominant flow patterns were stratified, plug and slug flow. The slug characteristics namely velocity,

frequency and length were measured and a correlation was proposed for slug frequency. A slug flow model for gas/power-law fluid was validated against experimental data.

Mansour et al. [80] performed experiments with nitrogen gas and different concentrations of PAA solutions (0.1, 0.2 & 0.4 wt%) to understand slug flow in a horizontal rectangular microchannel of hydraulic diameter 0.235×10^{-3} m, width 0.24×10^{-3} m and depth 0.23×10^{-3} m. A comparison of the experimental data with gas/water two-phase flow showed that the rheological properties significantly affected the measured flow parameters, flow patterns, bubble length, bubble velocity, liquid slug length and frictional pressure drop.

Santoso et al. [81] experimented with 0.4 wt% PAA solutions to understand the effect of non-Newtonian liquid properties on two-phase flow characteristics across a sudden expansion in a horizontal rectangular mini-channel. The larger channel had a width of 3.09×10^{-3} m, height of 2.79×10^{-3} m and hydraulic diameter of 2.94×10^{-3} m, and the narrow channel with a hydraulic diameter of 3.95×10^{-3} m is 5.98×10^{-3} m wide and 2.95×10^{-3} m high. The observed flow patterns were bubbly, slug and annular flow in both the channels. The void fraction, bubble velocity and length were also measured.

Picchi and Poesio [82] developed a unified model to predict flow pattern transitions during the simultaneous flow of gas and shear-thinning liquid through horizontal and slightly inclined pipes. They also proposed and validated a steady fully developed two-fluid model for the annular flow pattern.

5.2 NUMERICAL INVESTIGATIONS

Numerical studies on gas-liquid flow are relatively few and recent. One of the earliest studies was reported by Luo and Ghiaasiaan [83] who examined the effect of the liquid-phase rheological properties on hydrodynamic and interphase mass transfer processes using oxygen as the transferred species between aqueous solutions of polyacrylamide and notrogen gas.They used two different Plexiglass tubes of diameter 1.9×10^{-2} m and 5.08×10^{-2} m and length 3.31 m and 1.54 m respectively. The results indicated that the volumetric mass transfer coefficient decreased with increasing polymer concentration at both high and low liquid flow rate while the effect was negligible in churn flow.

Pillapakkam and Singh [84] developed a finite-element code based on level-set method to study deformation of bubbles in gravity driven flows and drops in simple shear and pressure driven flows over a wide range of Capillary and Deborah number. For a Newtonian bubble rising in a quiescent viscoelastic liquid, limiting values of capillary and Deborah number were identified above which the bubble assumed a cusp-like tail while the nose remained spherical under the influence of local viscoelastic and viscous stresses. On the other hand, in pressure driven flows, the droplet was stretched to form a rounded nose located closer to the channel centre while the tail was sharper and closer to the channel wall.

Dimakopoulos and Tsamophulos [85] examined the transient displacement of viscoelastic fluids by gas in straight cylindrical tubes of finite length.They used mixed finite element method in combination with quasi-elliptic grid generation scheme for discretizing the highly deforming domain of the liquid and the discontinuous Galerkin (DG) method for calculating the polymeric stresses.

Using high resolution 2-D numerical simulation of the motion of deformable bubbles in non-Newtonian liquids, Radl et al. [86] noted fluid elasticity to have a pronounced effect on bubble rise velocity and mass transfer. The simulation results obtained a higher overall mass transfer coefficient in non-Newtonian liquids at the same Reynolds number.

Bandyopadhyay et al. [87] used Fluent 6.3 to model the interaction between gas and non-Newtonian liquid during two phase flow through helical coils of 1.762 to 2.667 m diameter and 15m length. The Eulerian–Eulerian, viscous flow model was adopted and the liquid was described by a power law model. A comparison with experimental data showed that more accurate results were achieved for the hexagonal as compared to the tetrahedral grid.

Jia et al. [88] focused on drag reduction of non-Newtonian liquid by gas injection.They considered two flow regimes namely stratified and slug flow of gas/shear-thinning liquids and used three dimensional CFD simulations to predict drag reduction ratio and pressure gradient. The technique was not applicable under high liquid flow rates.

Ratkovich et al. [89] analysed void fraction and pressure drop during two phase flow of Newtonian and non-Newtonian liquids in a vertical pipe of 3.4 m length and 0.01905 m internal diameter. They implemented star CCM+ using volume of fluid (VOF) method and showed void fraction predictions from CFD to be more accurate compared to empirical correlations.

Bandyopadhyay and Das [90] used Fluent 6.3 and adopted the two phase Eulerian-Eulerian approach to analyse single and two phase flow of SCMC solution through elbows. They noted the pressure to be highest at the outer wall. Thus due to centrifugal forces, the liquid phase flowed along the outer wall and air with lower density was pushed along the inner wall. The pressure drop was more for 45° as compared to 135° elbow and the contour plot of velocity showed the maximum velocity to be near the inner wall of the elbow.

Castillo et al. [91] presented a stabilized finite element method to solve immiscible fluid flow problem for viscoelastic liquids by the level set method.The method exhibited good stability and robustness.

Tricha et al. [92] used the two fluid model to reveal the effect of rheological and elastic parameters during pulsating two phase flow through an elastic tube. A numerical finite difference method using Crank Nicholson scheme was employed to determine pressure and velocity profiles. The study was intended to model blood flow in small vessels.

Sontti and Atta [93] performed an extensive CFD analysis in a circular microchannel of 0.5×10^{-3} m diameter and 5×10^{-3} m length to explore the influence of surface tension, inlet velocity and apparent viscosity on the length, shape and velocity of Taylor bubble and the film thickness around the bubble during co-flow. They used different mass concentration of CMC solutions (0%, 1%, 3% and 6%) and reported the bubble length to decrease with increasing capillary number, inlet gas-liquid velocity ratio and CMC concentration. They also noted an increase in bubble velocity with increasing film thickness around the bubble.

5.3 CORRELATIONS TO PREDICT PRESSURE DROP AND IN-SITU GAS VOIDAGE

Several correlations have been proposed in literature to predict pressure drop and in situ void fraction during gas-non Newtonian liquid flow through pipelines. Most of these correlations are extensions of the information available in literature on gas-Newtonian liquid flow through the same geometry. For example, based on Lockhart and Martinelli's correlation for pressure drop in horizontal pipes, several models have been proposed to predict the two phase frictional multiplier during gas-non Newtonian liquid flow where the frictional multiplier is defined as

$$\phi_l^2 = \left(-\left(\frac{dp}{dz}\right)_{TP} \Big/ -\left(\frac{dp}{dz}\right)_L \right) \qquad (12)$$

and $\left(-\frac{dp}{dz}\right)_{TP}$ and $\left(-\frac{dp}{dz}\right)_L$ refer to the respective pressure gradient for two phase flow and flow of liquid only through the same pipe. $\left(-\frac{dp}{dz}\right)_G$ is the pressure gradient flow of gas only through the same geometry. Table 1 lists the correlations proposed for ϕ_l^2 and the correlations proposed to predict gas voidage are summarised in Table 2.

In addition, Das et al. [46] have adopted the Lockhart-Martinelli [94], Zuber-Findlay [95] and Farooqi-Richardson [96] correlations to estimate two phase pressure drop in pipes, irrespective of the prevailing flow pattern.

Table-1 : Empirical Correlations to predict two phase multiplier for gas/non-Newtonian liquid flow

Reference	Proposed Correlation	Range of validity and/or relevant assumptions
[1]	$\phi_L^2 = \frac{(-\Delta p_{TP}/L)}{(-\Delta p_L/L)}$ $\phi_G^2 = \frac{(-\Delta p_{TP}/L)}{(-\Delta p_G/L)}$	Laminar conditions(not considered any effects due to expansion of gas) For n<1 and λ_L<1, ϕ_L^2<1.

[72]	$\phi_L^2 = \lambda_L^{(1-n)}$	Ideal plug model (Negligible slip and pressure drop across the gas slug)
[97]	$\phi_L^2 = \left(J + \frac{C_0}{\chi} + \frac{J}{\chi^2} \right)$	Laminar flow (error of ± 40 %) $2.9 \times 10^{-3} \leq D \leq 0.207$ m, $0.17 \leq U_{SL} \leq 2$ ms^{-1}, $0.11 \leq U_{SG} \leq 23$ ms^{-1}
[98]	$\phi_L^2 = \left[\frac{1+1.036X10^{-4}\left(\frac{D^n u_{TP}{}^{(2-n)} \rho_{TP}}{8^{(n-1)}k} \right)^{1.235}}{1+1.036X10^{-4}\left(\frac{D^n u_L{}^{(2-n)} \rho_L}{8^{(n-1)}k} \right)^{1.235}} \right] \lambda_L$	Considered single phase fluid flows (error of ± 15 %) $2.9 \times 10^{-3} \leq D \leq 0.207$ m, $0.17 \leq U_{SL} \leq 2$ ms^{-1}, $0.11 \leq U_{SG} \leq 23$ ms^{-1}

In Table 1, subscripts L, G and TP denote liquid, gas and two phase mixture respectively. U , -Δp and ρ refer to velocity, pressure drop due to friction and density of the phase denoted by the subscript respectively and D is the conduit diameter. Likewise, U_{SL} and U_{SG} denote the superficial velocities (volumetric flow rate per unit conduit cross sectional area) of the corresponding phase. The mixture velocity $U_{TP} = U_{SL}+U_{SG}$ and mixture density expressed in terms of in-situ gas voidage, α is

$$\rho_{TP} = \alpha \rho_G + (1-\alpha)\rho_L \tag{13}$$

λ_L is the inlet volume fraction of the liquid phase $\left(\frac{u_{SL}}{u_{SG}+u_{SL}} \right)$ and J is the ratio of superficial liquid velocity to critical value of superficial liquid velocity for the transition from laminar to turbulent flow, viz, $J = \left(\frac{U_{SL}}{U_{SL,crit}} \right)^{(1-n)}$

and

$$\chi = \left[\left(\frac{dp}{dl} \right)_G \Big/ \left(\frac{dp}{dl} \right)_L \right]^{1/2} \tag{14}$$

n and k is the flow behaviour index and flow consistency index of the shear thinning liquid as mentioned earlier.

Two-Fluid model [72]

$$\left(\frac{dp}{dl} \right)_{TP} = \left(\frac{dp}{dl} \right)_{SL} + \left(\frac{dp}{dl} \right)_{SG} \tag{15}$$

Developed model [72] for slug Flow

$$\left(\frac{dp}{dl} \right)_{TP} = \left(\frac{dp}{dl} \right)_{SL} = 2\frac{f_{SL}}{D} \rho_{SL} U_{TP}{}^2 K \tag{16}$$

Where $\rho_{SL} = (1-\alpha_S)\rho_G + \alpha_S \rho_L$ is the average density within the liquid slug. α_s denotes the liquid slug holdup and K is the ratio of the length of liquid slug zone to the length of the total slug unit which includes a Taylor bubble.

Table-2 : Empirical Correlations to predict gas voidage for gas/non-Newtonian liquid flow

Reference	Proposed Correlation	Test Conditions	Range of Validity
[46]	$\alpha_g = 1 - \exp\left[-1.4\times10^{-2}\mathrm{Re}_g^{0.7}\mathrm{Re}_L^{-0.64}N_{PL}^{-0.18}\right]$ $\mathrm{Re}_L = \frac{\rho_L D U_{TP}}{\mu_{eff}}$ $\mathrm{Re}_G = \frac{\rho_G D U_{TP}}{\mu_{eff}}$ $N_{PL} = \left(g\mu_{eff}^4 / \rho_L \sigma_L^3\right)$ $\mu_{eff} = 8^{n-1} D^{1-n} U_{SL}^{n-1} k$	Horizontal slug flow for air-CMC solution	$0.19\times10^{-4} \le U_{SG} \le 4.38\times10^{-4}$ m^3/s $0.4\times10^{-4} \le U_{SL} \le 2.84\times10^{-4}$ m^3/s
[46]	$\alpha_g = 1 - \exp\left[-4.25\times10^{-3}\mathrm{Re}_g^{0.86}\mathrm{Re}_L^{-0.6}N_{PL}^{-0.18}\right]$	Vertical slug flow for air-CMC solution	$0.49\times10^{-4} \le Q_G \le 4.56\times10^{-4}$ m^3/s $0.84\times10^{-4} \le Q_L \le 2.84\times10^{-4}$ m^3/s
[57]	$\alpha = 0.7892\left(\frac{U_{SG}}{U_{TP}+U_d}\right)^{0.87} J^{0.2682}$ $J = \left(\frac{U_{SL}}{U_{SL,crit}}\right)^{(1-n)}$ $U_d = (gD)^{0.5}(0.35\sin\theta + 0.65\cos\theta)$ θ - angle of inclination from horizontal g - acceleration due to gravity $U_{SL,crit}$ - critical superficial liquid velocity when laminar flow ceases to exist (setting Re=2000).	Intermittent flow in upward inclined pipe of 0.06 m ID	U_{SL} = 0.89 ms^{-1} U_{SG} = 0 to 2.5 ms^{-1}
[63]	$\alpha = \frac{1}{\pi}\bar{h}\left[\cos^{-1}\left(2\bar{h}-1\right) - \left(2\bar{h}-1\right)\left(1-\left(2\bar{h}-1\right)^2\right)^{1/2}\right]$ $\bar{h} = \frac{h}{D} = k\frac{U_{SL}^{0.35}}{U_{SG}^{0.65}}$ $k = 1.5(\mathrm{m/s})^{0.3}$, $\bar{h} = \frac{h}{D}$ h - average film thickness, D - conduit diameter	Stratified flow in downward inclined pipe of 0.06 m ID	U_{SL} = 0 to 0.8 ms^{-1} U_{SG} = 0 to 2.5 ms^{-1}
[70]	$\alpha = \frac{(1-\sin\theta)^{0.05}}{1+3.166\times10^{-5}\mathrm{Re}_L^{1.225}}$	Slug flow in upward inclined pipe of 0.06 m ID	$0° \le \theta \le 75°$ U_{SG} = 1.82 m/s U_{SL} = 0.61 m/s

6. LIQUID-LIQUID FLOW IN CLOSED CONDUITS

The past survey shows that liquid-liquid flow is relatively less explored compared to gas-liquid flow and the results for gas-liquid flow cannot be extended to liquid-liquid systems in a straightforward manner. The density and viscosity ratio are much larger and either of the two phases can wet the conduit wall. In addition, liquid-liquid flow exhibits the unique phenomenon of phase inversion where the dispersed phase inverts to form the continuous phase and vice versa. On the other hand, biphasic liquid flows are increasing in importance as they are commonly encountered in a diverse range of applications including oil and gas well cementing, restart of waxy crude oil pipeline, enhanced oil recovery, biomedical applications, biofilms, food processing etc. Thus there is ample scope of future research on Newtonian/non-Newtonian liquid-liquid flow in closed conduits. The few studies reported till date are presented in this section.

A numerical study on duct flow of multiple visco-plastic fluids such as cement slurry and drilling mud is performed by Gonzalez and Frigaard [99] using regularisation techniques and the augmented Lagrangian method.

Firouzi and Hashemabadi [100] analytically solved the momentum balance equations for steady state, laminar, fully developed, stratified two-phase flow in horizontal pipe using appropriate boundary conditions. They used one Newtonian and one Bingham plastic fluid and investigated pressure drop, velocity distribution and location of plug region. The studies showed significant

influence of the rheological properties on two-phase velocity profiles. Subsequently, the same group, Firouzi and Hashemabadi [101] predicted the dimensionless velocity distribution, Martinelli correction factor and liquid holdup and concluded that the rheological properties have significant effects on pressure drop for large values (liquid–liquid flow) and small values (gas–liquid flow) of viscosity ratio.

Li and Sundararaj [102] reported experiments on the steady state deformation of a viscoelastic drop (Boger fluid) in a Newtonian liquid at high capillary number under shear flow. A specially designed Couette apparatus was used for visualization from two perpendicular directions. The observations revealed drop deformation both along flow and in the vorticity direction and for the same capillary number, the deformation was more significant for the bigger drop.

Hormozi et al. [103] reported an intersting study of visco-plastic lubrication of visco-elastic fluid where elasticity prevented interfacial instabilities and stabilized core annular flow. The test liquids were polyethylene oxide solution (viscoelastic) as the core liquid and carbopol solution, a visco-plastic fluid with yield stress, as the lubricant. Subsequently, Hormozi et al. [104] performed a theoretical analysis of viscoplastic lubrication using energy stability methods and demonstrated exponential decay of suitable energy functional for sufficiently small Reynolds & Weissenberg number and small finite restrictions on shear stress & elastic stress perturbations.

Ebrahimi et al. [105] attempted to understand the effect of thixotropic behavior on the viscous fingering phenomenon in a rectangular Hele-Shaw cell where the displacing liquid was Newtonian and the displaced liquid obeying the Moore model was a thixotropic liquid. The researchers used lubrication theory and showed that the shape of the finger was significantly affected by the thixotropic behavior of the displaced liquid.

Swain et al. [106] performed a lattice Boltzmann simulation of pressure-driven displacement of a viscoplastic material by a Newtonian liquid. They validated the code by comparing the results obtained using different regularized models from literature to model the viscoplasticity of the displaced material. They concluded that an increase in Bingham number and flow index decreased the interfacial instabilities and the speed of the propagating finger.

Fu et al. [107] observed slug, droplet, parallel and jet flow during their experiments in T-shaped rectangular microchannels. The dimensions of the micro channels were 400 µm × 400 µm, 400 µm × 600 µm, 400 µm × 800 µm and the test liquids were cyclohexane as the dispersed phase and different concentration of aqueous carboxyl methyl cellulose (CMC) solution as the continuous phase. The researchers noted a larger range of droplet and parallel flow at a higher concentration of CMC solution and in a smaller channel dimension.

Tripathi et al. [108] achieved good agreement between theory and simulation of core annular flow (CAF) using CFD software Ansys Fluent 14.5 in a horizontal pipe. They used highly viscous shear-thickening oil as the core liquid and water as the annular liquid and reported CAF to be more stable for lower interfacial tension.

Alba and Frigaard [109] performed experimental and theoretical investigations on the dynamics of removal of a viscoplastic liquid by a heavier Newtonian liquid in an inclined pipe. The effect of inclination angle (β), Reynold number (Re), Froude number (Fr) and Bond number (Bo) were discussed for center and slump type flows which were classified based on the density difference of the liquids. In centre-type displacement, the displacing liquid flowed through the centre of the pipe with an approximately uniform layer of the lighter liquid on the wall and for the slump-type displacement, the displacing liquid flowed beneath the lighter liquid. The transition between the two flow regimes was in the range $600 < Re/Fr < 800$.

Roumpea et al. [110] reported an experimental study on liquid-liquid plug flow with xanthan gum solution and silicone oil flowing in a quartz microchannel of 200 µm ID. The increase in xanthan gum concentration produced longer, bullet-shaped plugs and resulted in increased thickness of the surrounding liquid film.

7. SUMMARY AND AVENUES OF FURTHER STUDY

The past survey suggests that although two phase flow involving Newtonian liquid is explored to a satisfactory extent, not much is known about two phase flow involving non-Newtonian liquids. In the last six decades, most of the investigations have been for gas-liquid flow. Studies on liquid-liquid flow are relatively scarce. The investigations are mostly experimental with some CFD studies reported in recent years. Most of the numerical studies focus on 2D simulations. Detailed 3D simulation and analytical models are scarce. Further, till date the analysis are primarily based on information available for two phase flow involving Newtonian liquids. Additional insight is necessary to understand the hydrodynamics of flow, particularly the influence of viscoelasticity on flow parameters. The studies on the effect of viscoelasticity are confined mainly to the film region around a Taylor bubble in order to understand the film thickness and velocity profile. One of the major challenges in understanding non-Newtonian two phase flow is to estimate the relative importance of elastic and viscous components under actual flow conditions. This depends on the characteristic time of a process as is evident from the definition of De and the same fluid pair may exihibit different characteristics under different flow conditions and for different flow distributions.
The significant lacunae in literature can be summarized as follows :-

- Most of the studies are for shear thinning liquids. The behavior of dilatant and visco elastic liquids are not much investigated.

- Till date, the flow patterns reported for gas-liquid flow are similar to those observed in gas-Newtonian two phase flow. Studies to understand the influence of rheology on flow morphology is scarce.
- The majority of the studies are in horizontal and vertical conduits. The coupled effect of inclination and flow rheology is not much investigated.
- Biphasic flow through pipe fittings, inevitable in practical applications, is not much explored.
- Multiphase microfluidics involving non-Newtonian liquid has rarely been explored and is of much relevance in the current era of miniaturization.
- Although bubble interaction is well studied in applications pertaining to bubble columns, interaction of Taylor bubbles with each other and with small bubbles in gas-liquid flow needs to be investigated for a more accurate analysis of gas-liquid slug flow.
- The motion of bubbles in non-circular conduits and inclined conduits is not much explored. The past researchers have reported that the maximum rise velocity of Taylor bubble occurs within a range of angle of conduit inclination similar to the observations in Newtonian media. Further information on upslope and downslope flow are necessary for a complete understanding.
- Liquid-liquid two phase flow involving non-Newtonian liquid is less understood and less explored compared to gas-non Newtonian liquid flow and there is ample potential to exploit non-Newtonian liquids for visco plastic lubrication, drag reduction, etc.
- Dimensionless numbers are not well defined for two phase non-Newtonian flow. For example, in single phase flow, Reynolds number is defined in terms of apparent viscosity, μ_{app}. Likewise in two phase flow, the definition of equivalent viscosity to be used in Re should be the shear viscosity at the shear rate for the particular flow condition. It is difficult to estimate the velocity gradient under given input conditions. Several studies have adopted PIV technique but more needs to be known to correlate shear rate with flow rheology under two phase flow conditions.

Based on the identified lacunae, the avenues for further research can be summarized as :-

- Development of suitable instrumentation for objective identification of flow pattern and pattern transition.
- Identification of suitable non-dimensional parameters to describe the flow physics.
- Detailed study to estimate in-situ holdup and pressure drop as function of input parameters and fluid properties.
- Development of analytical and numerical models to predict flow hydrodynamics from input parameters.

8. FUNDING

The entire research did not receive any external grant from funding agencies other than scholarship from IIT Kharagpur.

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