THE MATHEMATICAL MODEL BASED ON THE BATTLE OF UKRAINE AND RUSSIA.

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***Abstract*: In this paper, we use the strategy of mathematical model based on the battle of Ukraine and Russia. According to the linear payoff function, the distribution of the same type weapon to different types of targets is invalid. That's to say, when the Russia use the whole of i type weapon to fight with Ukraine j type target, we analyse the best distribution strategy is that we should distribute the whole type weapon against another type weapon of enemy.**

***Keywords*: Mathematical model, Lanchester model, Weapons, Target.**

I. Introduction

During the war, F. W. Lanchester model for proposed several military operations research and combat modelling apply mathematical models to analyze a variety of military conflicts and obtain insights about these phenomena. One of the earlilest and most important set of models used for combat modelling is the **Lanchester equations.** Legacy Lanchester equations model the mutual attritional dynamics of two opposing military forces and provide some insights regarding the fact of such engagements. In this paper, we review recent developments in **Lanchester modelling** focusing on contemporary conflicts in the world. Specifically, we present models that capture irregular warfare. Such as insurgencies, highlights the role of target information in such conflicts, and capture multicateral situations where several players are involved in the conflict (Such as the current war of Ukraine and Russia).

The Russia-Ukraine war is an ongoing, the war between **Russia** (Together with **Pro-Russian separatist forces**) and Ukraine. It began in February 2014 following the Ukraine **Revolution of Dignity,** and initially focused on the status of **Crimea** and part of the **Donbas,** internationally recognised as part of Ukraine the first eight years of the conflict included the **Russian annexation of crimea (2014)** and the **War in Donbas** (2014- present) between Ukraine nd Russia – backed separatists, as well as **Naval incidents, cyberwarfare,** and political tensions. Following **a Russian military build-up** on the **Russia-Ukraine border** from late 2021, the conflict expanded significantly. When Russia launched a **Full-scale invansion of Ukraine** on 24 february 2022. Table 1 shows both joint military strength, Table 2 shows both joint total ammunition, Table 3 shows total number of dead in Ukraine **(day 1 to 15)**

Table: 1 both of joint military strength:

	Russia	Ukraine	Correlation
Attack aircraft	**1,511**	**98**	**15.418:1**
Tanks	**12,240**	**2,596**	**4.715:1**
Towed artillery	**7,571**	**2,040**	**3.711:1**

Table: 2 Ukraine and Russia total ammunition:

	RUSSIA	UKRAINE
TOTAL TROOPS	**2,900,000**	**1,00,000**
ACTIVE PERSONNEL	**9,00,000**	**2,00,000**
RESERVE PERSONNEL	**2,000,000**	**9,00,000**
ATTACK AIRCRAFT	**1,511**	**98**
ATTACK HELICOPTER	**544**	**34**
TANKS	**12,240**	**2,596**
ARMOURED VEHICLES	**30,122**	**12,303**
TOWED ARTILLERY	**7,571**	**2,040**

Table 3: Total number of dead in Ukraine (day 1 to 15)

DAY	SOLDIRES	CIVILIANCE
1	40	351
2	57	137
3	64	198
4	69	102
5	89	352
6	70	300
7	98	350
8	90	260
9	86	138
10	498	350
11	300	364
12	334	133
13	427	474
14	700	549
15	683	1170

II. Using conventional Lanchester model

We use F. W. Lanchester model to describe the Ukraine and Russia Battle. Which is often conventional warfare. Therefore, we want to discuss the following problem based on previously collected data.

- Based on the above data, we can speculate the amount of ammunition supplies required for simple machanized battle, that is, predict the consumption of munitions in the war.
- Finland will send weapons and ammunitions to Ukraine, prime minister sanna mairn said on Monday.(February 28)
- Ukrainians need food and ammunition, Russia needs gas.
- We want to discuss how to amount of ammunition impact on both sides of the battlefield. There is consumption model about artillery ammunitions of both sides.

$$\frac{dz}{dt} = -bg(t) + p(t)$$

$$\frac{dg}{dt} = -cz(t) = q(t)$$

Then we use data related to the battle between Russia and Ukraine to detect the model.

Here Z is the amount of artillery consumption of the Russia, and g is the amount of artillery consumption of the Ukraine. P(t) is the amount of ammunition supply Russia of the day, and q(t) is the amount of ammunition supply Ukraine of the day. Z(t) is the amount of artillery consumption of Russia of the day and g(t) the amount of artillery consumption of the Ukraine of the day. b is the loss of soldiers of the Russia and c is the loss of soldiers of the Ukraine .

Z=7,571
G=2,040
P(t)=90%
q(t)=30%
z(t)=540.785
g(t)=145.714
b=10,000
c=15,000

III. Solving model

We derive equation (1) we have,

$$\frac{d^2g}{dt^2} = -c\frac{dz}{dt} = cbg - cp(t)$$
$$\frac{d^2g}{dt^2} - bcg = -cp(t)$$

Equation (2) Which has driving force "-cp(t)" is a 2^{nd} order linear ODE's with constant coefficient, coefficients. We can solve (2) by using numerical method of single second order equation and have.

$$g(t) = g_o \cos h(\beta t) - \frac{z_o}{r} \sin h(\beta t) - \frac{1}{r}\int_0^1 \sin h\, \beta(t-s)\, p(s) ds$$

Means the amount or artillery of the Ukraine and Russia before battle Z=0 because before battle $\beta = \sqrt{x},\ r = \frac{\sqrt{b}}{c}$, in integrated (3),we can have.

$$z(t) = -rg_{o\ \sin h(\beta t)} + \int_0^1 \cos h\, \beta(s-t) p(s)\, ds$$

Therefore, we can get the transformation model of ammunition consumption with time of days.

$$z(t) = -rg_o \sin h(\beta t) + \int_0^1 \cos h\, \beta(s-t) p(s)\, ds$$

$$g(t) = g_o \cos h(\beta t) - \frac{z_o}{r} \sin h(\beta t) - \frac{1}{r}\int_0^1 \sin h\, \beta(t-s) p(s)\, ds$$

According to the historical data and ammunition storage of the Russia before the battle, it is reasonable to suppose that the Ukraine army have a total of (2,040) artillery before the table and (7,571) artillery of Russia.

Then g is the number of artillery in Ukraine and Z is the number of artillery in Russia.

g= 2,040

In historical records the total troops about Russia in (2,900.000) and total troops about Ukraine in (1,00,000). The value of b and c are the total consumption ammunition is g=2,040. The total artillery ammunition rate which we have collected, the result is shown.

b=4000, C=6000, P(t)=90% =90×1/100 =0.900 in substitute the equation (1)

$$\frac{dz}{dt} = -bg(t) + P(t)$$

$$= -10{,}000.\, g(t) + 0.900$$

$$\frac{dg}{dt} = -15{,}000.\, Z(t)$$

IV. One - On – One Combat and Lanchester's linear law:

A simple model for this style of combat, first devised by Lanchester, assumes the fighting between the forces occur one-one-one. This means that every man fights only his opposite number and all troops not fighting wait their turn to fill in the ranks at the front. Allowing troop numbers to be continuous in time t, the equation for the rate of change of the T(t) is Russian troop and S(t) is Ukraine troop.

$$\frac{ds}{dt} = -k_t N \quad \ldots\ldots\ldots (1)$$

$$\frac{dT}{dt} = -k_s N \ldots\ldots (2)$$

Here, N is the total number of soldier dead of each side.

T=Russia=10,000

(day 1 to 15)

S=Ukraine=15,000

=10,000+15,000

N=25,000

The value of k_t and k_s determine the artillery strength of each opposing army in table(1).

k_t=7,571 ....(3)

k_s=2,040 ....(4)

Dived (3) and (4) in (1) b (2)

$$\frac{ds}{dt} = \frac{7571}{2,040} = 3.711 \qquad \ldots\ldots\ldots\ldots (5)$$

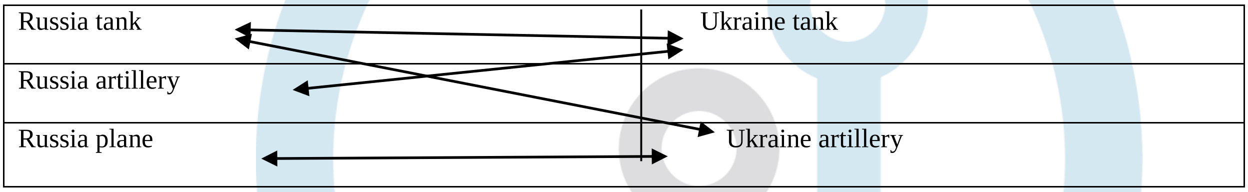
Integrating the above, and using S and T we get.

$$k_s(S(t) - 15,000) = k_t(T(t) - 10,000)$$

V. Improving model

Model improvement

In the modelling for different weapons of the service in battle, we give appropriate symbols for the weapons and establish the total model. Let i=1,2,......., j=1,2,...... and i, j stand for the different types of Russia and Ukraine weapons. z_i=$z_{i(t)}$ stand for the i type unit of Russia at a time. z_{io} is the initial number of i type unit of Russia . $g_{j=}g_{j(t)}$stand for the number of j type unit of Ukraine at t time. g_{io} in the initial number of j type of Ukraine. β_{ji} is the consumption rate Russia. γ_{ji} is the proportion or probability of j type weapon of Ukraine.Ukraine hitting i type target of Russia. p_{ij} the consumption rate of Ukraine; δ_{ij} Is the proportion or probability of i type weapon of Russia hitting i type target of Ukraine, A is the efficiency of the plane and S is the number of time each plane take off per unit time. Based on the literature data and assumption, the tanks are remarked (z_1 ,g_1), artillery (z_2 , g_2), planes (z_2,g_2) between the Russia and Ukraine on the battlefield has the following interdependent relationship.



VI. Conclusion

Analysing the above model, we get the following conclusion;

According to the linear payoff function, the distribution of the same type weapon to different types of targets is invalid. That's to say, when the Russia use the whole of i type weapon to fight with Ukraine j type target, there is no need to distribute fire at the same type weapon. The best distribution strategy is that we should distribute the whole type weapon against another type weapon of enemy.

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