Financial Securities and Its Application in the Financial Market and the Effect of Volatility on an European call and put Option

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ABSTRACT

This paper provides a brief introduction to financial Securities. We also describe the learning of financial option valuation. Mathematical finance is the branch of applied mathematics concerned with the application of mathematical techniques in finance, like Asset pricing (derivative securities), Hedging and risk management, portfolio optimization, and structured products. Finance is defined as the provision of money at the time when it is required. It is the life hood of any enterprise involved in a financial transaction. Traditionally finance has been classified into two parts, public finance, and private finance. Public finance deals with requirements, receipts, and disbursements of funds in government institutions like state, local self-government, and central government whereas private finance deals with requirements, receipts, and disbursement of funds in case of an individual, a profit-seeking enterprise, and a non-profit seeking organization. It is the theory of portfolio to judge investment strategies, in this paper, we describe the concept of trying to identify the best individual stock for investment in financial securities, the continuous evolution of the financial market, there has been a continuous development of different financial instruments. In recent years trading volume has increased in the stock market which has led to the high volatility in the option prices.

Thus in this paper describe the fact of Black -Scholes model with dividends and non-dividend paying stock, it will derive and extend the mathematical or numerical models suggested by financial economics, while a financial economist might study the structural reasons why a company may have a certain share price, a financial mathematician may take the share price as a given, and attempt to use stochastic calculus to obtain the fair value of derivatives of the stock.

Keywords:- Financial Securities, black-Scholes Option Pricing

Introduction

A stock is a security that gives its owner the right to a proportion of any profits that might be distributed (rather than reinvested) by the firm that issues the stock and to the corresponding part of the firm in case it decides to close down and liquidate. The owner of the stock is called the stockholder. The profits that the company distributes to the stockholders are called dividends.

A share is the smallest ownership unit of the stocks. They may be bought or sold on or off an exchange. It is a unit of ownership interest in a corporation or financial asset. While owning shares in a business does not mean that the shareholder has direct control over the business's day-to-day operations, being a shareholder does entitle the possessor to an equal distribution in any profits, if any are declared in the form of dividends. The two main types of shares are common shares and preferred shares.

Financial securities are classified into two major types' namely Basic securities and derivatives, as shown in Figure 1. The basic securities can be further classified into two types namely, fixed-income investments like bank account and bonds, etc, and equities like stocks. The derivatives are also classified into two types namely option & Futures.

The basic characteristics of each type, its use from the point of view of an investor, and its organization in different markets are given below in successive sections.

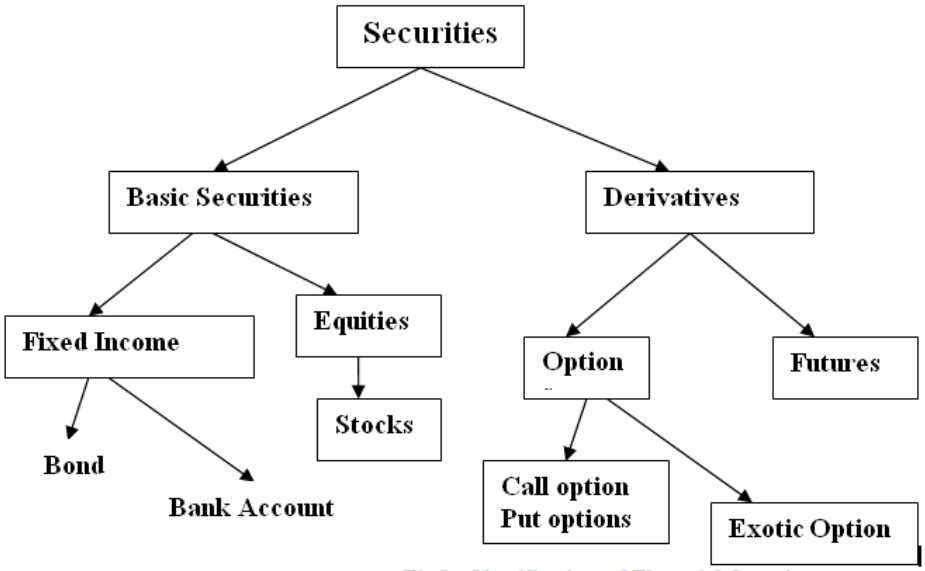

Fig.1 Classification of Financial Security

Bond

The bond is a fixed income basic security that gives its owner the right to a fixed, predetermined payment, at a future, predetermined date, called maturity. The amount of money that a bond will pay in the future is called nominal value, face value, par value, or principal. A bond is nothing but a tradable loan agreement. Bonds are issued by governments, private and public corporations, and financial institutions. There are two types of a bond, zero-coupon bonds and coupon bonds. A zero-coupon bond is a simplest possible bond. It promises a single payment at a single future date, the maturity date of the bond. Bonds that promise more than one payment when issued are referred to as coupon bonds. Bonds depend on their maturity. Bonds are classified into short-term bonds(bonds of maturity no greater than one year), and long-term bonds, when their maturity exceeds one year.

Derivative

A derivative is a product whose value depends on some other underlying basic assets. This includes metals such as gold and silver, commodities such as wheat and orange juice, energy resources such as oil and gas, and financial assets such as shares, bonds, and foreign currencies. In all cases, the link between the derivative and the underlying commodity or financial asset is one of value. There are three main types of derivative product, forwards and futures, swaps and options. These derivative products are discussed in the subsequent subsection given below. There are three types of derivatives trading in the security exchange markets. Speculation, Hedging, Arbitrage

Speculation

Derivatives are very well suited to speculating on the prices of commodities and financial assets and on key market variables such as interest rates, stock market indices and currency exchange rates. It is much less expensive to create a speculative position using derivatives than by actually trading the underlying commodity or asset. As a result, the potential returns are that much greater. A classic application is the trader who believes that increasing demand or reduced production is likely to improve the market price of a commodity. As it would be too expensive to buy and store the physical commodity, the trader buys an exchange-traded futures contract, agreeing to take delivery on a future date at a fixed price. If the commodity price increases, the value of the contract will also rise and can then be sold back into the market at a profit. Speculation is the betting on the movement of a stock (whether the stock goes up in value or goes down). The characteristic that differentiates options from other investment types is the ability to make a profit even if the stock goes down in value, or goes sideways. This is made possible by the sophisticated properties of Options.

Hedging

Hedging means reducing or controlling risk. This is done by taking a position in the futures market that is opposite to the one in the physical market with the objective of reducing or limiting risks associated with price changes. Hedging comes from the term "to Hedge" and is any technique designed to reduce or eliminate financial risk. Hedging is the calculated installation of protection and insurance into a portfolio in order to offset any unfavorable moves.

Hedging is a two-step process. A gain or loss in the cash position due to changes in price levels will be countered by changes in the value of a futures position. For instance, a wheat farmer can sell wheat futures to protect the value of his crop prior to harvest. If there is a fall in price, the loss in the cash market position will be countered by a gain in futures position.

In fact, hedging is not restricted only to financial risks. Hedging is in all aspects of our lives. We buy insurance to hedge against the risk of unexpected medical expenses. We prepare fire extinguishers to hedge against the risk of fire and we sign contracts in business to hedge against the risk of non-performance. Hence, hedging is the art of offsetting risks. In this type of transaction, the hedger tries to fix the price at a certain level with the objective of ensuring certainty in the cost of production or revenue of sale.

Arbitrage

In economics and finance, arbitrage is the practice of taking advantage of a price difference between two or more markets, striking a combination of matching deals that capitalize upon the inequity, the profit being the difference between the market prices. In principle and in academic use, an arbitrage is risk-free, in common use, as in statistical arbitrage, it may refer to expected profit, though arbitrage losses may occur, and in practice, there are always risk in arbitrage, some minor (such as fluctuation of prices decreasing profit margins), some major (such as devaluation of a currency or derivative). In academic use, an arbitrage involves taking advantage of differences in price of a single asset or identical cash-flows,

A simple example occurs when a trader can purchase an asset cheaply in one location and simultaneously arrange to sell it in another at a higher price. Such opportunities are unlikely to persist for very long, since arbitrageurs would rush in to buy the asset in the 'cheap' location, thus closing the pricing gap. In the derivatives business arbitrage opportunities typically arise because a product can be assembled in different ways out of different building blocks. If it is possible to sell a product for more than it costs to buy the constituent parts, then a risk-free profit can be generated. In practice the presence of transaction costs often means that only the larger market players can benefit from such opportunities.

Forward Contracts

A forward contract is a contractual agreement made directly between two parties. One party agrees to buy a commodity or a financial asset on a date in the future at a fixed price. The other side agrees to deliver that commodity or asset at the predetermined price. There is no element of optionality about the deal. Both sides are obliged to go through with the contract, which is a legal and binding commitment, irrespective of the value of the commodity or asset at the point of delivery. Since forwards are negotiated directly between two parties, the terms and conditions of a contract can be customized. However, there is a risk that one side might default on its obligations. Forward contract is an agreement to buy or sell at a specified future time a certain amount of an underlying asset at a specified price. Forward contract is an agreement to replace a risk with a certainty. The

buyer in the contract is said to hold a long position, and the seller is said to hold a short position. The specified price in the contract is called the delivery price and the specified time is called maturity. Let E-delivery price, and T-maturity, then a forward contract's payoff V_T at maturity is.

$V_T = S_T - E$ (long position)

$V_T = E - S_T$ (short position)

Where S_T denotes the price of the underlying asset at maturity $t = T$.

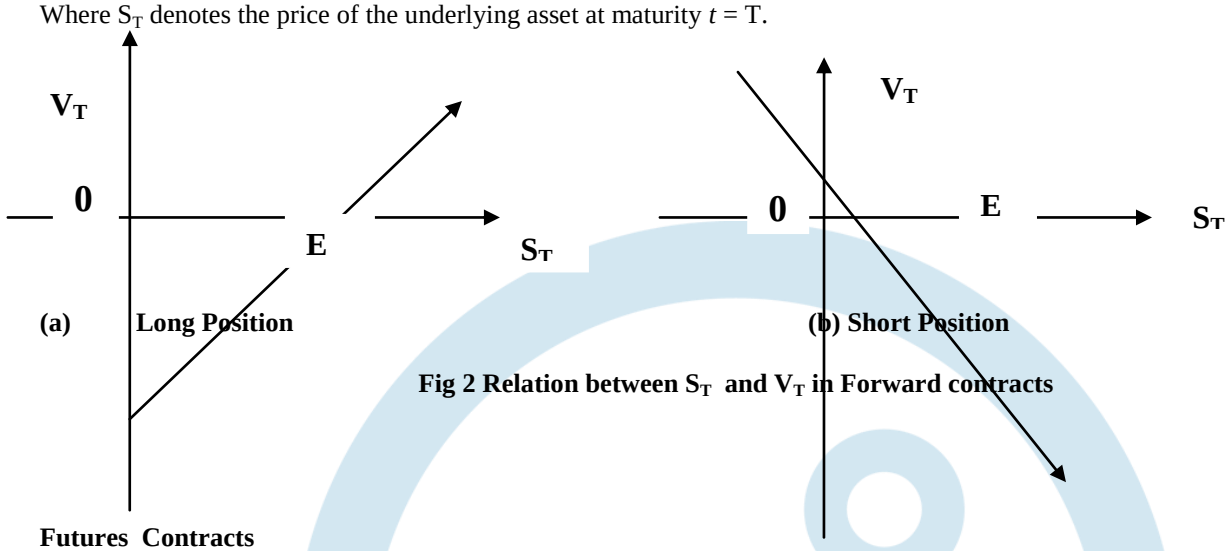

Fig 2 Relation between S_T and V_T in Forward contracts

Futures Contracts

A futures contract is essentially the same as a forward, except that the deal is made through an organized and regulated exchange rather than being negotiated directly between two parties. One side agrees to deliver a commodity or asset on a future date (or within a range of dates) at a fixed price, and the other party agrees to take delivery. The contract is a legal and binding commitment. There are three key differences between forwards and futures. First is futures contract is guaranteed against default. Second, futures are standardized, in order to promote active trading. Third, they are settled on a daily basis.

Swaps And Options

A swap is an agreement made between two parties to exchange payments on regular future dates, where the payment legs are calculated on a different basis. As swaps are over- the- counter (OTC) deals, there is a risk that one side or the other might default on its obligations. Swaps are used to manage or hedge the risks associated with volatile interest rates, currency exchange rates, commodity prices and share prices. A typical example occurs when a company has borrowed money from a bank at a variable rate and is exposed to an increase in interest rates, by entering into a swap the company can fix its cost of funding. Although it is often considered as one of the most basic types of derivative product, a swap is actually composed of a series of forward contracts.

Options

Financial options are contingent claims or financial derivatives because their value depends on the underlying asset. Options give their holder the right to buy or sell the underlying asset. With the rapid growth and deregulation of the economy, a variety of derivatives are designed by financial institutions to satisfy the needs of their customers. Although financial derivatives are getting more and more complicated, options are still one of the most important financial instruments and have wide applications in the market .

Mathematical finance is an old science but it has become a major topic for numerical analysts since Merton and Black-Scholes modeled financial derivatives. Since the Black-Scholes model relies on stochastic differential equations, option pricing became rapidly an attractive topic for specialists in the theory of probability, and stochastic methods were developed first for practical applications, along with analytical closed formulas. But soon, with the rapidly growing complexity of the financial products, other numerical solutions became attractive, in particular method based on partial differential equations .

Option pricing theory has a long and illustrious history, but it also underwent a revolutionary change in 1973. At that time, Fischer Black and Myron Scholes presented the first completely satisfactory equilibrium option pricing model. In the same year, Robert Merton extended their model in several important ways. These path-breaking articles have formed the basis for many subsequent academic studies. These studies have shown that option pricing theory is relevant to almost every area of finance.

Moneyness For The American And European Options

Options Moneyness is definitely the most important stock options concept in option trading. It describes the relationship between options strike price and the price of the underlying asset and determines if intrinsic value exists in an option. Moneyness is described using one of the three states, In The Money (ITM), At The Money (ATM) or Out Of The Money (OTM). As the price of the underlying asset changes, an option moves from one moneyness state to another, affecting the value of your option trading position upon expiration.

In the money is the state of options moneyness where intrinsic value exists in an option. Call Options are in the money when its strike price is lower than the price of the underlying asset and Put Options are in the money when its strike price is higher than the price of the underlying asset. At the money is the state of options moneyness where the strike price of an option is equal to the price of the underlying asset. Out of the money is the state of options moneyness where no intrinsic value exists and that the price of an option consists only of extrinsic value. A call option is out of the money when its strike price is higher than the price of the underlying asset and a put option is out of the money when its strike price is lower than the price of the underlying asset.

In the case of a call (American or European), when the stock price is larger than the strike price, S (t) > E, we say that the option is **in the money.** If S(t) < E we say that the option is **out of the money.** When the stock and the strike price are equal, S (t) = E, we say that the option is **at the money.** When the call option is **in the money**, we call the amount S (t) − E the intrinsic value of the option. If the option is not in the money, the intrinsic value is zero.
For a put (American or European), we say that it is in the money if the strike price is larger than the stock price, E > S (t), out of the money if E < S (t), and at the money when S (t) = E. When in the money, the put's intrinsic value is E − S (t). Moneyness has two types of option values namely intrinsic value and time value. Which are discussed in successive subsections.

Intrinsic Value
The intrinsic value is the gross profit that would be realized upon immediate exercise of the option which is either greater than zero or equal to zero. In other words, intrinsic value is the amount by which the portion is in-the-money. An option that is out-of-the- money or at-the-money has no intrinsic value.
For the Call option, Intrinsic value= stock price - exercise price
For the Put option, Intrinsic value= exercise price - stock price
Time Value
Time value is the difference between the option price and the intrinsic value. Time value is also known as extrinsic value, or instrumental value.
Time Value = Option Value - Intrinsic Value.
The option's time value reflects the probability that the option will gain in intrinsic value or become profitable to exercise before it expires. The time value of an option is not negative (because the option value is never lower than the intrinsic value), and converges towards zero with time. At expiration, where the option value is simply its intrinsic value, time value is zero. Prior to expiration, the change in time value with time is non-linear, being a function of the option price.

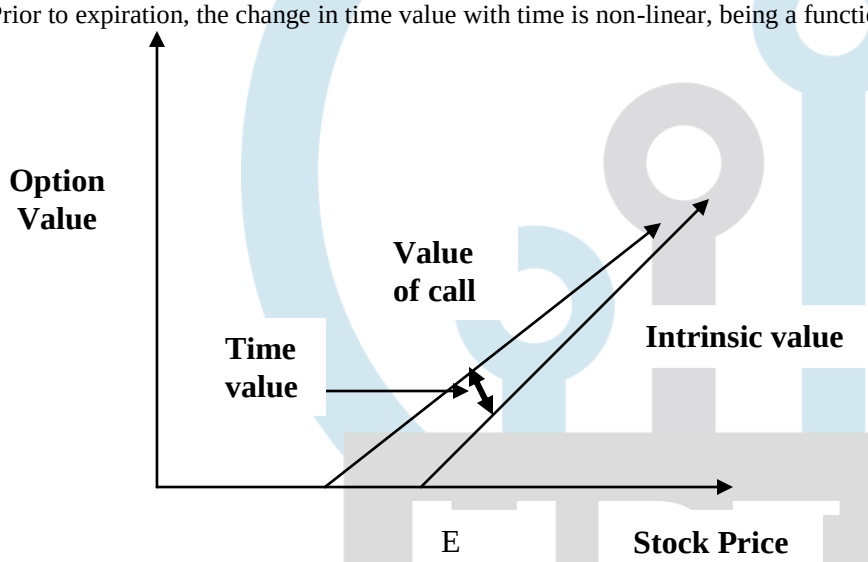

Fig: 3 –Relation between Option Value and Stock Price depicting Time Value

Black Scholes Model

The Black–Scholes model is a mathematical model of the market for an equity, in which the equity's price is a stochastic process. The Black Scholes Option Pricing Model determines the fair market value of European options but may also be used to value American options. And Black–Scholes PDE is a partial differential equation which (in the model) satisfies the price of a derivative on the equity.The Black-Scholes Model was originally created for the pricing and hedging of European Call and Put options as the American Options market. The six assumptions of the Black-Scholes Model are

(i) Stock pays no dividends
(ii) Option can only be exercised upon expiration.
(iii) Market direction cannot be predicted, hence "Random Walk".
(iv) No commissions are charged in the transaction.
(v) Interest rates remain constant.
(vi) Stock returns are normally distributed; thus, volatility is constant over time.

Mathematical Representation Of Black-Scholes Model

The model is based on two types of stocks.

(i) Non-Dividend paying stocks.

(ii) Dividend paying stock
(iii) The mathematical expressions for above two types of stocks are presented in subsequent subsections.

Non Dividend Paying Stock

Non dividend means no income to the shareholders. While dividends are the only direct income, the total return of holding a stock is the dividend plus the capital gain of the stock price. If the price of the Underlying instrument *S* follows a geometric Brownian motion with constant drift μ and volatility σ then we have

$$dS = S\mu dt + S\sigma dW \qquad (1)$$

The graph for the equation (1) depicting relation between stock price and time is shown in Figure below. The curves in figure indicate Brownian motion for variation of stock price with respect to time.

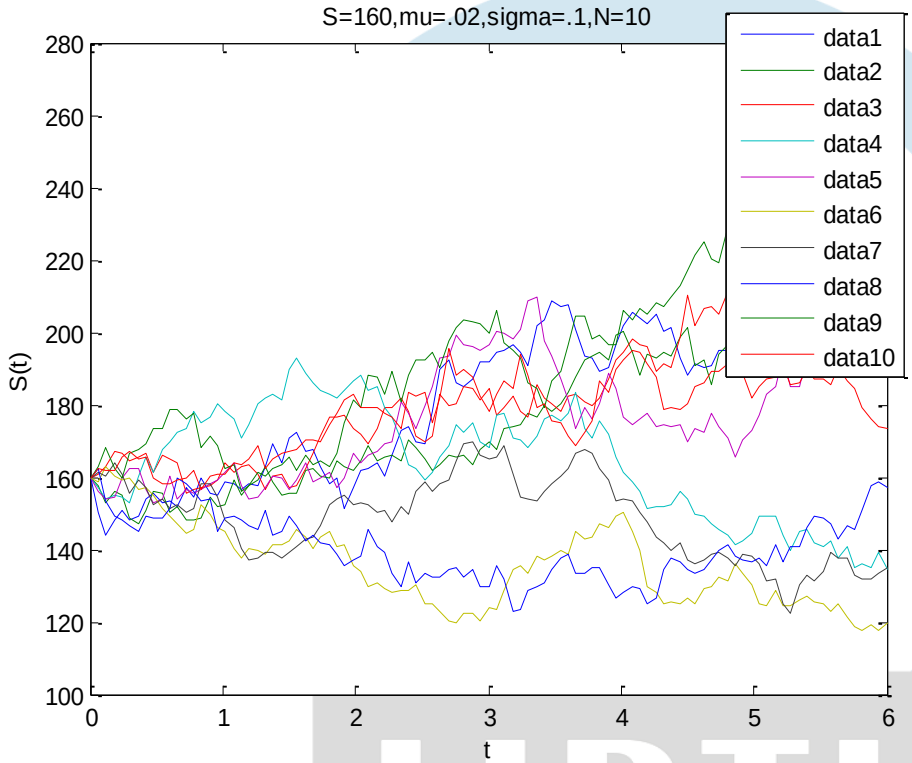

Fig. 4-Graph between time (months) and Stock Price

We use the Wiener process with the help of following properties.

$E(dX) = 0$ and $E(dX)^2 = dt$ (2)

We know that σ is proportional to Var (*dS*), therefore the expectation and the variance are calculated as.

$$\begin{cases} E(dS) = \mu S dt \\ \mathrm{var}(dS) = \sigma^2 S^2 dX^2 \end{cases} \qquad (3)$$

Since $E(S^2 dXdt) = 0$, the standard deviation is the square-root of the variance, so σ is Proportional to $\dfrac{\sqrt{\mathrm{var}(dS)}}{S}$.

The value of an option is dependent on the time to expiry and the value of the underlying asset. Let *V* denote the value of an option, either a call or a put. *V* is a function of S and t (that means it depends on both S and t), *dt* denotes the change in time t and *dV* denotes the change in *V*.

Now here a Taylor Series expansion is employed, to model *dV*. The first few terms of an infinite Taylor's series are being used as an approximation for *dV*. as given below .

$$dV = \frac{\partial V}{\partial S} dS + \frac{\partial V}{\partial t} dt + \frac{1}{2}\frac{\partial^2 V}{\partial S^2} dS^2 + \ldots\ldots\ldots\ldots \qquad (4)$$

From equation (1) we get

$$dS^2 = \mu^2 S^2 dt^2 + \sigma^2 S^2 dX^2 + 2\sigma\mu S^2 dXdt \qquad (5)$$

Now applying Ito's Lemma and the assumptions

$dX^2 \to dt$, as $dt \to 0$, $dS^2 \to \sigma^2 S^2 dt$
in equation (5) and substituting in equation (4) we get

$$dV = \sigma S \frac{\partial V}{\partial S} dX + (\mu S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t})dt \qquad (6)$$

This equation represents the change in the asset price and the change in the option value. Now, consider a portfolio consisting of one option with value V and a number $-\Delta$ of the underlying asset. A portfolio is a collection of assets in this case, we will consider a portfolio consisting of an option and shares of stock.

Let $\Pi = V - \Delta S$ (7)

Where

Π - The value of the portfolio,

V- Value of the option. Then we have

$$d\prod = \sigma S(\frac{\partial V}{\partial S} - \Delta)dX + (\mu S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu \Delta S)dt \qquad (8)$$

If $\Delta = \frac{\partial V}{\partial S}$ (9)

The change in risk free portfolio should equal the exponential growth of placing money in the bank. Therefore

$$d\prod = r\prod dt = (\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t})dt \qquad (10)$$

Now using equation (7) in equation (10) and dividing by *dt*, the famous Black-Scholes equation for valuing an option with value V is obtained as given below .

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \qquad (11)$$

This is the Black–Scholes partial differential equation (PDE) for a price of European option for a non-dividend paying stock. It is a relationship between *V*, *S*, *t* and certain partial derivatives of *V*.

One of the important drawbacks of this model is that the volatility is assumed to be a constant function. But for many options the Black-Scholes model can still be successfully used. The improved asset models have an important impact on the equations for option prices. When a constant dividend payment is assumed there will be only a slight change to the general Black-Scholes equation. Taking the same function $\prod$ as in equation (7) and a constant dividend payment δS, We express d $\prod$ as-

$$d\prod = dV - \Delta dS - \delta S \Delta dt \qquad (12)$$

Therefore the Black-Scholes PDE becomes-

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-\delta)S\frac{\partial V}{\partial S} - rV = 0 \qquad (13)$$

Parabolic equation (11) and (13) are second order partial differential equations in S and first order in time. Here two boundary conditions are necessary. The Black-Scholes equation (11) is a diffusion reaction equation. Here the value of a call option will be denoted by C and the value of a put option by P. The boundary conditions follow from economical arguments. If S = 0, the value of the call option equals zero. For S tends to infinity, the holder will exercise and the value of his option will be (S-*E*). Thus we have

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0 \qquad (14)$$

And the boundary conditions are.

$$C(0,t) = 0 \text{ And } C(S,T) = (S-E)^+ \qquad (15)$$

Solution of the equation (14) for European call option and the Black-Scholes price for a European Call option on a non-dividend-paying stock trading at time *t* is.

$$C(S,t) = SN(d_1) - Ee^{-r(T-t)}N(d_2) \qquad (16)$$

Where

$$d_1 = \frac{\log S - \log E + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \qquad (17)$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \qquad (18)$$

Where N (d_1) and N (d_2) are the cumulative distribution value for a standard normal random variable with values d_1 and d_2, E is the strike price, r is the risk-free rate of interest, and T is the time of expiration.

Similarly the Black-Scholes solution for the price of European Put option on a non-dividend-paying stock trading at time t is.

$$P(S,T) = -SN(-d_1) + Ee^{-r(T-t)}N(-d_2) \qquad (19)$$

Where $d1$ and $d2$ are given by expressions (17) and (18) above

Hence $N(d_1)$ and $N(d_2)$ are the cumulative normal distribution function defined by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy$$

The Black–Scholes call and put values depend on S, E, r, T-t and σ^2. Among these five quantities, only the volatility σ cannot be observed directly. One approach is to find the volatility from the observed market data and given option value, and knowing S, t, E, r and T, find σ, which leads to this value. Having found σ, we may use the Black–Scholes formula to value other options on the same asset. A σ computed in this way is known as an implied volatility.

Dividend Paying Stocks

If we assume that 'with dividend rate δ', then the Black-Scholes equation becomes

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-\delta)S\frac{\partial V}{\partial S} - rV = 0$$

In case of dividend paying stock, the additional shares are given by company. Thus the parameter of dividend is incorporated in the model which is expressed as given below.

$C = Se^{-\delta T}N(d_1) - Ee^{-rT}N(d_2)$

And

$P = Ee^{-rT}N(-d_2) - S_0\, e^{-\delta t}N(-d_1)$

Where

$d_1 = \frac{\left[\log\left(S_0/E + (r-\delta+\sigma^2/2)\right)T\right]}{\sigma\sqrt{T}}$

And

$d_2 = d_1 - \sigma\sqrt{T}$ Where

δ = dividend of underlying assets.

C= price of a call option

S=current stock price underlying assets

E= Option exercise price

e= 2.71828, the base of the natural log function

r= Risk-free interest rate (annualized continuously compounded rate on a safe asset with the same maturity as the expiration of the option usually the money market rate for a maturity equal to the option's maturity)

T=Current time until expiration.

σ =standard deviation of stock

N = cumulative normal distribution function

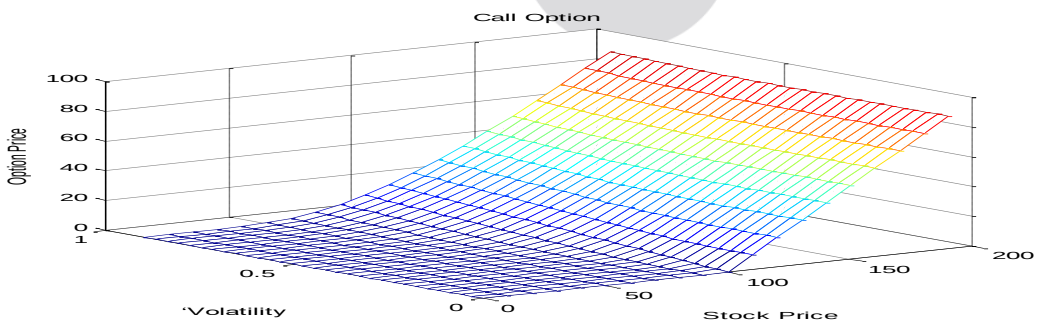

Fig.5(a)- Relation among volatility (90%), Stock Price and call Option Price

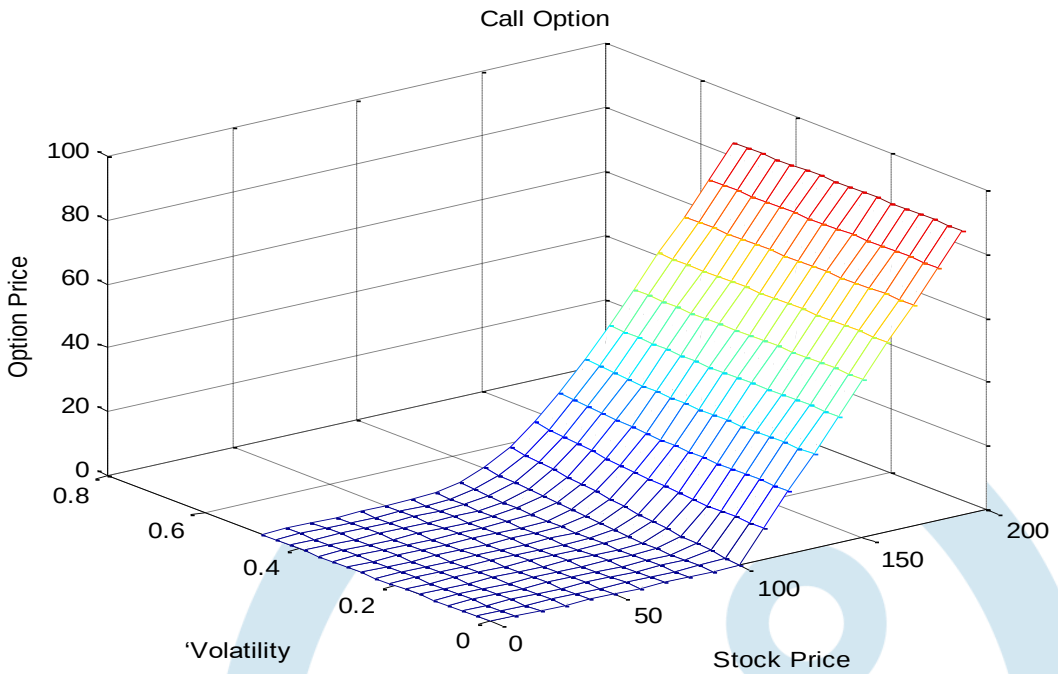

Fig.5(b)- Relation among volatility (50%), Stock Price and call Option Price

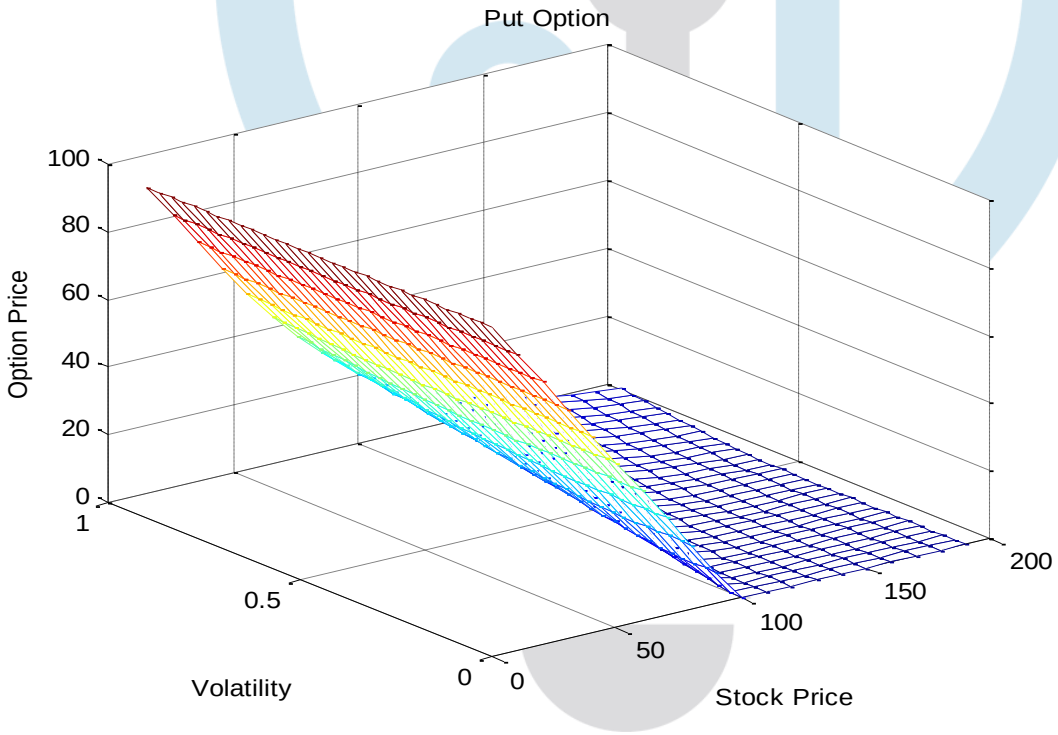

Fig.6(a)- Relation among volatility(90%), Stock Price and Put Option Price

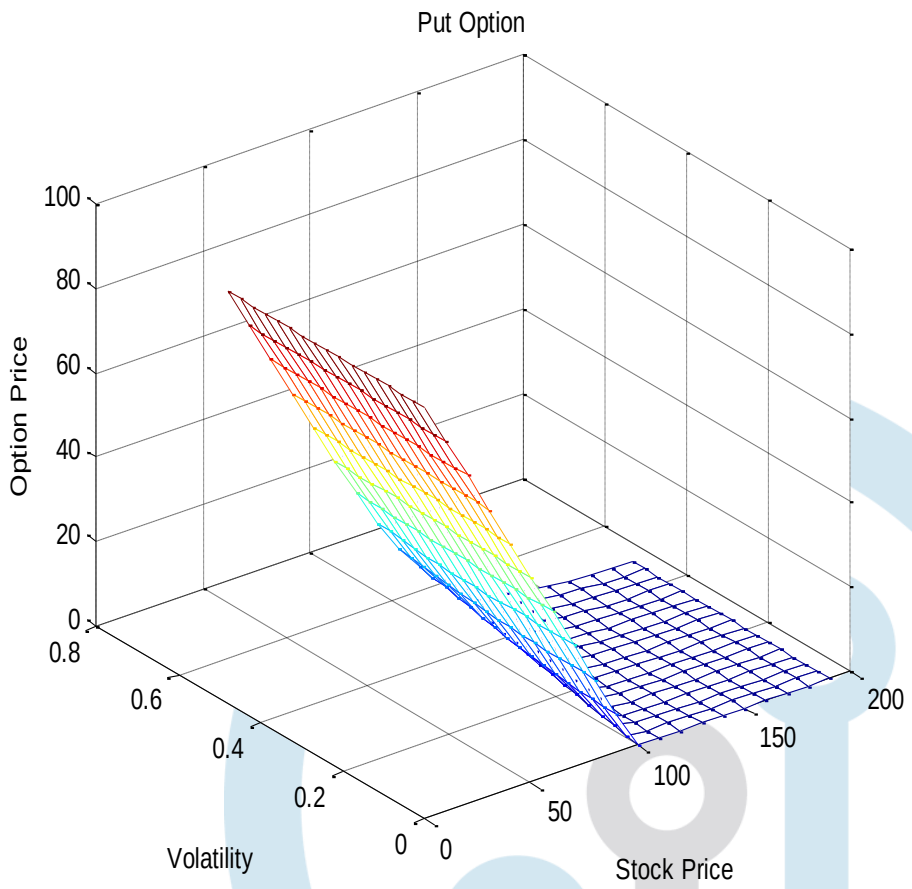

Fig.6(b)- Relation among volatility(50%), Stock Price and Put Option Price

Numerical Results & Discussion

We consider the current price of an asset is Rs.200 units, the exercise price of the option is Rs.100 units, the risk-free interest rate is 6%, the time to maturity of the option is 0.4 years, and the volatility of the asset is 90% and 50% .

In figure 5(a) when volatility is zero, the stock holder starts exercising his option when stock price becomes equal to strike price 100 units i.e stock is at the money. The option price increases with increase in stock price and the option price reaches its maximum price 80 units when stock price becomes 200 units i.e stock is in the money. This relation is linear when volatility is zero. But this relation becomes non-linear when volatility becomes greater than zero and increases up to 90%. In Figure 5(b) on the low volatility option price slowly increases as compared to that in Figure 5(a).

In Figure 6(a) & 6(b) it is it is observed that this relation between option price and stock price is linear when volatility is zero, but it becomes non linear when volatility increases. When volatility is zero , option price starts increasing when stock price is 100 units and option price goes an increasing upto 40 units. But when volatility is 90%,the option price in Figure6 (a) starts increasing when stock price becomes around 150 units. In the same way in Figure 6(b) when volatility is 50% the option price starts increasing when stock price is around 130 units.

Concluding Remark

The financial management is one of the most challenging task for any organization or an individual. It involves lot of planning, scheduling and controlling. If one plans to invest in financial securities then he has to optimize between fixed income securities and derivatives. The derivatives being more risky require very careful planning, which can be achieved using mathematical modelling. The basic concept and processes involved in financial management are presented in this paper which forms the basic foundation for mathematical modelling of financial problems.

In this paper the effect of volatility on option price has been obtained and it is concluded that the volatility has significant effect in the price of European options. The price of option increases significantly with the increase in volatility.

Refferences

1. Andrew M. Chisholm, Derivatives Demystified (2004). “A Step- by-Step Guide to Forwards, Futures, Swaps and Options” John Wiley & Sons Ltd.
2. Angelos Dassios & Jayalaxshmi Nagaradjasarma(2004) “Pricing Of Asian Options on Interest Rates in the CIR Model” Proceeding (437) Financial Engineering and Applications.
3. Ahoniemi Katja (2006) “Modeling and Forecasting Implied Volatility an Econometric Analysis of the VIX Index” HECER Discussion Paper No. 129, ISSN 17950562.
4. Alobiadi G, (2000) “American options and their strategies” Ph.D. thesis, University of Western Ontario, London, Canada,

5. Arora, K. and Sharma, M. (2013) 'Analysis of option pricing using implied volatility of equity option contracts', Emerging Trends in Business and Economy, Excel Books, pp.13–28.
6. Black. F., Scholes M., (1973)," The pricing of options and corporate liabilities", Journal of Political Economy vol. 81 pp 637–659.
7. Chung-Ki Cho, Sunbu Kang, Taekkeun Kim And Yonghoon Kwon, (2006). "Parameter Estimation Approach to the Free Boundary for the Pricing of an American Call Option" An International Journal Computers And Mathematics With Applications Vol.51 713-720.
8. Claus Munk (2005) " fixed income analysis, security pricing and risk management" Ph.D thesis of Department of Accounting and Finance, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark .
9. Cox J.C., Ross S.A., Rubinstein M.(1979) " Option pricing, A simplified approach" Journal of Financial Economics vol7, 229–263.
10. Chen Chao-Jung "Pricing European and American Options with Extrapolation" Ph.d thesis, Institute of Finance National Taiwan University.
11. Choi S. and Marcozzi M.D. (2001), "A Numerical Approach to American Curreny Option Valuation", Journal of Derivatives vol. 19,
12. Chang Lo-Bin & Palmer Ken (2007)" Smooth convergence in the binomial model" Finance Stoch vol. 11 pp. 91–105
13. Carlson Mark (2006) "A Brief History of the 1987 Stock Market Crash" with a Discussion of the Federal Reserve Response.
14. Christiane Lemieux & Pierre L'Ecuyer (1998) "Efficiency improvement by lattice rules for pricing asian options" Proceedings of the Winter Simulation Conference
15. Cornelis A. Los & Rossitsa Yalamova (2004) "Multifractal Spectral Analysis of the 1987 Stock Market Crash" International Research Journal of Finance and Economics ISSN 1450-2887 Issue 4
16. Donald Aingworth, Rajeev Motwani (1999) & Jeffrey D. Oldham "Accurate Approximations for Asian Options"
17. Duffie D, Pan. J, and Singleton K(2000) "Transform analysis and asset pricing for affine jump-diffusions" Econometrica, 68, pp. 1343–1376.
18. Duffie D (1996) "Dynamic Asset Pricing Theory", Princeton University Press,.
19. Nagendran, R. and Venkateswar, S. (2014) 'Validating Black-Scholes model in pricing Indian stock call options', Journal of Applied Finance & Banking, Vol. 4, No. 3, p.5.