Radiative Effects on Double Diffusive Free Convection Flow Past a Vertical Oscillating Plate in the Presence of Chemical Reaction with Variable Mass Transfer

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ABSTRACT: This paper aims to investigate the influence of chemical reaction, radiation and the combined effects of heat and mass transfer on flow over a vertical oscillating plate with constant heat flux. The basic equations governing the flow, heat transfer and concentration are solved by Laplace transform technique to obtain an exact solution of the problem. Numerical results for the velocity, temperature, concentration, skin friction are shown graphically for different values of physical parameters involved. The physical aspects of the problem are also discussed.

KEY WORDS: Free convection, mass transfer, chemical reaction, radiation, oscillating plate.

INTRODUCTION: The natural convection flows are particularly important in atmospheric and oceanic circulation, in the design of spaceships. Besides the importance of natural convection flows in many areas of interest in technology and nature, a study of natural convection processes is important in the problems of heat rejection and removal in many devices, processes, and systems.Natural convection has been analyzed extensively by many investigators. Some of them are Ibrahim et.al. [1], Amer et.al. [2].

Mass transfer is mass in transit due to a species concentration gradient in a mixture. By concentration gradient, we mean a spatial difference in the abundance of the chemical species. There are countless industrial applications, however one of the most common is used in petrochemical refining. Fractional distillation, whereby crude oil components are separated out, is a form of mass transfer technology. A number of investigations have already been carried out mass transfer flow under the assumption of different physical situations. Mohamed et.al.[3] and Hussanan et.al.[4] analyzed mass transfer flow past a vertical plate. Flow past a vertical oscillating plate with mass transfer is investigated by Chandrakala et. al. [5] and Adeyemi et.al. [6]

Chemical reaction can be codified as either homogeneous or heterogeneous processes. Chemical reaction has many applications in different chemical engineering processes and other industrial applications such as polymer production, manufacturing of ceramics . Mohamed et. al.[7] analyzed the flow over a surface in the presence of chemical reaction by finite element method. Unsteady flow past a vertical plate with chemical reaction is analyzed by Raghunath et.al.[8] and Sehra et.al. [9].

In the above mentioned studies the effects of radiation on flow has not been considered. Flow past a vertical plate with chemical reaction and radiation is analyzed by Nazibuddin et. al. [10] , Ramana et.al.[11] . Effects of radiation and chemical reaction on flow with variable temperature is pioneered by Rajput et.al. [12]. Sweta et.al. [13] studied radiation effects on free convection mass transfer fluid flow past a surface. Jain [14] studied the unsteady double diffusive convective flow past a vertical plate with radiation by laplace transform technique.

MATHEMATICAL ANALYSIS: We consider a two-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical plate. The x'-axis is taken on the infinite plate and parallel to the free stream velocity and y'-axis normal to it. Initially, the plate and the fluid are at same temperature T'_∞ with concentration level C'_∞ at all points. At time t' > 0, the plate concentration is changed to $C'_\infty + (C'_w - C'_\infty)\frac{u_R^2 t}{\upsilon}$ with heat supplied at a Constant rate to the plate and it starts oscillating with a velocity $U_R \cos \omega' t'$ in its own plane. It is assumed that viscous dissipation , heat produced by chemical reaction and assuming there exist a homogeneous chemical reaction of first order with constant rate K_l between the diffusing species and the fluid. Since the plate is infinite in extent therefore the flow variables are the functions of y' and t' only. The fluid is considered to be gray absorbing-emitting radiation but non scattering medium .The radiative heat flux in the x'-direction is considered negligible in comparison with of y'-direction. Then neglecting variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations:

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \qquad (1)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - k_l C' \qquad (2)$$

$$\frac{\partial u'}{\partial t'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta\,(T' - T'_\infty) + g\beta_c\,(C' - C'_\infty) \qquad (3)$$

with following initial and boundary conditions

$$u'=0,\ T'=T'_\infty,\ C'=C'_\infty \text{ for all } y',t'\le 0 \qquad (4)$$

$$u'=u_R\cos(\omega't'),\ \frac{\partial T'}{\partial y'}=\frac{-q}{\kappa},\ C'=C'_\infty+(C'_W-C'_\infty)\frac{u_R^2 t}{\upsilon} \quad \text{at } y'=0,\ t'>0$$

$$u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \qquad \text{as } y'\to\infty,\ t'>0$$

The radiative heat flux term, by using the Rosseland's approximation is given by

$$q_r=-\frac{4\sigma'}{3\kappa^*}\frac{\partial T'^4}{\partial y'} \qquad (5)$$

We assume that the temperature differences within the flow are such that T'^4 may be expressed as a linear function of the temperature T'. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms

$$T'^4\simeq 4T'^3_\infty T'-3T'^4_\infty \qquad (6)$$

By using equations (5) and (6), equation (1) gives

$$\rho C_p\frac{\partial T'}{\partial t'}=\kappa\frac{\partial^2 T'}{\partial y'^2}+\frac{16\sigma' T'^3_\infty}{3\kappa^*}\frac{\partial^2 T'}{\partial y'^2} \qquad (7)$$

Introducing the following dimensionless quantities

$$t=\frac{t'}{t_R},\ y=\frac{y'}{L_R},\ u=\frac{u'}{U_R},\ \omega=\omega' t_R,\ k=\frac{U_R^2 k_1}{\nu^2},$$

$$Pr=\frac{\mu C_p}{\kappa},\ Sc=\frac{\nu}{D},\ \theta=\frac{T'-T'_\infty}{\frac{q\nu}{\kappa U_R}},\ G=\frac{g\beta q\nu^2}{\kappa U_R^4}\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gm=\frac{\nu g\beta_C(C'_w-C'_\infty)}{U_R^3},\ k=\frac{\nu k_1}{U_R^2}$$

$$,\ R=\frac{\kappa^*\kappa}{4\sigma' T^3_\infty}$$

$$\Delta T=T'_w-T'_\infty,\ U_R=(\nu g\beta\Delta t)^{1/3},$$

$$L_R=\left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3},\ t_R=(g\beta\Delta T)^{-2/3}\nu^{1/3} \qquad (8)$$

On solving equations (7) ,(2) and (3) by Laplace-transform, we get

$$\theta=\sqrt{\frac{tH}{\pi Pr}}\,\mathrm{erfc}\left(\frac{-\eta^2 Pr}{H}\right)-\eta\sqrt{t}\ \mathrm{erfc}\left(\eta\sqrt{\frac{Pr}{H}}\right) \qquad (9)$$

$$C=\frac{t}{2}\{\exp(2\eta\sqrt{kSc\,t})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})+\exp(-2\eta\sqrt{kSc\,t})\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt})\}$$
$$+\frac{\eta\sqrt{Sct}}{2\sqrt{k}}\{\exp(2\eta\sqrt{kSc\,t})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})-\exp(-2\eta\sqrt{kSc\,t})\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt})\} \qquad (10)$$

For $Pr=Sc\neq 1$

$$u=\frac{\exp(i\omega t)}{4}\{\exp(2\eta\sqrt{i\omega t})\,\mathrm{erfc}(\eta+\sqrt{i\omega t})+\exp(-2\eta\sqrt{i\omega t})\,\mathrm{erfc}(\eta-\sqrt{i\omega t})\}$$
$$+\frac{\exp(-i\omega t)}{4}\{\exp(2\eta\sqrt{-i\omega t})\,\mathrm{erfc}(\eta+\sqrt{-i\omega t})+\exp(-2\eta\sqrt{-i\omega t})\,\mathrm{erfc}(\eta-\sqrt{-i\omega t})\}$$
$$+\sqrt{\frac{H}{Pr}}\frac{G\,t^{3/2}}{3\left(\frac{Pr}{H}-1\right)}\left\{\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2)-\eta(6+4\eta^2)\,\mathrm{erfc}(\eta)\right\}$$
$$+\sqrt{\frac{H}{Pr}}\frac{Gt^{\frac{3}{2}}}{3\left(1-\frac{Pr}{H}\right)}\left\{\frac{4}{\sqrt{\pi}}\left(1+\eta^2\frac{Pr}{H}\right)\exp\left(-\eta^2\frac{Pr}{H}\right)-\eta\sqrt{\frac{Pr}{H}}\left(6+4\eta^2\frac{Pr}{H}\right)\mathrm{erfc}\left(\eta\sqrt{\frac{Pr}{H}}\right)\right\}$$
$$+\frac{Gm}{kSc}\left\{t(1+2\eta^2)\,\mathrm{erfc}(\eta)-\frac{2\eta e^{-\eta^2}}{\sqrt{\pi}}\right\}+\frac{Gm(1-Sc)}{k^2Sc^2}\,\mathrm{erfc}(\eta)$$
$$-\frac{Gm(1-Sc)}{2k^2Sc^2}\exp\left(\frac{kSct}{1-Sc}\right)\left\{\exp\left(2\eta\sqrt{\frac{kSct}{1-Sc}}\right)\mathrm{erfc}\left(\eta+\sqrt{\frac{kSct}{1-Sc}}\right)+\exp\left(-2\eta\sqrt{\frac{kSct}{1-Sc}}\right)\mathrm{erfc}\left(\eta-\sqrt{\frac{kSct}{1-Sc}}\right)\right\}$$
$$-\frac{Gmt}{2kSc}\{\exp(2\eta\sqrt{kSct})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})+\exp(-2\eta\sqrt{kSct})\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt})\}$$
$$+\frac{Gm\eta}{2kSc}\sqrt{\frac{tSc}{k}}\{\exp(2\eta\sqrt{kSct})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})-\exp(-2\eta\sqrt{kSct})\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt})\}$$

$$-\frac{Gm(1-Sc)}{2k^2Sc^2}\left\{\exp\left(2\eta\sqrt{kSct}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{kt}\right)+\exp\left(-2\eta\sqrt{kSct}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{kt}\right)\right\}$$
$$+\frac{Gm(1-Sc)}{2k^2Sc^2}\exp\left(\frac{kSct}{1-Sc}\right)\left\{\exp\left(2\eta\sqrt{\frac{kSct}{1-Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{\frac{kt}{1-Sc}}\right)+\exp\left(-2\eta\sqrt{\frac{kSct}{1-Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{\frac{kt}{1-Sc}}\right)\right\} \quad (11)$$

Where

$$H = 1+\frac{4}{3R}$$

SKIN-FRICTION: From velocity field , skin-friction at the plate in non dimensional form is expressed as:

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\tau = \frac{\exp(i\omega t)}{2\sqrt{t}}\left\{\sqrt{i\omega t}\,\mathrm{erf}(\sqrt{i\omega t})\right\}+\frac{\exp(-i\omega t)}{2\sqrt{t}}\left\{\sqrt{-i\omega t}\,\mathrm{erf}(\sqrt{-i\omega t})\right\}$$
$$+\frac{G\,t\,H}{(Pr-H)}\sqrt{\frac{H}{Pr}}\left(1-\sqrt{\frac{Pr}{H}}\right)+\frac{1}{\sqrt{\pi t}}$$
$$+\frac{Gm(1-Sc)}{k^2Sc^2}\left(\exp\left(\frac{kSc\,t}{1-Sc}\right)\right)\sqrt{\frac{kSc}{1-Sc}}\left\{\mathrm{erf}\left(\sqrt{\frac{kSc\,t}{1-Sc}}\right)+\mathrm{erf}\left(\sqrt{\frac{kt}{1-Sc}}\right)\right\}$$
$$-\frac{Gm\,t}{k\,Sc}\left\{\sqrt{kSc}\,\mathrm{erf}\left(\sqrt{kt}\right)+\sqrt{\frac{Sc}{\pi t}}\,e^{-kt}\right\}$$
$$-\frac{Gm(1-Sc)}{k^2Sc^2}\left\{\sqrt{kSc}\,\mathrm{erf}\left(\sqrt{kt}\right)\right\}+\frac{Gm}{2k^{3/2}\sqrt{Sc}}$$
$$+\frac{Gm(1-Sc)}{k^2Sc^2\sqrt{\pi t}} \quad (12)$$

DISCUSSION: Figure 1 reveals temperature profiles against η (distance from the plate). It is clear from the figure that the magnitude of temperature is maximum at the plate and then decays to zero as $\eta \to \infty$. Further, the magnitude of the temperature for air is greater than that of water. This is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting a decrease in thermal boundary layer thickness. It is also seen that it decreases steeply for Pr = 7 than that of Pr = 0.71. Furthermore, it is cleared that an increase in radiation parameter to 4, 15 the temperature decreases but it increases with increasing time.

The species concentration profiles verses η is presented in figure 2. It is observed that the concentration at all points in the flow field decreases with η and tends to zero as $\eta \to \infty$. Furthermore, an increase in the value of Sc leads to a decrease in concentration boundary layer thickness. Physically, it is true since increase of Sc means decrease of molecular diffusivity which results in decrease of concentration boundary layer. Hence, the concentration of species is higher for small values of Sc. It is also observed that concentration increases with an increase in time on the other hand it decreases with an increase in chemical reaction parameter k. It is because increase in k means increase in Sc so the same effect is observed as that in the case of increase of Sc.

Figure 3 illustrates the effect of ωt and Pr on the velocity. It is evident from the figure 3 that the velocity increases and attains its maximum value in the vicinity of the plate ($\eta \leq 0.5$) and then tends to zero asymptotically. It is also observed that both the velocity and the penetration for Pr = 0.71 is higher than that of Pr = 7. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. It is also seen that the velocity decrease with an increase in ωt. Figure 4 elucidates the influences of Sc, t, and Pr on the velocity. It is noticed that the maximum velocity attains near the plate then decreases and vanish far away from the plate. when Pr = 7 and 0.71 it decreases continuously to asymptotic value. Further, they also increases with an increase in time at each point in the flow field for Sc=022. Moreover, the effect of time on the velocity is more dominant than other parameters. Finally, we observed that the velocity for Hydrogen(Sc=0.22) is higher than that of water vapour(Sc=0.60) and carbon di oxide(Sc=0.96) for both air and water.

Figure 5 elucidates the effects of parameters Sc, k, Gm, ωt on skin friction at the plate at different values of t. It is cleared from the figure that skin- friction decreases with an increase in time and for ωt=0 it becomes negative means separation of boundary later occurs for higher values of t (t>0.3). The figure depicts that skin friction grow more for heavier particles (Sc=0.96) than for lighter particles. Physically, it is correct since an increase in Sc serves to increase momentum boundary layer thickness. Further, it decreases with an increase in value of Gm and marginally increases with an increase in k.

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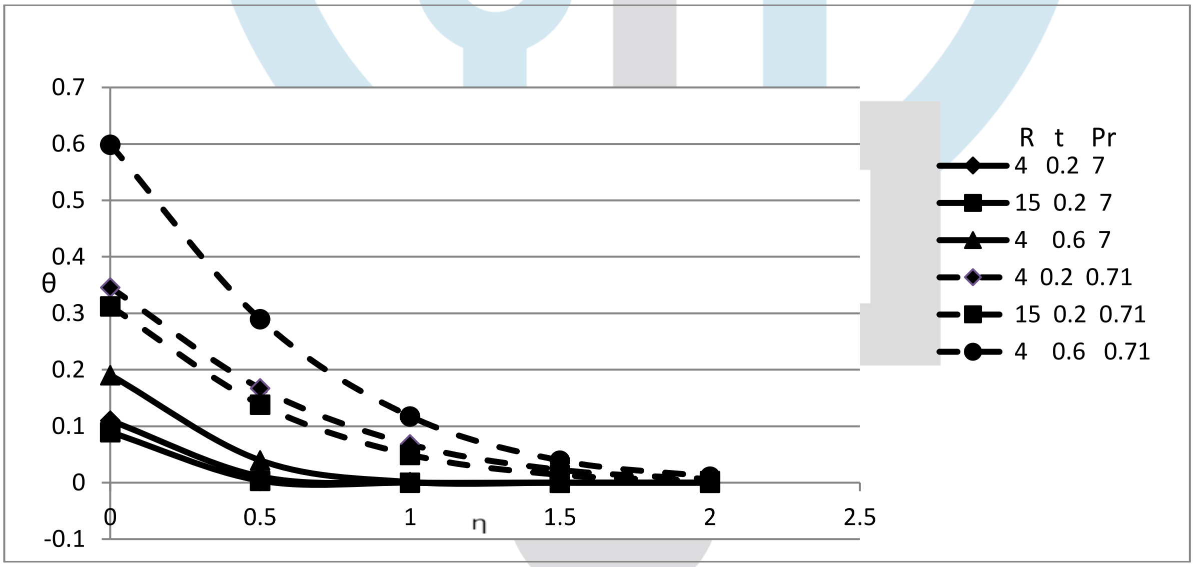

Figure1: Temperature profile

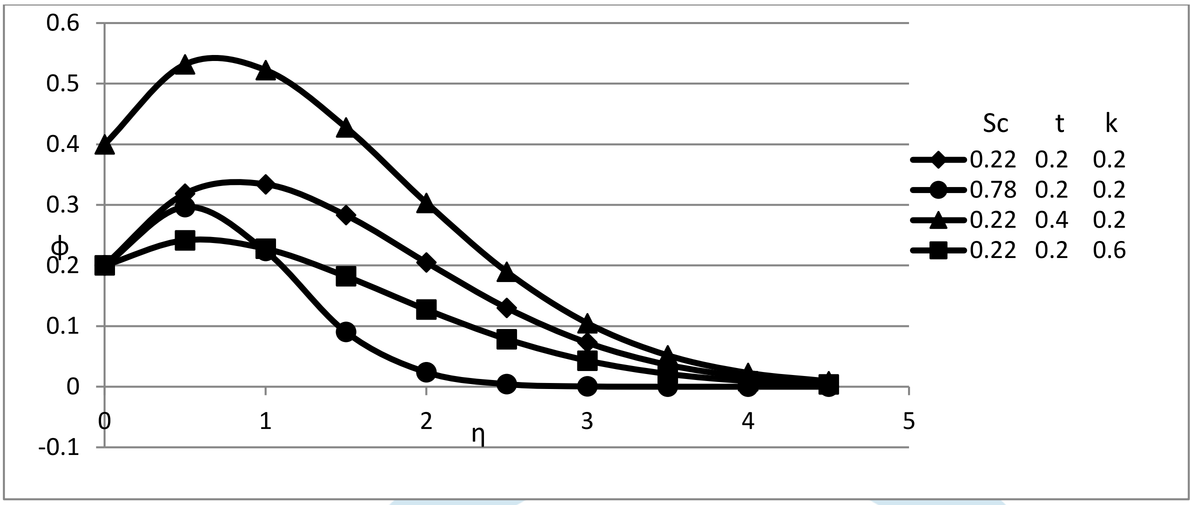

Figure 2: Concentration profile

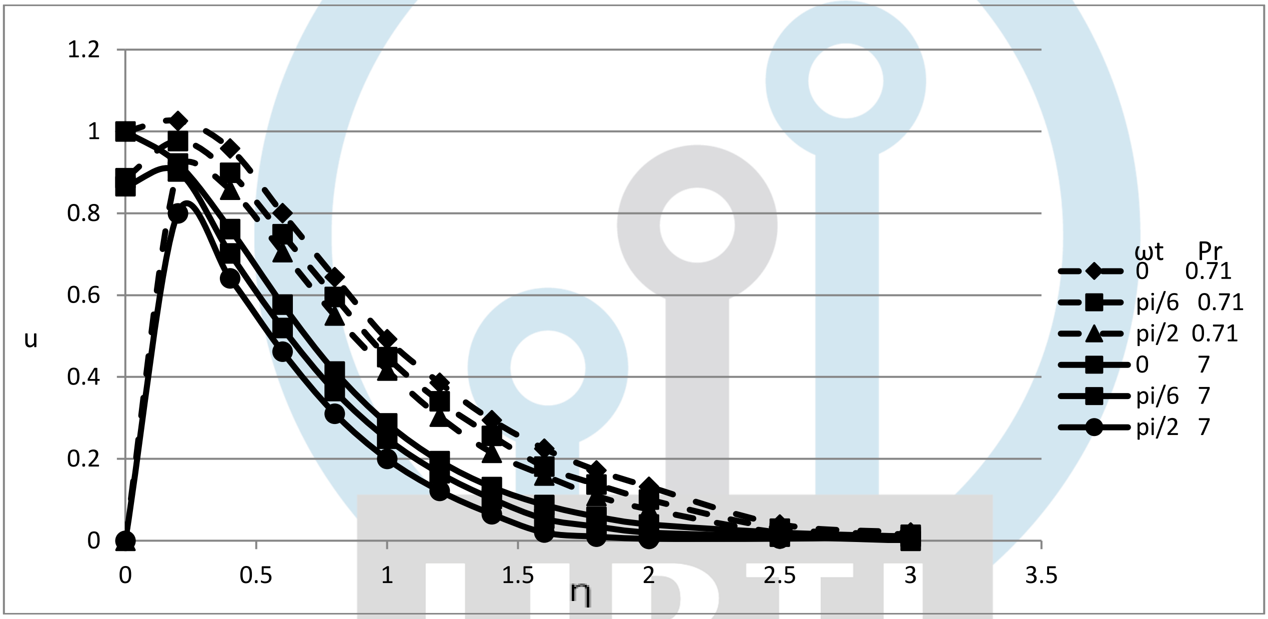

Figure 3 : Velocity profile for sc=0.22, t=0.2, R=4, k=0.2, G=5, Gm=2

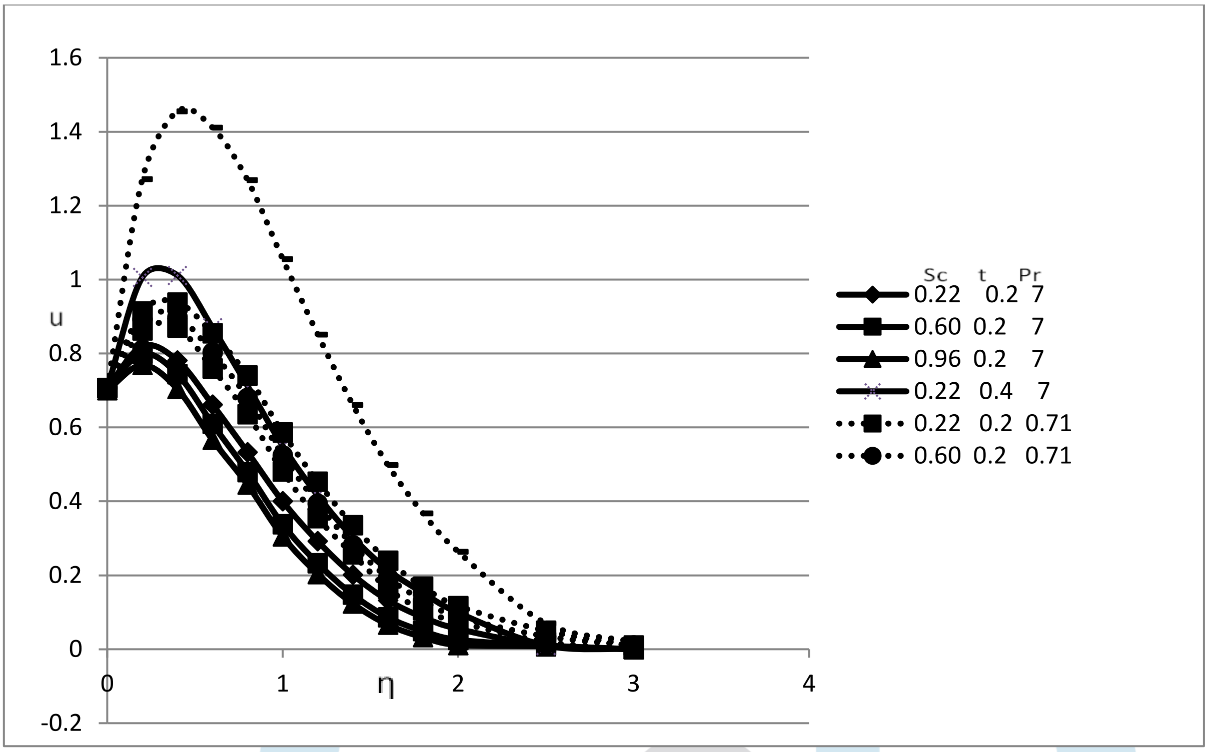

Figure 4: Velocity profile ωt=π/4, R=4, k=0.2, G=5, Gm=2

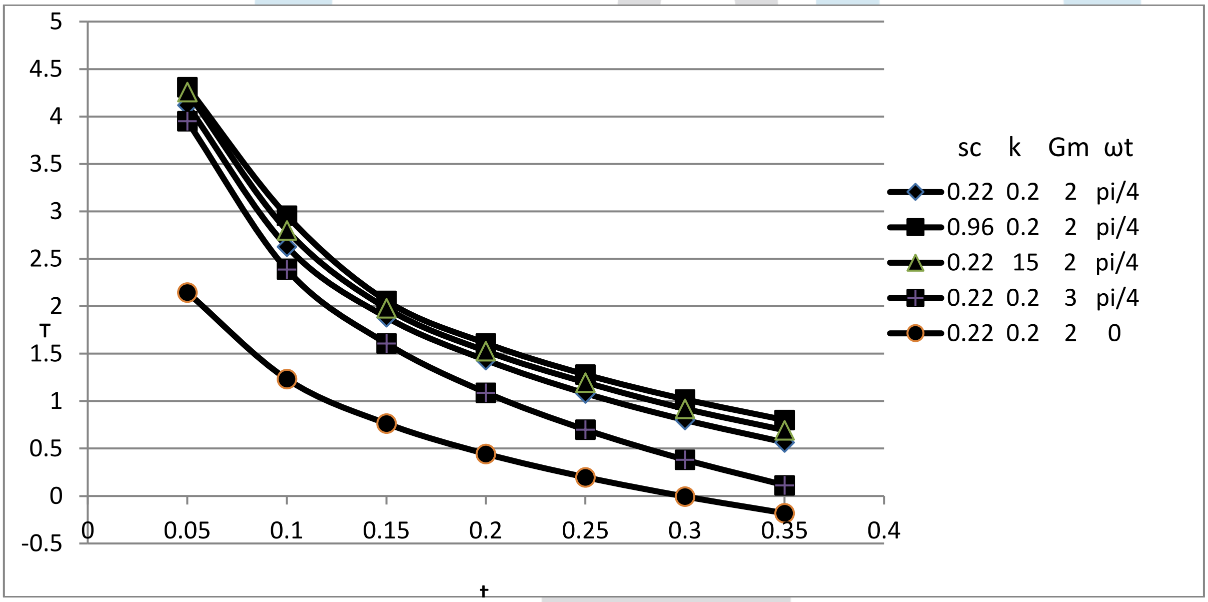

Figure 5: Skin friction for R=4, G=5, Pr=7