AN ANALYTICAL STUDY ON MATHEMATICAL MODELS OF INVISCID FLUID ITS APPLICATIONS AND IMPLEMENTATION

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***Abstract*: Mathematical modelling helps us understand physical processes in science and engineering by providing a mechanism to describe those processes. The other inquiries can also be addressed using models of fluid motion. This work discusses the use of mathematical modelling to study the movement of inviscid Fluid. The Navier-Stokes equations for viscous fluid or Euler's equations for inviscid fluid are the basic equations used to represent the motion of a fluid. These can be expressed in a variety of ways, such as in a rotating frame of reference, leading to a hierarchy of distinct equations under various balancing circumstances, each of which is a reliable approximation in a variety of real-world contexts. Considering the basic mathematical characteristics of these several sets of equations can provide insight into the mechanisms driving the fluid flow being represented. Each of these types of equations operates slightly different. Free boundary issues, nonlinear surface waves, compressible fluid, shock waves, approximations for compressible flow, and complicated variable approaches are some of the subjects that are taken into consideration. These subjects are thoroughly studied from both a mathematical and physical perspective.**

***Keywords*: Inviscid, Viscosity, Hydrodynamics, Incompressibility, Turbulent, Superfluidity.**

INTRODUCTION

Fluid flows are the focus of fluid dynamics, a branch of fluid mechanics. It can be used to predict climate patterns, solve a wide range of engineering issues involving Fluid in movement, comprehend the intricate dynamics of gaseous stars and the interstellar space, and even solve some traffic issues once the traffic can be regarded as a continuous liquid, to name just a few applications. The majority of theoretical research focuses on mathematical equations of idealised incompressible flows, even though the majority of these users with compressible or at least marginally compressible Fluid (Jeon, et al. 2016). The comprehensive approach of the classical Navier-Stokes system of equations, which describes the motion of an incompressible viscous fluid, is still largely unsolved despite a coordinated and sustained effort of centuries of excellent mathematicians; for more information, see the survey of Fefferman. Simple mathematical models, such as those based on the idea of incompressibility, can be seen as the boundaries of more complicated systems, where particular distinctive parameters become insignificant or predominate. In many situations, these boundaries can be explicitly derived (Marchioro, et al. 2012). The fundamental physical concepts of mass, momentum, and energy efficiency as well as general thermodynamic relationships between the related macroscopic system parameters are reflected in the common primitive formulas used in mathematical hydrodynamics. The system must be expressed in terms of non-dimensional variables adjusted by an appropriate system of distinctive units in order to reflect the distinctive characteristics of a particular fluid flow. This makes it easy to tell apart an equation system created to predict the development of a gaseous star from one created to explain the rate of mass flow of petroleum in a pipeline (John, et al. 2017).

Generally, fluid dynamics explains how Fluid, including gases and liquids, flow. In relation to this thesis work, hydrodynamics—the study of moving liquids—is of special importance. The so-called "continuum assumption" is based on the representation of the flow on a macroscopic scale. Although we refer to liquids as continuous medium rather than discrete ones even if in reality they are made up of molecules that clash with one another or the area boundary, for instance (Bhatti, et al. 2020). Since functions specified on the domain are used to represent system attributes and variables like density, pressure, and flow velocity, they are well-defined at infinitesimal volume elements of the fluid domain and correlate to insignificantly averaged real tangible values. Therefore, differential equations that capture the "continuous evolution" of these parameters can be used to describe movement of fluid. In particular, the conservation of mass and the conservation of (linear) momentum are two basic conservation rules that these phenomena result from mathematically (Monaghan, 2012).

The movement of an inviscid fluid, in which the fluid's viscosity is equal to zero, is referred to as inviscid flow. Although inviscid Fluid, also referred to as super Fluid, are rare, fluid dynamics uses inviscid flow extensively. As the viscosity gets closer to zero, the Reynolds number of an inviscid flow hits infinity. The Navier-Stokes equation can be reduced to a method known as the Euler equation when viscous forces are disregarded, as in the scenario of inviscid flow. This condensed equation can be used to describe both inviscid flow and movement with low viscosity and a Reynolds number significantly higher than one (Ockendon, et al. 2013). Many low viscosity fluid dynamics issues can be easily solved using the Euler equation, but the assumption of marginal viscosity is no longer applicable in the fluid areas close to a solid boundary or, more commonly, in areas with large velocity profiles that are obviously followed by viscous forces. Potential flows (also known as irrotational flows) and rotational inviscid flows are the two primary categories used to describe inviscid flows (Eldredge, 2019).

In the fields of gas dynamics, acoustics, electro- and magneto-gas dynamics, restricted gas dynamics, plasma dynamics, etc., inviscid flows are studied. A quantitative description of the behaviour of the fluid particles making up the medium is employed in the concept of restricted gases and plasma. With the use of modest, volume-averaged quantities of mass, momentum, and energy, the flow may be deemed to fall within the purview of the ongoing medium model in other circumstances (Sharipov, 2012). The model equation must be used to link a fluid's velocity, pressure, and density (at locations in space where they are consistent) if its viscosity and thermal conductivity are neglected:

$$\rho \frac{d\vec{v}}{dt} = \rho\vec{F} - \mathrm{grad}\,p,$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\mathrm{div}\,\vec{v},$$

$$\rho \frac{d}{dt}\left(U + \frac{v^2}{v}\right) = \rho\vec{F}\vec{v} - \mathrm{div}\,p\vec{v} + \rho q.$$

The first equation, known as Eider's equation, connects a fluid particle's acceleration to an external body push and the compression force exerted on its side by adjacent fluid particles. The generalisation of Newton's second law to the movement of fluid particles is represented by this equation (Jayawardena, 2021). The second equation demonstrates the law of mass conservation, which states that the rate of change in a fluid particle's density is equal to its rate of change in volume with the sign switched. A change in a fluid particle's internal and kinetic energy is caused by the action of imposed mass forces and surface forces (pressure p), as well as by an influx of heat with intensity q from an external source (de Jong, 2014).

Density can fluctuate significantly as a result of a large internal heat release or an external heat influx. The main characteristic of many issues that the inviscid flow model can be used to tackle is the compaction of a material. To determine the rates of underlying physicochemical transitions, chemical reactions between elements of a mixture, dissociation or ionisation, as well as for the stimulation of intrinsic degrees of freedom, etc., kinetic formulas must be added to the integrals for an inviscid flow in a number of challenges (Yang, et al. 2013).

On some substrates inside the flow area, the characteristics of gas can experience abrupt changes. Irregular surfaces are divided into two categories. An irregular surface is said to be tangential if no flow can pass through it. A shock wave is a common illustration of an irregular surface of the second type. The formulas for the finite gas volume are stated in integral form in this instance because the flow of fluid particles contacts the irregular surface (Zeng, 2017). Difficulties resolved with the use of an inviscid flow model are separated into internal and exterior problems based on geometrical parameters. Internal flows include those that occur in nozzles, gas turbine blades, aerodynamic wind tunnels, etc. Such a dilemma pertains to a class of extrinsic inviscid flows if a streamlined body is submerged in a liquid whose velocity at infinity is characterised by a homogeneous profile (Huang, et al. 2020).

Air and water, the two most significant Fluid, have sizable coefficients of viscosity. The inviscid flow model, however, provides a fair representation of working mechanism at a distance from the enclosing solid substrate even for them. The Navier-Stokes equations for fluid movement, which, in contrast to Euler's equations, account for internal friction or tangential pressures between nearby layers as well as for the medium's molar thermal conductivity, are required for a more rigorous formulation of the problem (Boyer, 2012). The degree of the differential equations decreases even if the Navier-Stokes equations turn into Euler ones at the limit when the Reynolds number Re approaches infinity. As a result, the boundary conditions of a full system of equations will not be satisfied by the solution of simpler equations (Buckmaster, 2019). Additionally, it can be assumed that in the special case of Re is infinity, the Navier-Stokes equations' solutions rely on the notion that the entire area of flow can be split into two regions: an external flow region that is isolated from the body by the thin coating and proliferates beyond it in the form of a thin wake. It is well recognised that this split is incorrect in a number of situations, such as when the border layer is separated (Boyer, et al. 2012).

The self-similarity structure is represented by a homogenous supersonic flow revolving around an endless thin cone around stationary inviscid flows. In this situation, it is difficult to determine a typical linear measurement. The picture stays the same regardless of how many times the flow field around the cone vertex is extended or compressed; it is unchanged. Fluid flow in an axially-symmetric movement around the cone is delayed first in a conical shock wave (OS) trailing at the cone vertex, and then in a compacted layer of gas existing between a shock wave and the body, turning the flow towards the path of the cone base (Zhu, et al. 2013).

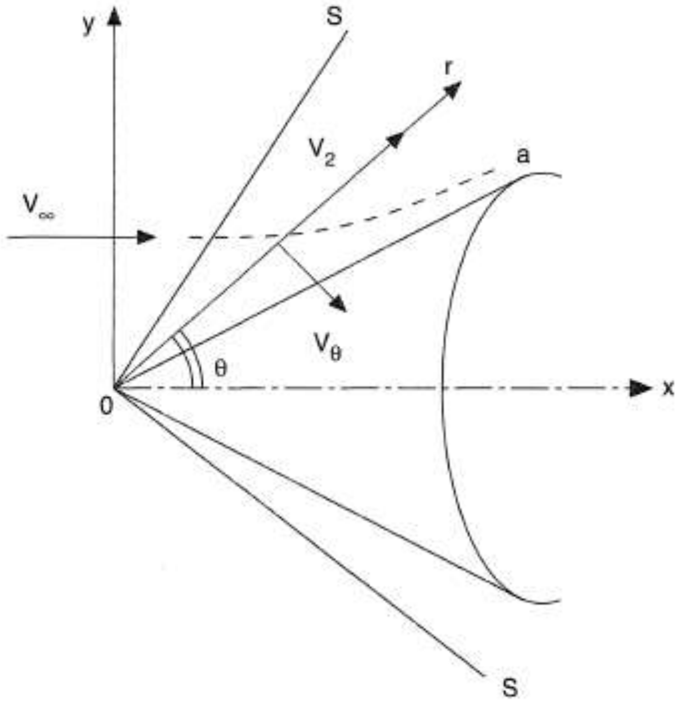


Figure: Homogeneous supersonic flow around an infinite thin cone.

Prandtl hypothesis

The surface was given its modern definition by Ludwig Prandtl. According to his theory, shear forces caused by viscosity are only visible at thin sections at the fluid's border that are close to hard substrates. Viscous shear forces are negligible outside of these zones and in areas with a good pressure gradient, therefore the fluid flow field can be taken to be identical to the movement of an inviscid fluid (Georgievskii, et al. 2019). By considering inviscid flow and examining the irrotational flow characteristics around the solid body, the Prandtl hypothesis can be used to predict the movement of a real fluid in areas of favourable differential pressure. Inviscid flow cannot accurately simulate the separation of boundary layers and turbulent wakes that happen in real Fluid. It is erroneous to utilise inviscid flow to calculate the flow of a real fluid in areas of unfavourable pressure gradient because dissociation of the boundary layer typically happens where the pressure gradient changes from favourable to unfavourable (Roy, et al. 2018).

Reynold's Number

In hydrodynamics and engineering, a dimensionless statistic known as the Reynolds number (Re) is frequently utilised. George Gabriel Stokes first described it in 1850, and Osborne Reynolds popularised it. After Reynolds, Arnold Sommerfeld called the idea in 1908. The Reynolds number is determined as follows (Cramer, et al. 2014):

$R_e = l_c v_p / \mu$

The ratio of inertial to viscous forces in a fluid is represented by the value, which can be used to assess the relative significance of viscosity. Since there are no viscous forces in an inviscid flow, the Reynolds number tends to infinity. The Reynolds number is significantly higher than one when viscous forces are minimal. Assuming inviscid flow in these circumstances (Re>>1) can be helpful under streamlining numerous fluid dynamics issues (Constantin, et al. 2018).

To depict fluid motion, analytical solutions in hydrodynamics are presented. The Euler and Navier-Stokes equations, which apply to inviscid and viscous Fluid, correspondingly, are the fundamental calculations for Newtonian incompressible Fluid (Sukhinov, et al. 2020). These nonlinear partial differential equations (PDEs) depict the development in time of the velocity and pressure at each point along the fluid, considering the initial velocity and proper boundary constraints, for a particular set of body pressures exerted on the fluid. The hydrodynamic equations present difficult mathematical difficulties, such as establishing the existence and distinctiveness of solutions, figuring out their uniformity, asymptotic behaviour for long periods of time, and sustainability (Tadmor, 2012).

Stochastic analysis is incorporated into hydrodynamics to shed some light on fluid behaviour. There have been several attempts to define a tumultuous regime. However, because the dynamics in a turbulent flow regime are chaotic and extremely unstable, it is difficult to analyse individual solutions that govern the flow at any moment, for a particular initial state. This specific chaotic movement has several distinctive statistical properties (Maharshi, et al. 2018). The goal of a statistical measure of a turbulent flow is to highlight some important cumulative flow characteristics that, ideally, enable understanding of the key dynamics. Stochastic hydrodynamics is relevant to the kinetic gas concept in this way. The velocity field's lack of spatial uniformity is yet another trait of turbulent flows. Since they are first order PDEs rather than second-order PDEs, the Euler equations are a peculiar limit of the Navier-Stokes equations. It comes as no surprise if they use various mathematical methods (Kim, et al. 2017).

The significance of molecular and eddy viscosity to the transmission of momentum can frequently be disregarded for gas flow at intermediate pressures at and above the velocity of sound. In these circumstances, the model equations govern the conservation of mass, the conservation of energy, and the conservation of momentum (without a viscous term). Since the eddy viscosity is not considered, a turbulence model is not required. Heat transfer by conduction can be compared to viscous momentum transfer in the energy equations (Davies, 2012). In reality, the same mechanism in gases that causes viscosity also causes thermal conductivity, and the eddy diffusivity for momentum transfer is utilised to calculate the eddy diffusivity for heat exchange. As a result, in the energy equations, we can typically neglect heat transmission through conduction in situations where we can disregard viscous momentum transfer (Yang, et al. 2017).

Incompressible inviscid flow:

Streamlines, Stream Tubes, and Stream Filaments

A streamline is a distinction made in a fluid whose tangent at each site is parallel to the local fluid velocity. The constant flow pattern is made up of the total number of streamlines at any particular time. A stream tube is made up of the streamlines that are drawn through every point on a closed curve. The definition of a stream filament is completed by stating that it is a stream tube whose cross-section is a curve with microscopic dimensions (Sakajo, et al. 2014).

The arrangement of the stream tubes and filaments varies periodically if the flow is erratic. The stream tubes and filaments, on the other hand, are immobile if the flow is constant. In the latter scenario, a stream tube mimics a tube that a fluid is actually flowing through. This arises because the flow is, by default, always tangential to these walls, hence there can be no flow past the walls and into the tube. Furthermore, because the motion is constant, the walls are fixed in both space and time. Therefore, if the walls were replaced with a stiff frictionless barrier, the fluid's motion inside the tube would remain unaffected (Spurk, et al. 2020).

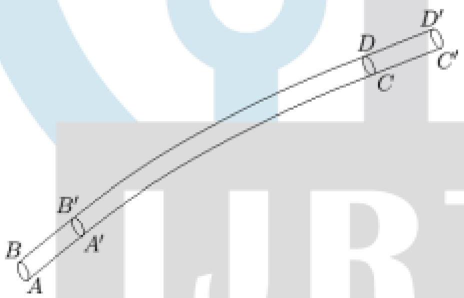
Consider a steady-moving stream thread of an incompressible fluid. Assume that the filament's cross-sectional region is so small that the fluid velocity is constant across the cross-section. Allow the cross-section to be normal to the direction of this shared velocity everywhere as well. Assume that the flow rates at two places on the filament with the cross-sectional areas of S_1 and S_2, respectively, are v_1 and v_2. Think about the portion of the filament that lies between these locations. Due to the fluid's incompressibility, the same amount of fluid must move into one end of the section and out of the other in a specific amount of time, implying that (Coco, 2020)

$V_1 s_1 = V_2 s_2$

Bernoulli's theorem

The most basic version of Daniel Bernoulli's (1700–1783) principle says that the quantity is consistent along a streamline in the constant supply of an inviscid fluid, where p is the pressure, ρ is the density, and T is the total energy per unit mass (Misaiko, et al. 2014).

$$\frac{p}{\rho} + \mathcal{T}$$



Euler equations:

It is customary to refer to the conservation equations for inviscid flow and minimal thermal conductivity as the Euler equations in honour of the renowned Swiss mathematician who initially proposed them. These are the Euler equations (Toda, et al. 2015):

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) &= -\nabla p + \mathbf{F} \\
\frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}u^2\right)\right] + \nabla \cdot \left[\rho \mathbf{u}\left(e + \frac{1}{2}u^2\right)\right] &= \nabla \cdot (-p\mathbf{u}) + \mathbf{u} \cdot \mathbf{F} + Q
\end{aligned}$$

Helmholtz's Theorem:

Helmholtz's theorems, which bear his name, explain the three-dimensional motion of fluid near vortex filaments in fluid mechanics. These axioms hold for inviscid flows as well as flows where the impact of viscous forces is negligible and may be disregarded (Heras, 2016).

The three theorems of Helmholtz are as follows (Kustepeli, 2016):

- First Helmholtz theorem- A vortex filament's strength is consistent throughout its length.
- The second theorem of Helmholtz- A vortex filament must reach the fluid's edges or create a closed channel in order to not end in the fluid.

- The third theory of Helmholtz- A fluid that is originally irrotational stays irrotational in the absence of rotating external forces.

For inviscid flows, Helmholtz's theorems are applicable. When vortices in actual Fluid are observed, their strength always decreases progressively as a result of the energy dissipation power of viscous forces. Helmholtz's theorems can be used to comprehend (Wu, 2018):

- μLift production on an air-foil
- beginning vortex
- Equine vortex
- Wingtip vortices.

Navier-Stokes Equation

Another significant set of equations, now referred to as the Navier-Stokes equations, was published by George Gabriel Stokes in 1845. The equations were initially created by Claude-Louis Navier applying molecular theory, and Stokes later confirmed them using continuum theory. The fluid motion is described by the Navier-Stokes equations (Braack, et al. 2014):

$\rho\ D_v/D_t = -\Delta p + \mu\Delta^2 v + \rho g$

The Navier-Stokes equation can be reduced to its simplest form, the Euler solution, when the fluid is inviscid or its viscosity can be considered to be negligible. This approximation makes the problem considerably simpler to answer and is applicable to many different forms of flow where viscosity is minimal. Ocean currents, upstream movement over ramps in a river, and flow around an aeroplane wing are a few examples. When $\mu = 0$, the Navier-Stokes equation is reduced to the Euler equation. $\Delta^2 v = 0$ results in a "inviscid flow arrangement," which is another criterion that eliminates viscous force. These flows are observed to resemble vortices (Tao, 2019).

Solid boundaries

It is significant to note that near solid borders, as in the case of an aeroplane wing, insignificant viscosity can no longer be maintained. Viscosity may often be ignored in turbulent low regimes (Re >> 1), although this is only true when far from solid contacts. It is useful to divide flow into four different areas when thinking about flow close to a solid surface, like flow through a pipe or around a wing (Zhang, et al. 2014).

Main turbulent stream: Viscosity can be disregarded farthest from the surface.

Viscosity is only marginally significant in the inertial sub-layer, which is where the main chaotic stream begins.

Buffer layer: The transitional layer between the viscous and inertial layers.

Nearest to the surface is the viscous sub-layer, where viscosity is significant (Liu, et al. 2017).

It's crucial to keep in mind that these boundaries are very arbitrary, even if they can be a valuable tool for demonstrating the importance of viscous forces close to solid contacts. Many hydrodynamic issues can be solved by maintaining inviscid flow, however when solid barriers are involved, this hypothesis necessitates careful analysis of the fluid sub layers (Adami, et al. 2012).

Super-Fluid

The state of matter known as superfluid, commonly referred to as inviscid flow, is characterised by frictionless flow and zero viscosity. The only fluid that displays superfluidity that has been identified so far is helium. When helium-4 is cooled to below the lambda point, which is 2.2K, it turns into a superfluid. Helium exists as a liquid that displays typical fluid dynamic characteristics above the lambda point. When cooled to below 2.2K, it starts to behave in a quantum way. For instance, the heat capacity increases significantly near the lambda point and then sharply decreases when the temperature is further lowered. Additionally, superfluid helium has a very high thermal conductivity, which enhances its superior cooling capabilities. Similarly, it is discovered that helium-3 becomes superfluid at 2.491 mK (Reeves, et al. 2015).

The construction of numerous models for the streamline modeling relied heavily on the Prandtl-Batchelor hypothesis and free streamline theory, two existing streamline concepts. In closed streamline conditions, the Prandtl-Batchelor theorem holds true for flows with large Reynolds numbers, but it is incorrect in the boundary layer domain. Contrarily, freestream line theory makes use of Laplace transformations and conventional Helmholtz free border theory. Relying on it, a few interesting mathematical models for a variety of challenging hydrodynamic and aerodynamic scenarios, including wake models, cavity streams, free-surface disturbances brought on by viscosity, and turbulence, have recently been constructed (Templalexis, 2014). Many concepts have utilised mass and energy conservation and dissipation strategies in the past. These methods provide an overall fluid flow within the specified boundary parameters. They make it difficult to describe local parameters inside the fluid flow, though. Regarding distinct circumstances of fluid flow and various kinds of Fluid, several types of solvers employ diverse methodologies and equations. To analyze local parameters that contain flaws, the majority of them use differential equations and partial differential equations. In recent decades, streamline simulation has grown in popularity among researchers as a possible alternative to more established techniques. Additionally, experimental values are always needed because the computational techniques, fundamental equations, and computing power all play a significant role in how reliable CFD results are (Chen, et al. 2016).

Although the body of available evidence suggests that shear stress and dynamic pressure patterns over a body's surface in a stream are the causes of streamline formation. In terms of the fluid, the flow, the structure, and the dimensions of the body, there is effectively no mathematical formula that can describe how or why streamlines are formed. A mathematical theory known as "Streamline's Shape Theory" makes the claim that it can explain how streamlines form when a Newtonian fluid flows over an object in an open circuit. Theoretically, when a solid body is strong enough to endure the deformation and dislocation, laminar flow of a Newtonian fluid over it will cause the fluid's particles to move. The fluid's dynamic pressure as well as the distribution of shear stress over the body help to generate a streamline (Watt, et al. 2012).

CONCLUSION

Following an overview of multi-phase and multi-fluid flows and their significance for superconductivity, we provided the likely most straightforward one-dimensional two fluid model. Through the continuity equations, where the dissimilarities of the fluid fields were adjusted with the void fraction, the Euler and Navier-Stokes equations are coupled together. Evidence that studying hydrodynamic systems is vital for human culture and society does not need to be proven. The second piece of evidence is the practically unlimited variety of flows that exist in both nature and engineering, some of which are perfect Fluid and others of which are viscous. We could focus on systems with multi-phase flows when a liquid moves alongside its vapour or another non-condensable substance. To make the current model more sophisticated, additional work needs to be done.

REFERENCES

1. Adami, S., Hu, X.Y. and Adams, N.A., 2012. A generalized wall boundary condition for smoothed particle hydrodynamics. *Journal of Computational Physics*, *231*(21), pp.7057-7075.
2. Bhatti, M.M., Marin, M., Zeeshan, A. and Abdelsalam, S.I., 2020. Recent trends in computational fluid dynamics. *Frontiers in Physics*, *8*, p.593111.
3. Boyer, F. and Fabrie, P., 2012. *Mathematical Tools for the Study of the Incompressible Navier-Stokes Equations andRelated Models* (Vol. 183). Springer Science & Business Media.
4. Braack, M. and Mucha, P.B., 2014. Directional do-nothing condition for the Navier-Stokes equations. *Journal of Computational Mathematics*, pp.507-521.
5. Buckmaster, T. and Vicol, V., 2019. Nonuniqueness of weak solutions to the Navier-Stokes equation. *Annals of Mathematics*, *189*(1), pp.101-144.
6. Chen, S. and Sun, Q., 2016. A quasi-one-dimensional model for hypersonic reactive flow along the stagnation streamline. *Chinese Journal of Aeronautics*, *29*(6), pp.1517-1526.
7. Coco, A., 2020. A multigrid ghost-point level-set method for incompressible Navier-Stokes equations on moving domains with curved boundaries. *Journal of Computational Physics*, *418*, p.109623.
8. Constantin, P. and Vicol, V., 2018. Remarks on high Reynolds numbers hydrodynamics and the inviscid limit. *Journal of Nonlinear Science*, *28*(2), pp.711-724.
9. Cramer, M.S. and Bahmani, F., 2014. Effect of large bulk viscosity on large-Reynolds-number flows. *Journal of fluid mechanics*, *751*, pp.142-163.
10. Davies, J.T., 2012. *Turbulence phenomena: An introduction to the eddy transfer of momentum, mass, and heat, particularly at interfaces*. Elsevier.
11. de Jong, W., 2014. Conservation: Mass, momentum, and energy balances. *Biomass as a Sustainable Energy Source for the Future: Fundamentals of Conversion Processes*, p.71.
12. Eldredge, J.D., 2019. *Mathematical modeling of unsteady inviscid flows* (Vol. 50, p. 461). New York, NY, USA: Springer International Publishing.
13. Georgievskii, D.V., Müller, W.H. and Abali, B.E., 2019. Thin-layer inertial effects in plasticity and dynamics in the Prandtl problem. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, *99*(12), p.e201900184.
14. Heras, R., 2016. The Helmholtz theorem and retarded fields. *European Journal of Physics*, *37*(6), p.065204.
15. Huang, W., Wu, H., Yang, Y.G., Yan, L. and Li, S.B., 2020. Recent advances in the shock wave/boundary layer interaction and its control in internal and external flows. *Acta Astronautica*, *174*, pp.103-122.
16. Jayawardena, A.W., 2021. *Fluid Mechanics, Hydraulics, Hydrology and Water Resources for Civil Engineers*. CRC Press.
17. Jeon, S. and Heinz, U., 2016. Introduction to hydrodynamics. *Quark–Gluon Plasma 5*, pp.131-187.
18. John, V., Linke, A., Merdon, C., Neilan, M. and Rebholz, L.G., 2017. On the divergence constraint in mixed finite element methods for incompressible flows. *SIAM review*, *59*(3), pp.492-544.
19. Kim, C., Nonaka, A., Bell, J.B., Garcia, A.L. and Donev, A., 2017. Stochastic simulation of reaction-diffusion systems: A fluctuating-hydrodynamics approach. *The Journal of chemical physics*, *146*(12), p.124110.
20. Kustepeli, A., 2016. On the Helmholtz theorem and its generalization for multi-layers. *Electromagnetics*, *36*(3), pp.135-148.
21. Liu, C. and Hu, C., 2017. An immersed boundary solver for inviscid compressible flows. *International Journal for Numerical Methods in Fluid*, *85*(11), pp.619-640.
22. Maharshi, K., Mukhopadhyay, T., Roy, B., Roy, L. and Dey, S., 2018. Stochastic dynamic behaviour of hydrodynamic journal bearings including the effect of surface roughness. *International Journal of Mechanical Sciences*, *142*, pp.370-383.
23. Marchioro, C. and Pulvirenti, M., 2012. *Mathematical theory of incompressible nonviscous Fluid* (Vol. 96). Springer Science & Business Media.
24. Misaiko, K. and Vesenka, J., 2014. Connecting the dots: Links between kinetic theory and Bernoulli's principle. In *Phys. Edu. Research Conf. Proc.* (Vol. 52, pp. 257-260).
25. Monaghan, J.J., 2012. Smoothed particle hydrodynamics and its diverse applications. *Annual Review of Fluid Mechanics*, *44*, pp.323-346.
26. Ockendon, H. and Tayler, A.B., 2013. *Inviscid fluid flows* (Vol. 43). Springer Science & Business Media.
27. Reeves, M.T., Billam, T.P., Anderson, B.P. and Bradley, A.S., 2015. Identifying a superfluid Reynolds number via dynamical similarity. *Physical review letters*, *114*(15), p.155302.

28. Roy, S., Pathak, U. and Sinha, K., 2018. Variable turbulent Prandtl number model for shock/boundary-layer interaction. *AIAA Journal*, *56*(1), pp.342-355.
29. Sakajo, T., Sawamura, Y. and Yokoyama, T., 2014. Unique encoding for streamline topologies of incompressible and inviscid flows in multiply connected domains. *Fluid Dynamics Research*, *46*(3), p.031411.
30. Sharipov, F., 2012. Benchmark problems in rarefied gas dynamics. *Vacuum*, *86*(11), pp.1697-1700.
31. Spurk, J.H. and Aksel, N., 2020. Equations of Motion for Particular Fluid. In *Fluid Mechanics* (pp. 103-160). Springer, Cham.
32. Sukhinov, A.I., Chistyakov, A.E., Protsenko, E.A., Sidoryakina, V.V. and Protsenko, S.V., 2020. Accounting method of filling cells for the solution of hydrodynamics problems with a complex geometry of the computational domain. *Mathematical Models and Computer Simulations*, *12*(2), pp.232-245.
33. Tadmor, E., 2012. A review of numerical methods for nonlinear partial differential equations. *Bulletin of the American Mathematical Society*, *49*(4), pp.507-554.
34. Tao, T., 2019. Searching for singularities in the Navier–Stokes equations. *Nature Reviews Physics*, *1*(7), pp.418-419.
35. Templalexis, I.K., 2014. The importance of force terms modeling within the streamline curvature through-flow method. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, *228*(7), pp.825-835.
36. Toda, A.A. and Walsh, K., 2015. The double power law in consumption and implications for testing Euler equations. *Journal of Political Economy*, *123*(5), pp.1177-1200.
37. Watt, S.D., Sharpe, G.J., Falle, S.A. and Braithwaite, M., 2012. A streamline approach to two-dimensional steady non-ideal detonation: the straight streamline approximation. *Journal of Engineering Mathematics*, *75*(1), pp.1-14.
38. Wu, J.C., 2018. Theorems of Helmholtz and Kelvin. In *Elements of Vorticity Aerodynamics* (pp. 17-33). Springer, Berlin, Heidelberg.
39. Yang, L.M., Shu, C., Wu, J., Zhao, N. and Lu, Z.L., 2013. Circular function-based gas-kinetic scheme for simulation of inviscid compressible flows. *Journal of Computational Physics*, *255*, pp.540-557.
40. Yang, L., Xu, J., Du, K. and Zhang, X., 2017. Recent developments on viscosity and thermal conductivity of nanoFluid. *Powder Technology*, *317*, pp.348-369.
41. Zeng, H., 2017. Global resolution of the physical vacuum singularity for three-dimensional isentropic inviscid flows with damping in spherically symmetric motions. *Archive for Rational Mechanics and Analysis*, *226*(1), pp.33-82.
42. Zhang, Y. and Zhou, C.H., 2014. An immersed boundary method for simulation of inviscid compressible flows. *International Journal for Numerical Methods in Fluid*, *74*(11), pp.775-793.
43. Zhu, Y.Z., Yi, S.H., Tian, L.F., He, L. and Chen, Z., 2013. Visualization of mach 3.0/3.8 flow around blunt cone with supersonic film cooling. In *Applied Mechanics and Materials* (Vol. 419, pp. 432-437). Trans Tech Publications Ltd.