Stability criteria for Mathematical Model of novel Coronavirus infected in Local and Non-Local peoples

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Abstract. In the present manuscript, we propose a stability analysis for the mathematical model of COVID-19 disease on a system based on ordinary differential equations. The model canbe described by the people being divided into two categories, namely local and non-local people, and let us consider local people who are super-spreaders and determine the equilibrium analysis for both local (Wuhan City, China) and non-local (other countries) cases, then find stability by using the Jacobian matrix techniques for the first and second wave, and also give the comparison results for both waves. Eventually, deals with data analysis for both cases graphically.

Keywords: Ordinary Differential Equation, COVID-19, Local and non-local cases, Equilibrium point, Eigenvalues, Stability Analysis, Jacobian matrix.

1 Introduction

The umpteen coronaviruses were first discovered in domestic poultry in 1930's causes of respiratory gastrointestinal, liver, and neurologic diseases in animals. Among the family of coronaviruses only seven are known the symptoms of diseases in humans and from the seven, fourof them most oftentimes symptoms of a common cold. Coronaviruses 229E and OC43 cause the common cold. The serotypes NL63 and HUK1 also coincide with the symptoms of common cold [6]. The extant coronaviruses disease 2019 (COVID-19) is outset from the Wuhan city in china and pro- liferate worldwide. From the data report of the World Health Organization(WHO), the COVID-19affected case worldwide is 18,700,000, recovered cases are 11,300,000, and death cases are 705000. The coronaviruses are a family of concomitant viruses diseases in mammals and birds. This COVID-19 can be clique from the other two viruses namely Middle East Respiratory Syndrome(MERS-CoV) and Severe Acute Respiratory Syndrome(SARS-CoV).

MERS-COV was distinguished in 2012, Saudi Arabia and it is disseminated from dromedary camels to humans and SARS-COV was distinguished near the end of 2002 in Guandong, China [10] and it is transmitted from civet cats to humans. In a novel type of viruses namely SARS-COV-2 causes much more severe and sometimes lethal, respiratory infections in human than other coronaviruses and havecaused the major outburst of destructive pneumonia in the 21st century. Zoonotic means transmitted between animals and humans. The COVID-19 is a zoonotic pathogen that begins infected animalsand is transmitted from animals to person and then transfers from humans to humans. From these viruses, peoples are highly affected because of Lock down, no work, no economy, and also infected cases are highly increased day by day.

From this factor, we can define a mathematical model and explain the model in a visualized manner. The peoples are split into two categories one is the local case(Wuhan city, China) and the other one is non-local cases (other countries) then we have considered as local peoples aresuper-spreaders. The mathematical models are important tools for analyzing the spread and control of infectious diseases. The mathematical models are nature of converting the problems form anapplication area into a docile mathematical formulation and it can be used in many areas such as natural science and engineering and it is important tools for understanding the difficult structure and optimize the industrial process.

The equilibrium points of the system should be solved for the independent variables while

equating the derivative to zero. In mathematics, stability means stipulation in which a slight disturbance in a system does not develop high disturbances an effect on that system. In this modelto find the stability of the system by using the equilibrium point and the jacobian matrix. we can say the stable or unstable of the system by using the jacobian matrix and the eigenvalues. The eigenvalue of the matrix is all values are negative then we say that the system is stable, unless at least one of the values is positive then the system is unstable.

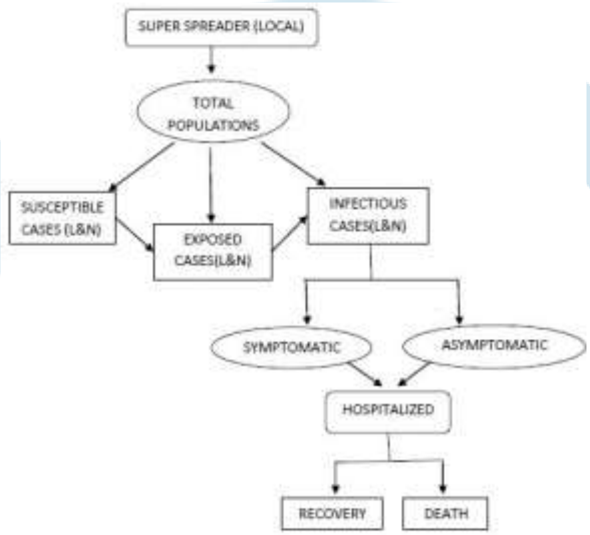

Figure 1

In the present manuscript first section, we will deal with the explanation of the mathematical model and describes a system of the model. The second part deals with stability analysis for a local infected case, the third part discusses stability analysis for the non-local infected case. Till of my knowledge,no manuscript will deal about stability analysis for the mathematical modeling of COVID-19 in alocal and non-local infected case by using the Jacobian matrix and eigenvalues of our assuming model. Finally gives a data analysis is made from south china morning post and daily observation cases rateby WHO(World Health Organization) for both cases.

2 Mathematical Model:

This mathematical model can be described by the total populations that are categorized into local(L) and non-local(N) infected cases and we have to consider local peoples who are super-spreadersand which are represented peoples are living in Wuhan city, China and non-local peoples are represented other countries. Infected cases are detach into eight cases for local peoples such as super-spreaders(ϖ) susceptible cases for local (δ_L), exposed cases for local (ϑ_L), infectious cases for local(ι_L), symptomatic cases for local(Δ_L), asymptomatic cases for local(Θ_L), hospitalized cases for local (γ_L), recovery cases for local(ω_L) and death cases for local(Λ_L) and for non-local peoples detach into different cases such as susceptible cases for non-local (δ_N), exposed cases for non-local (ϑ_N), infectious cases for nonlocal(ι_N), symptomatic cases for non-local(Δ_N), asymptomatic cases for nonlocal(Θ_N), hospitalized cases for non-local (γ_N), recovery cases for non-local(ω_N) and death cases for nonlocal(Λ_N), T represents Total populations and the total population of local people is 1.11 crores and non-local is 778.89 crores. H^1 represents Infected peoples are recovered from hospitalized, H^2 represents Infected peoples are recovered without hospitalized and Let us consider the infected peoples are recovered from hospitalized is greater than infected peoples are recovered without hospitalized ($H^1 > H^2$). P_1 represents the probability of susceptible case to becomes a super spreader, P_2 is represents the probability of exposed case to becomes a super spreader, $1+P_1+P_2$ represents the probability of both susceptible and exposed case to becomes a super spreader.Let us consider 'd' be a positive derivative of susceptible case is greater than super-spreader(ϖ) and also assume that death case cannot have any equilibrium point for both local and non-local cases. Let us assume to be the probability of local case is $P_1 > 1$ and $P_2 > 1$, the non-local case is $P_1 = 0$ and $P_2=0$ and the affected each case of the population at a time t respectively. β represents Humanto Human transmission for local peoples, β' represents Human to Human transmission for non-local peoples and an average rate of both local and non-local peoples are denoted as follows,

$$L_{avg} = \frac{\delta_L+\vartheta_L+\iota_L+\Delta_L+\Theta_L+\Upsilon_L+\Omega_L+\Lambda_L+\varpi}{T}$$

$$N_{avg} = \frac{\delta_N+\vartheta_N+\iota_N+\Delta_N+\Theta_N+\Upsilon_N+\Omega_N+\Lambda_N}{T}$$

Local Peoples Infected Cases Model:

$$\frac{d\delta_L}{dt} = T - d\delta_L - \beta\iota_L\vartheta_L + \varpi\delta_L \tag{1}$$

$$\frac{d\vartheta_L}{dt} = (1 + P_1 + P_2 + \gamma_L - \delta_L)\vartheta_L \tag{2}$$

$$\frac{d\iota_L}{dt} = (\beta + (H^1L_{avg} - H^2L_{avg}))\iota_L \tag{3}$$

$$\frac{d\gamma_L}{dt} = (\iota_L - \vartheta_L)\gamma_L + \delta_L\beta\Theta_L \tag{4}$$

$$\frac{d\Delta_L}{dt} = (T - d\delta_L + d\vartheta_L)\Delta_L \tag{5}$$

$$\frac{d\Theta_L}{dt} = L_{avg}(H^1 - H^2)\Theta_L \tag{6}$$

$$\frac{d\omega_L}{dt} = (\gamma_L + \vartheta_L - \Delta_L)\omega_L \tag{7}$$

$$\frac{d\Lambda_L}{dt} = T - \iota_L\beta - \vartheta_L\beta + \omega_L\beta - \gamma_L\beta - \varpi\beta. \tag{8}$$

Non-Local Peoples Infected Cases Model:

$$\frac{d\delta_N}{dt} = T - d\delta_N - \beta'\iota_N\vartheta_N \tag{9}$$

$$\frac{d\vartheta_N}{dt} = (1 + \gamma_N - \delta_N)\vartheta_N \tag{10}$$

$$\frac{d\iota_N}{dt} = (\beta' + (H^1N_{avg} - H^2N_{avg}))\iota_N \tag{11}$$

$$\frac{d\gamma_N}{dt} = (\iota_N - \vartheta_N)\gamma_N + \delta_N \beta' \Theta_N \tag{12}$$

$$\frac{d\Delta_N}{dt} = (T - d\delta_N + d\vartheta_N)\Delta_N \tag{13}$$

$$\frac{d\Theta_N}{dt} = N_{avg}(H^1 - H^2)\Theta_N \tag{14}$$

$$\frac{d\omega_N}{dt} = (\gamma_N + \vartheta_N - \Delta_N)\omega_N \tag{15}$$

$$\frac{d\Lambda_N}{dt} = T - \iota_N \beta' - \vartheta_N \beta' + \omega_N \beta' - \gamma_N \beta'. \tag{16}$$

3 Stability Analysis for Local Cases:

In this section, we have to find the equilibrium point for the locally infected cases and applying the values of the equilibrium point in the jacobian matrix then find the eigenvalues for the matrix andfrom this, we can say the system is stable or unstable.

Theorem 1. *The local infected case model system is stable if and only if every eigenvalue of the matrix L^1 has negative real parts.*

Proof. First we have to find equilibrium point for each cases for local infected peoples,
Case 1:(Susceptible)

$$\frac{d\delta_L}{dt} = T - d\delta_L - \beta\iota_L\vartheta_L + \varpi\delta_L = 0$$

$$\frac{d\delta_L}{dt} = 0$$

$$(-d + \varpi)\delta_L = \beta\iota_L\vartheta_L - T$$

$$\delta_L = \frac{\beta\iota_L\vartheta_L - T}{-d + \varpi}$$

Case 2:(Exposed)

$$\frac{d\vartheta_L}{dt} = (1 + P_1 + P_2 + \gamma_L - \delta_L)\vartheta_L$$

$$(1 + P_1 + P_2 + \gamma_L - \delta_L)\vartheta_L = 0$$

$$\vartheta_L = 0.$$

Case 3:(Infected)

$$\frac{d\iota_L}{dt} = (\beta + (H^1L_{avg} - H^2L_{avg}))\iota_L$$

$$(\beta + (H^1L_{avg} - H^2L_{avg}))\iota_L = 0$$

$$\iota_L = 0.$$

Case 4:(Hospitalized)

$$\frac{d\gamma_L}{dt} = (\iota_L - \vartheta_L)\gamma_L + \delta_L\theta\Theta_L$$

$$(\iota_L - \vartheta_L)\gamma_L + \delta_L\theta\Theta_L = 0$$

$$\gamma_L = -\frac{\delta_L\theta\Theta_L}{(\iota_L - \vartheta_L)}.$$

Case 5:(Symptomatic)

$$\frac{d\Delta_L}{dt} = (T - d\delta_L + d\vartheta_L)\Delta$$

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$$(T - d\delta_L + d\vartheta_L)\Delta_L = 0.$$

$$\Delta_L = 0.$$

Case 6:(Asymptomatic)

$$\frac{d\Theta_L}{dt} = L_{avg}(H^1 - H^2)\Theta_L$$

$$L_{avg}(H^1 - H^2)\Theta_L = 0.$$
$$\Theta_L = 0.$$

Case 7:(Recovery)

$$\frac{d\omega_L}{dt} = (\gamma_L + \vartheta_L - \Delta_L)\omega_L$$
$$(\gamma_L + \vartheta_L - \Delta_L)\omega_L = 0.$$
$$\omega_L = 0.$$

Next, We can find the jacobian matrix for each case in locally infected peoples.

$$L = \begin{bmatrix} -d+\varpi-\beta\iota_L & -\beta\vartheta_L & 0 & 0 & 0 & 0 & 0 & 0 \\ -\vartheta_L & \rho & 0 & \vartheta_L & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta+(H^1-H^2)L_{avg} & 0 & 0 & 0 & 0 & 0 \\ \beta\Theta_L & -\gamma_L & \gamma_L & \iota_L-\vartheta_L & 0 & \delta_L\beta & 0 & 0 \\ -d\Delta_L & d\Delta_L & 0 & 0 & T-d\delta_L+d\vartheta_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{avg}(H^1-H^2) & 0 & 0 \\ 0 & \omega_L & 0 & \omega_L & -\omega_L & 0 & \gamma_L+\vartheta_L-\Delta_L & 0 \\ 0 & -\beta & -\beta & -\beta & 0 & 0 & \beta & 0 \end{bmatrix}$$

where, $\rho = 1 + P_1 + P_2 + \gamma_L - \delta_L$

Then we substitute equilibrium point for the above jacobian matrix for local cases and we get the following matrix,

$$L^1 = \begin{bmatrix} -d+\varpi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1+P_1+P_2+\frac{T}{d-\varpi} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta+(H^1-H^2)L_{avg} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta\left(\frac{T}{d-\varpi}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & T-d\left(\frac{T}{d-\varpi}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{avg}(H^1-H^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta & -\beta & -\beta & 0 & 0 & \beta & 0 \end{bmatrix}$$

Finally, we have to show that eigenvalues of the matrix L^1 and from this, we can say the system is stable or unstable as defined in the hypothesis.

$$L^1 = \begin{bmatrix} A-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C-\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0-\lambda & 0 & \beta\left(\frac{T}{d-\varpi}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & D-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0-\lambda & 0 \\ 0 & -\beta & -\beta & -\beta & 0 & 0 & \beta & 0-\lambda \end{bmatrix}.$$

where,

$$A = -d + \varpi \tag{17}$$

$$B = 1 + P_1 + P_2 + \frac{T}{d - \varpi} \tag{18}$$

$$C = \beta + (H^1 - H^2)L_{avg} \tag{19}$$

$$D = T - d\left(\frac{T}{d - \varpi}\right) \tag{20}$$

$$E = L_{avg}(H^1 - H^2) \tag{21}$$

Therefore, the eigenvalues of the above jacobian matrix is,

$$\lambda_1 = \varpi - d$$

$$\lambda_2 = 1 + P_1 + P_2 + \frac{T}{d - \varpi}$$

$$\lambda_3 = \beta + (H^1 - H^2)L_{avg}$$

$$\lambda_4 = 0$$

$$\lambda_5 = T - d\left(\frac{T}{d - \varpi}\right)$$

$$\lambda_6 = L_{avg}(H^1 - H^2)$$

$$\lambda_7 = 0$$

$$\lambda_8 = 0$$

In this characteristic equation have only one of the eigenvalue is negative namely λ_1 and other eigenvalues are positive and zero. Therefore from the above hypothesis the local infected case system of the model is unstable.

Remark 1 (2). The local case system model is unstable if and only if at least one of the eigenvalues of the matrix L^1 has positive real parts.

4 Stability Analysis for Non-Local Cases:

Theorem 2. *The Non-local infected case model system is stable if and only if every eigenvalue of the matrix N^1 has negative real part.*

Proof. First we have to find equilibrium point for each cases for non-local infected peoples.
Case 1:(Susceptible)

$$\frac{d\delta_N}{dt} = T - d\delta_N - \beta'\iota_N\vartheta_N$$
$$= 0$$

$$\delta_N = \frac{\beta'\iota_N\vartheta_N - T}{-d}.$$

Case 2:(Exposed)

$$\frac{d\vartheta_N}{dt} = (1 + \gamma_N - \delta_N)\vartheta_N$$

$$(1 + \gamma_N - \delta_N)\vartheta_N = 0$$
$$\vartheta_N = 0.$$

Case 3:(Infected)

$$\frac{d\iota_N}{dt} = (\beta' + (H^1 N_{avg} - H^2 N_{avg}))\iota_N$$

$$(\beta' + (H^1 N_{avg} - H^2 N_{avg}))\iota_N = 0$$
$$\iota_N = 0.$$

Case 4:(Hospitalized)

$$\frac{d\gamma_N}{dt} = (\iota_N - \vartheta_N)\gamma_N + \delta_N \beta' \Theta_N$$

$$(\iota_N - \vartheta_N)\gamma_N + \delta_N \beta' \Theta_N = 0$$
$$\gamma_N = -\frac{\delta_N \beta' \Theta_N}{(\iota_N - \vartheta_N)}.$$

Case 5:(Symptomatic)

$$\frac{d\Delta_N}{dt} = (T - d\delta_N + d\vartheta_N)\Delta_N$$

$$(T - d\delta_N + d\vartheta_N)\Delta_N = 0.$$
$$\Delta_N = 0.$$

Case 6:(Asymptomatic)

$$\frac{d\Theta_N}{dt} = N_{avg}(H^1 - H^2)\Theta_N$$

$$N_{avg}(H^1 - H^2)\Theta_N = 0.$$
$$\Theta_N = 0.$$

Case 7:(Recovery)

$$\frac{d\omega_N}{dt} = (\gamma_N + \vartheta_N - \Delta_N)\omega_N$$

$$(\gamma_N + \vartheta_N - \Delta_N)\omega_N = 0.$$
$$\omega_N = 0.$$

Next, We can find the jacobian matrix for each case in non-local infected peoples.

$$N = \begin{bmatrix} -d & -\beta'\iota_N & -\beta'\vartheta_N & 0 & 0 & 0 & 0 & 0 \\ -\vartheta_N & 1 + \gamma_N - \delta_N & 0 & \vartheta_N & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta' + (H^1 - H^2)N_{avg} & 0 & 0 & 0 & 0 & 0 \\ \beta'\Theta_N & -\gamma_N & \gamma_N & \iota_N - \vartheta_N & 0 & \delta_N \beta' & 0 & 0 \\ -d\Delta_N & d\Delta_N & 0 & 0 & T - d\delta_N + d\vartheta_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{avg}(H^1 - H^2) & 0 & 0 \\ 0 & \omega_N & 0 & \omega_N & -\omega_N & 0 & \gamma_N + \vartheta_N - \Delta_N & 0 \\ 0 & -\beta' & -\beta' & -\beta' & 0 & 0 & \beta' & 0 \end{bmatrix}$$

um point for the above jacobian matrix for non-local cases and we get

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Then we substitute equilib the following matrix,

$$N^1 = \begin{bmatrix} -d & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{T}{d} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta' + (H^1 - H^2)L_{avg} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta'(\frac{T}{d}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{avg}(H^1 - H^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta' & -\beta' & -\beta' & 0 & 0 & \beta' & 0 \end{bmatrix}$$

Finally, we have to show that eigenvalues of the matrix N^1 and from this, we can say the system is stable or unstable as defined in the hypothesis.

$$N^1 = \begin{bmatrix} -d-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{T}{d}-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta' + (H^1 - H^2)N_{avg} - \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0-\lambda & 0 & \beta'(\frac{T}{d}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{avg}(H^1 - H^2) - \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0-\lambda & 0 \\ 0 & -\beta' & -\beta' & -\beta' & 0 & 0 & \beta' & 0-\lambda \end{bmatrix}$$

Therefore the eigenvalues of Jacobian matrix is

$$\lambda_1 = -d$$
$$\lambda_2 = 1 - \frac{T}{d}$$
$$\lambda_3 = \beta' + (H^1 - H^2)N_{avg}$$
$$\lambda_4 = 0$$
$$\lambda_5 = 0$$
$$\lambda_6 = N_{avg}(H^1 - H^2)$$
$$\lambda_7 = 0$$
$$\lambda_8 = 0$$

In this characteristic equation have two of the eigenvalues are negative namely λ_1, λ_2. Therefore from the above hypothesis the non-local infected case system of the model is unstable.

5 Data Analysis:

We can summarize the reported data from Wuhan city, China for locally infected cases and other countries for a non-local infected case. The analysis can be done from the daily confirmation cases and the probability of susceptible cases becomes an infected case and the probability of exposed case to becomes the infected case then both cases are confirmation of infected from 14 days of symptoms onset. Initially, the local infected case is highly increasing but now compare to the non-local cases, the local cases are decreasing. The symptoms of COVID-19 are fever, cough, shortness of breath, muscle ache, confusion, headache, throat infection, chest pain, diarrhea, and vomiting. From these symptoms, we can analyze who is affected and non-affected peoples. From the above symptoms of COVID-19, we can analyze the stability of this manuscript both local and non-local model is unstable which can be already dealt in section 4. In local cases and non-local cases affected peoples, non-affected people, symptomatic, Asymptomatic cases, recovery cases, death cases, and also how those cases are increases is described by following results such as fig 1,2,3,4,5,6.

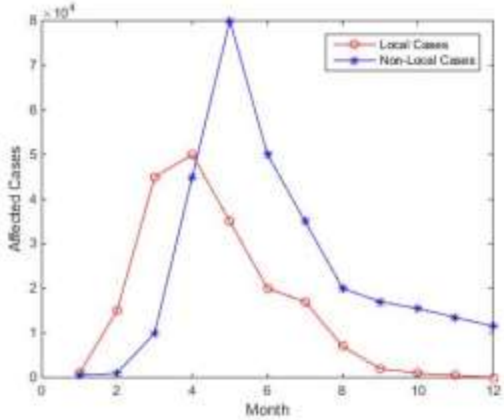

Figure 1

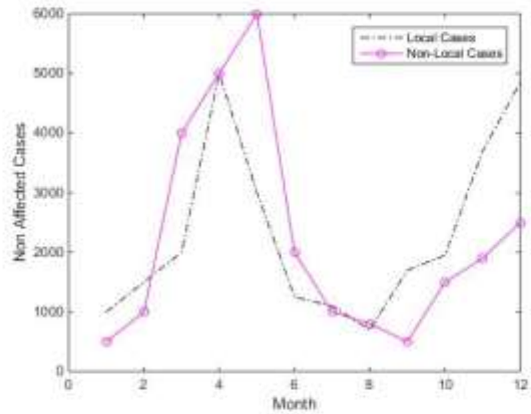

Figure 2

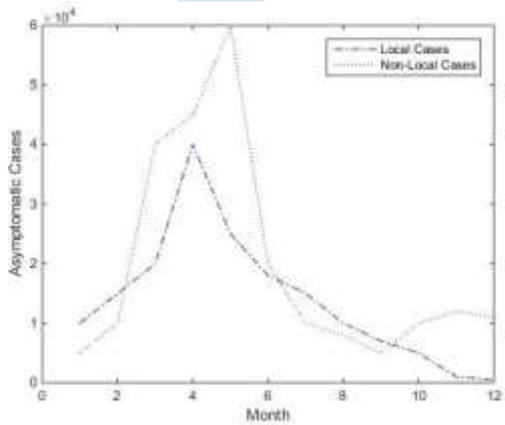

Figure 3

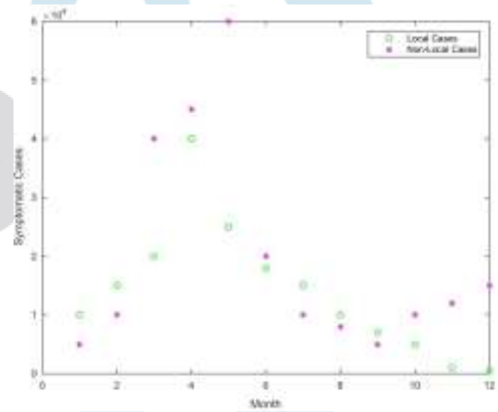

Figure 4

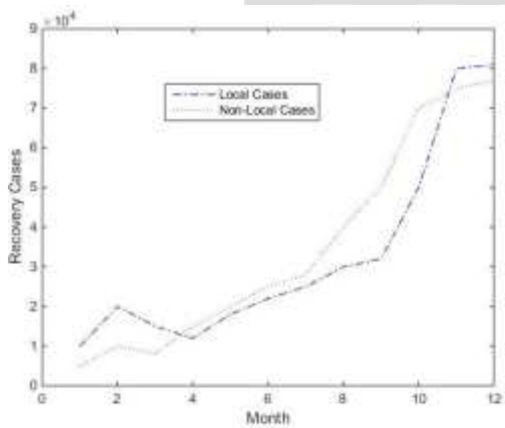

Figure 5

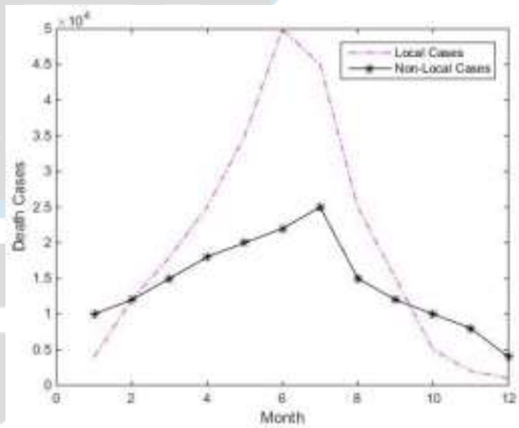

Figure 6

6 Results:

In this manuscript we can observe that the system of the local infected case and non-local infected case model is unstable because of the assumption of the model is ($H^1 > H^2$) (i.e) infected peoples are recovered from hospitalized is greater than without hospitalized and the eigenvalues of considering model are zeros and positive. Suppose, in this paper, we want to prove the assuming system of model is stable if infected peoples are recovered without hospitalized is higher than hospitalized cases and also the equilibrium point of the exposed case, infected case asymptomatic case, symptomatic case, and recovery case are all positive and non-zero in point of equilibrium and also eigenvalues are all negative real parts. The results were made for samples of reported data from both local peoples(Wuhan city, China)and nonlocal peoples(other countries) in the world health organization.

7 Conclusion:

In our mathematical model consider the two categories of Local and non-local infected cases and still now no paper can say the above model is stable or unstable but in our manuscript can say the system of model is unstable from the above results and the equilibrium point of the system. The system moves away from the equilibrium after the small disturbance or increases the affected case but now the situation of COVID-19 it cannot be returning to the equilibrium point is described as above data analysis and results.

8 Acknowledgement

This research work is supported by the Department of Science and Technology(DST)-Innovation in Science Pursuit for Inspired Research (INSPIRE Fellowship/[IF190368]), Government of India, Ministry of Science and Technology, Department of Science and Technology, Technology Bhawan, New Mehrauli Road, New Delhi-110016.

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