© 2022 IJRTI | Volume 7, Issue 12 | ISSN: 2456-3315

New Integrating Factor Depend on Independent Variable

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***Abstract*: In this research, a new integrating factor is proposed depending on the independent variable while the common integrating factor depends on the two variables (dependent and independent), to solve a non-exact differential equation from type $pydx + qxdy = 0$, we will prove a new integrating factor (μ_x) solve this type of differential equation, and support our research with an example of this case, we will prove that (μ_x) has good properties if compare it with the common integrating factor, we will present our conclusions and recommendations.**

***Keywords*: integrating factor, exact differential equation, non-exact differential equation, a necessary and sufficient condition.**

INTRODUCTION

The exact differential equation can be solved in several ways, if the differential equation is non-exact, we cannot solve it directly, here comes the idea of the integrating factor (μ) that enables us to solve it, so the integrating factor is very important in solving the non-exact differential equation, therefore, the integrating factors and the methods of finding them were numerous, due to the many types of non-exact differential equations. The exact differential equation of the type

$$pydx + qxdy = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

which we will convert from a non-exact differential equation to an exact one in order to solve it and we will discuss a proposed integrating factor for it and prove that it has become exact and we provide an example of this case.

Definition (1): The differential equation $Mdx + Ndy = 0$ is exact if there is a function $f(x, y)$ where $M = \frac{\partial f}{\partial x}$, $N = \frac{\partial f}{\partial y}$. [1]

Theorem (1): The necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ assuming that M , N and their partial derivatives are continuous functions of x and y within their region of existence R defined by $|x - x_0| \leq \propto$ and $|y - y_0| \leq \beta$. [2]

The exact differential equation is one of the cases of differential equations of the first order and the first degree, there are several ways to solve it, and these methods are not able to solve a non-exact differential equation.[3]

Definition (2): If a first-order and first-degree differential equation is multiplied by a factor and converted into an exact differential equation, this factor is called the (integrating factor), its conclusion depends on previous observation, experience, and information. [4],[5]

This means if $Mdx + Ndy = 0$ is non-exact [6], we multiply it by (μ) to become $\mu Mdx + \mu Ndy = 0$ and according to theorem (1), the last relationship is exact if $\frac{\partial \mu M}{\partial y} = \frac{\partial \mu N}{\partial x}$ [7],[8], integrating factor (μ) transformed the non-exact differential equation (which cannot be solved directly) into an exact differential equation that can be solved in several ways, equation (1) has a common integrating factor

$$\mu = x^{p-1}y^{q-1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This integrating factor depends on the variables *x* and *y* together [7]. The subject of the research starts from this point, so we will suggest another integrating factor to solve equation (1).

THE PROPOSED INTEGRATING FACTOR

$$\mu_x = x^{\frac{p-q}{q}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The proposed integrating factor will take the symbol (μ_x) to distinguish it from (μ) on the one hand and to know that it depends on the independent variable (*x*) only on the other hand.

PROPOSED INTEGRATING FACTOR AND THE NON-EXACT DEFFERENTIAL EQUATION

The proposed integrating factor (μ_x) in equation (3) is related to the non-exact differential equation $pydx + qxdy = 0$ and this will be shown by the following theorem

Theorem (2): Mathematical term $x^{\frac{p-q}{q}}$ is an integrating factor dependent on the independent variable of a differential equation of type $pydx + qxdy = 0$.

Proof: compare $pydx + qxdy = 0$ with the equation $Mdx + Ndy = 0$, so we get

$$M = py \quad , \; N = qx$$
$$\frac{\partial M}{\partial y} = p \;, \frac{\partial N}{\partial x} = q$$
$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

that is, $pydx + qxdy = 0$ is non-exact, so we multiply it by $x^{\frac{p-q}{q}}$ to get

$$pyx^{\frac{p-q}{q}}dx + qxx^{\frac{p-q}{q}}dy = 0$$

$$pyx^{\frac{p-q}{q}}dx + qx^{\frac{p}{q}}dy = 0 \quad \ldots\ldots (4)$$

compare equation (4) with $Mdx + Ndy = 0$, so we get

$$M = pyx^{\frac{p-q}{q}} \;,\; N = qx^{\frac{p}{q}}$$

$$\frac{\partial M}{\partial y} = px^{\frac{p-q}{q}} \quad \ldots\ldots (5)$$

$$\frac{\partial N}{\partial x} = q\frac{p}{q}x^{\frac{p-q}{q}} = px^{\frac{p-q}{q}} \quad \ldots\ldots (6)$$

From equations (5), (6), it is clear that equation (4) has become an exact differential equation (according to theorem (1)), that is, (μ_x) is an integrating factor of equation (1) (according to the definition of (2)).

SOLVE THE NON-EXACT DEFFERENTIAL EQUATION

The integrating factor (μ_x) is the proposed integrating factor for equation (1) (according to theorem (2)), that is, (μ_x) is the integrating factor that converts equation (1) from non-exact to exact, in order to solve equation (1) depending on the methods of solving exact differential equations, we write it by the formula

$$d\left(qyx^{\frac{p}{q}}\right) = 0 \quad \ldots\ldots (4a)$$

$$qyx^{\frac{p}{q}} = c$$

$$yx^{\frac{p}{q}} = \frac{c}{q}$$

$$yx^{\frac{p}{q}} = \text{constant} \quad \ldots\ldots (7)$$

Equation (7) is the general solution to equation (1).

Example (1): Solve the following non-exact differential equation

$$3ydx + 2xdy = 0 \quad \ldots\ldots (8)$$

Solution:

$$p = 3, q = 2$$

$$\mu_x = x^{\frac{p-q}{q}} = x^{\frac{3-2}{2}} = x^{\frac{1}{2}}$$

$$3yx^{\frac{1}{2}}dx + 2x^{\frac{3}{2}}dy = 0$$

$$d\left(qyx^{\frac{p}{q}}\right) = 0$$

$$d\left(2yx^{\frac{3}{2}}\right) = 0$$

$$2yx^{\frac{3}{2}} = c$$

$$yx^{\frac{3}{2}} = \frac{c}{p}$$

$$yx^{\frac{3}{2}} = \text{constant} \quad \ldots\ldots (9)$$

Equation (9) is the general solution to equation (8), when we compare it with the solution using the integrating factor (μ) mentioned in equation (2) where the solution is $(y^2x^3 = \text{constant})$ we notice *y* in equation (9) from first degree, this is a strong characteristic in favour of (μ_x), especially if we want to find the curve of the function (one of the applications of differential equations). [8]

CONCLUSION

We concluded the following from the research subject

1- A new integrating factor depends only on the independent variable.

2-When comparing the solution in equation (9) and the solution in the case that the integrating factor is $(\mu = x^{p-1}y^{q-1})$, the solution is $(y^2x^3 = \text{constant})$, then the solution in equation (9) is more accurate and this is a strong characteristic in favour of the proposed integrating factor (μ_x).

RECOMMENDATIONS

1- We recommend the proposed integrating factor (μ_x) to solve a non-exact differential equation (1).

2- Looking for the theme of integrating factors to suggest better than common integrating factors.

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