Regression Cum Exponential Type Estimator In Double Sampling

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Abstract
Auxiliary information is commonly used to improve the accuracy of the estimators, when estimating unknown population parameters. The present work suggested regression cum exponential quadratic product type estimator of the population mean in double sampling. The expression for the bias and mean square error (MSE) of the envisaged estimator were accomplished under first order of approximation. Further, empirical analysis exhibited that, when using a double sampling scheme, the envisaged estimator outperform other pertinent current estimators.

Key words: Estimator, Double sampling, Product type, Population mean

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1. Introduction

The usage of auxiliary information to increase the accuracy of the estimator is widely known to all. Several experts have offered several estimating strategies for the finite population mean of the study variable, based on their knowledge of the auxiliary variables, to help increase the estimator's precision when auxiliary information is lacking. In these situations, the estimators of the exponential product and ratio type are useful (Bahl and Tuteja, 1991). The exponential ratio estimators were envisaged by Singh and Vishwakarma (2007) followed the work of Bahl and Tuteja (1991) in two phase sampling methods. When two phase sampling provided for the acquisition of population parameters, Singh *et al.* (2008) proposed an unbiased estimator. Singh and Choudhury (2012) suggested a product type and an exponential ratio estimator in double sampling for estimating the finite population mean of study variables. In a two phase sample scheme, Sanaullah (2013) proposed the generalised exponential estimator for estimating population variance. Vishwakarma and Kumar (2014) suggested a method for estimating the research variable's finite population mean using a two-phase sampling procedure. A generalised synthetic estimator employing double sampling and auxiliary data was proposed by Bhal and Sangeeta (2015). Singh (2018) recommended an estimation of the population coefficient of variation of study variable Y where data on one auxiliary variable was provided. In order to estimate the population mean of the study variable using auxiliary information under double sampling, Vishwakarma and Zeeshan (2020) and Sharma and Kumar (2021) proposed a class of ratio cum product type estimator. It was concluded that the generated mean estimation estimators outperformed the conventional unbiased estimators.

Following the works mentioned above, we developed a regression cum exponential quadratic product type estimator of the population mean in double sampling that is more useful in generating accurate estimates of the population means of the studied variable.

Take into account a population of N units, $\underset{\sim}{U}= \underset{\sim}{U}_1, \underset{\sim}{U}_2, \ldots\ldots\ldots\ldots \underset{\sim}{U}_N$. Let Y and X denotes the study and auxiliary variables, respectively. Let, (yi, xi) I =1,2,…..n represents the n pairs of sample observations for the study variables and auxiliary variables obtained from population size of N using simple random sampling without replacement (SRSWOR). Let $\bar{X}$ and $\bar{Y}$ revealed the population mean of the auxiliary and study variables, respectively, and $\bar{x}$ and $\bar{y}$ represents the sample means.

Consider the following estimators of population mean developed in literature:

(1) $\bar{Y}= \bar{y}$=Sample Mean

MSE = $\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)S_y^2$

(2) Product estimator in double sampling

$\bar{Y}_{pd}= \bar{y}\frac{\bar{x}}{\bar{x}'}$

MSE = $\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)(S_y^2+R^2S_x^2+2R\rho S_xS_y)$

(3) Ratio estimator in double sampling

$\bar{Y}_{rd}= \bar{y}\frac{\bar{x}'}{\bar{x}}$

MSE = $\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)(S_y^2+R^2S_x^2-2R\rho S_xS_y)$

2. Proposed estimator

In this section, we propose a exponential type estimators in double sampling as:-

$\bar{Y}_{exp,dr}=\xi_1\bar{y}\ \exp\left(\frac{\bar{x}'-\bar{x}}{\bar{x}'+\bar{x}}\right)^2+\xi_2[\bar{y}+(\bar{x}'-\bar{x})b]$..........................(1)

Let us define,

$e_0=\frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1=\frac{\bar{x}-\bar{X}}{\bar{X}}$, $e_2=\frac{\bar{x}'-\bar{X}}{\bar{X}}$

So that E (e_0), E (e_1), E (e_2)=0

$E(e_0^2)=\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}$

$E(e_1^2)=\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_x^2}{\bar{X}^2}$

$E\,(e_2^2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2=\left(\frac{1}{n'}-\frac{1}{N}\right)\frac{S_x^2}{\bar{X}^2}$

$E\,(e_0e_1)=\left(\frac{1}{n}-\frac{1}{N}\right)C_{xy}=\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{xy}}{\bar{X}\bar{Y}}$

$E\,(e_0e_2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_{xy}=\left(\frac{1}{n'}-\frac{1}{N}\right)\frac{S_{xy}}{\bar{X}\bar{Y}}$

$E\,(e_1e_2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2=\left(\frac{1}{n'}-\frac{1}{N}\right)\frac{S_x^2}{\bar{X}^2}$

Now, substituting the expression for $\bar{y},\bar{x}$ and $\bar{x}'$ in terms of e_0, e_1 and e_2 in $\bar{Y}_{exp,dr}$, we get:

$\bar{Y}_{exp,dr}=\xi_1\bar{Y}(1+e_0)\ \exp\left(\frac{\bar{X}(1+e_2)-\bar{X}(1+e_1)}{\bar{X}(1+e_2)+\bar{X}(1+e_1)}\right)+\xi_2[\bar{Y}(1+e_0)+\{\bar{X}(1+e_2)-\bar{X}(1+e_1)b\}]$................(2)

$=\bar{Y}(1+e_0)+\xi_1\bar{Y}\frac{e_2}{2}-\xi_1\bar{Y}\frac{e_1}{2}+\frac{3}{8}\xi_1\bar{Y}e_1^2-\frac{1}{8}\xi_1\bar{Y}e_2^2-\xi_1\bar{Y}\frac{e_1e_2}{4}+\xi_1\bar{Y}\frac{e_0e_2}{2}-\xi_1\bar{Y}\frac{e_0e_1}{2}+\xi_2\bar{X}(e_2-e_1)b$.................(3) $(\xi_1+\xi_2=1, \text{where}\ \xi_1\ and\ \xi_2\ \text{are weights.})$

$\text{Bias}(\bar{Y}_{exp,dr})=E\,[\bar{Y}_{exp,dr}]-\bar{Y}=E[\bar{Y}(1+e_0)+\xi_1\bar{Y}\frac{e_2}{2}-\xi_1\bar{Y}\frac{e_1}{2}+\frac{3}{8}\xi_1\bar{Y}e_1^2-\frac{1}{8}\xi_1\bar{Y}e_2^2-\xi_1\bar{Y}\frac{e_1e_2}{4}+\xi_1\bar{Y}\frac{e_0e_2}{2}-\xi_1\bar{Y}\frac{e_0e_1}{2}+\xi_2\bar{X}(e_2-e_1)b]-\bar{Y}$

$=\frac{3}{8}\xi_1\bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{S_x^2}{\bar{X}^2}+\frac{1}{2}\xi_1\bar{Y}\left(\frac{1}{n'}-\frac{1}{n}\right)\frac{S_{xy}}{\bar{X}\bar{Y}}$

Mean square error (MSE) up to first order of approximation given, as

$\text{M.S.E.}(\bar{Y}_{exp,dr})=E[\bar{Y}_{exp,dr}-\bar{Y}]^2=E[\bar{Y}(1+e_0)+\xi_1\bar{Y}\frac{e_2}{2}-\xi_1\bar{Y}\frac{e_1}{2}+\frac{3}{8}\xi_1\bar{Y}e_1^2-\frac{1}{8}\xi_1\bar{Y}e_2^2-\xi_1\bar{Y}\frac{e_1e_2}{4}+\xi_1\bar{Y}\frac{e_0e_2}{2}-\xi_1\bar{Y}\frac{e_0e_1}{2}+\xi_2\bar{X}(e_2-e_1)b]^2+\bar{Y}^2-2\bar{Y}\,E[\bar{Y}(1+e_0)+\xi_1\bar{Y}\frac{e_2}{2}-\xi_1\bar{Y}\frac{e_1}{2}+\frac{3}{8}\xi_1\bar{Y}e_1^2-\frac{1}{8}\xi_1\bar{Y}e_2^2-\xi_1\bar{Y}\frac{e_1e_2}{4}+\xi_1\bar{Y}\frac{e_0e_2}{2}-\xi_1\bar{Y}\frac{e_0e_1}{2}+\xi_2\bar{X}(e_2-e_1)b]$

$=\left(\frac{1}{n'}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n}-\frac{1}{n'}\right)(S_y^2-\xi_1R\rho S_xS_y-2\xi_2b\rho S_xS_y+\frac{1}{4}\xi_1^2R^2S_x^2+\xi_1\xi_2RbS_x^2+\xi_2^2b^2S_x^2$ ....(4)

Differentiating equation (3) w.r.t. ξ_1, ξ_2 and b.we obtained optimum values of ξ_1, ξ_2 and b respectively,

$\xi_1=\frac{2}{R}\left(\rho\frac{S_y}{S_x}-\xi_2b\right)$

$\xi_2=\frac{1}{b}\left(\rho\frac{S_y}{S_x}-\xi_1\frac{R}{2}\right)$

$b=\frac{\rho}{\xi_2}\frac{S_y}{S_x}-\frac{1}{2}\frac{\xi_1}{\xi_2}R$

By substituting the optimum values of ξ_1, ξ_2 and b in equation (3), we get minimum mean square error of $\bar{Y}_{exp,dr}$ as follows:

$\text{MSE}(\bar{Y}_{exp,dr})_{min}=(1-\rho^2)\,S_y^2.$

REMARK: $\bar{Y}_{exp,dp}=\xi_1\bar{y}\ \exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'}\right)^2+\xi_2[\bar{y}+(\bar{x}'-\bar{x})b]$

They are equally efficient in case of product estimator because of quadratic function.

3. Efficiency comparison

In this section, we get some conditions by comparing MSE of the estimators under which $\bar{Y}_{exp,dr}$ better than the other existing estimators.

Comparison with simple mean per unit

MSE ($\bar{Y}_{exp,dr}$)<MSE ($\bar{Y}$)

If $\rho<\frac{(\xi_1R+2\xi_2b)}{4}\frac{S_x}{S_y}$

Comparison with product estimator

MSE($\bar{Y}_{exp,dr}$)< MSE ($\bar{Y}_{pd}$)

If $\rho>\frac{4R^2-\xi_1^2R^2-4\xi_1\xi_2Rb-4\xi_2^2b^2}{4(\xi_1R+2\xi_2b+2R)}\frac{S_x}{S_y}$

Comparison with ratio estimator

MSE($\bar{Y}_{exp,dr}$)< MSE ($\bar{Y}_{rd}$)

If $\rho<\frac{4R^2-\xi_1^2R^2-4\xi_1\xi_2Rb-4\xi_2^2b^2}{4(2R-\xi_1R-2\xi_2b)}\frac{S_x}{S_y}$

4. Empirical study

We have taken into account the following population data sets to evaluate the performance of the proposed estimator. The population is listed below:

Population 1: Cochran (1977)

Y= Food Cost, X= Family Income

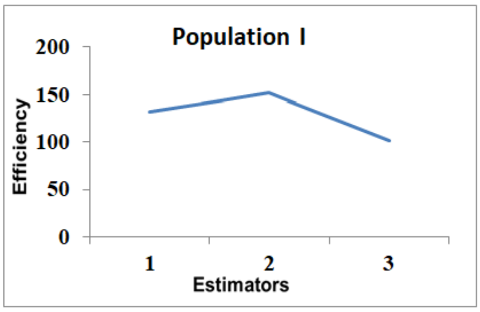
$\bar{Y}$=27.40, $\bar{X}$ = 39.63, $\rho = 0.52, S_x = 19.1150, S_y = 26.866$, n=16, N=33, n'= 22, R=0.377671, β=0.68.
Population 2: Singh and Agnihotri (2008)
$\bar{Y}$=39.068, $\bar{X}$ = 25.111, $\rho = 0.721, S_x = 40.67982, S_y = 56.45326$,n=16, N=278, n'= 25, R=1.555812, β=5.98.
Population 3: Kadilar and Cingi, (2006)
$\bar{Y}$=500, $\bar{X}$ = 25, $\rho = 0.90, S_x = 50, S_y = 7500$, n=50, N=200, n'= 22, R=20, β=0.002.
Percentage relative efficiency of estimators is calculated by using formula.

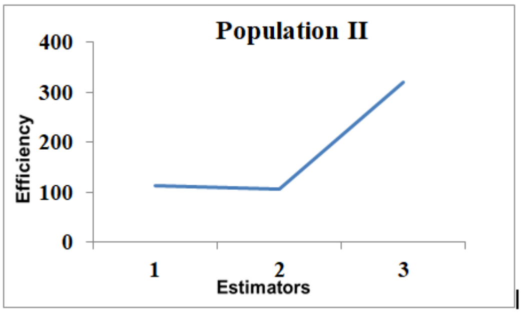
PRE $(\bar{Y}_k)=\frac{MSE(\bar{Y}_{exp,dr})}{MSE(\bar{Y}_k)}*100$

Where $\bar{Y}_k = \bar{Y}, \bar{Y}_{rd}, \bar{Y}_{pd}$.

Table 1: PRE of different estimator's w.r.t. $\bar{Y}_{exp,dr}$

S. No	Estimators	Population 1	Population 2	Population 3
1	$\bar{Y}$ $\xi_1 = 1.5, \xi_2$=-0.5	131.48	114.16	131.99
2	$\bar{Y}_{rd}$ $\xi_1 = 2, \xi_2$=-1	151.99	105.64	101.82
3	$\bar{Y}_{pd}$ ξ_1=0.9, ξ_2=0.1	101.08	320.90	201.11





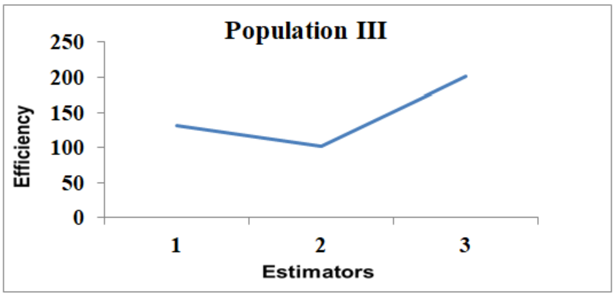


Figure1: Population-wise comparison of efficiencies of estimators

5. Conclusion

On the basis of PRE of the estimators $\bar{Y}$,$\bar{Y}_{rd}$,$\bar{Y}_{pd}$, as shown in Table (1), it has been observed that our proposed estimator is more efficient than existing estimators in several cases, exhibiting the utility of developed method in practice and indicating that it might operate well in practical surveys. Results of an empirical investigation are shown in Figure 1. It reveals that our suggested estimators get better as the value of $\xi_1$rises.

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