Plane wave propagation in functionally graded couple stress micropolar solid half space

[1]Sangeeta Malik, [2]Ankush Antil, [3]Krishan Kumar

[1]Professor, [2]Research Scholar, [3]Assistant Professor
[1]Department of Mathematics
Baba Mastnath University
Asthal Abohar, Rohtak, Haryana, India.

***Abstract*- The problem of reflection due to longitudinal plane wave incident at a free surface of functionally graded micropolar solid half is studied. It is found that the amplitude ratios of various reflected waves are functions of angle of incidence, frequency of incident wave and are influenced by the couple stress and graded properties of the medium. The expressions of amplitude ratios have been computed numerically and presented graphically. Some special cases of interest are also derived**.

***Keywords:* Couple stress, functionally graded, micropolar, microrotation, half space.**

1. INTRODUCTION

The classical elasticity theory is believed to be inadequate for the treatment of deformations and motions of a material possessing granular structure. In particular, the effect of granular structure, or microstructure, becomes important in transmitting waves of small wavelength and/or high frequency. When the wavelength is comparable with the average grain size, the motion of the grains must be taken into account. This introduces new types of waves not encountered in the classical theory. The theory of micromorphic materials introduced by Eringen and his co-workers [1]-[5] deals with a class of substances which exhibits certain microscopic effects arising from the local structure and micro motions of the media. These materials can support stress moments and body moments and are influenced by the spin inertia. The general theory is however very complicated and even in the case of constitutively linear elastic solids [2] the differential equations are not easily amenable to solution to simplify the theory of Eringin and Suhubi [2] introduced a theory of couple stress. In view of the fact that in the abundant recent literature the terminology of couple stress was used in a different context, they thought it may save the reader from confusion if they name the theory “Micropolar elasticity”. In contrast to the couple stress theory, in micropolar elasticity all components of the asymmetric stress tensor were determinate and the motion of media in fully described when the deformation and micro-rotation vectors are known. The concept of microrotation and corresponding field equation are totally absent in the couple stress theory.

The materials such as bone, blood, shale and soil can be considered as a micropolar materials since they exhibit micro-rotational motion. Parfitt and Eringen [6] investigated the effect of micro-structure in the propagation of plane waves and their reflections from a stress free flat surface of micropolar elastic halfspace. Ariman [7] investigated plane wave propagation and reflection of plane longitudinal displacement wave in an infinite micropolar elastic half space of fixed flat surface. The various problems on micropolar elasticity are found in [Boschi [8], Eringen [9], Hassanpour and Heppler [10], [Mirzajani et al. [11]] and [Gharahi and Schiavone [12]]. Wave propagation in a generalized thermoelastic media with additional parameters like diffusion, magnetic field, anisotropy, porosity, viscosity, microstructure, temperature and other parameters provide vital information about the existence of new or modified waves. This information is useful for experimental seismologists in correcting earthquake estimation.

Some relevant studies on wave propagation are studied by various authors. Plane waves reflection in micropolar transversely isotropic generalized thermoelastic half-space was presented by Kumar and Gupta [13]. Kumar et al. [14] studied effect of two temperatures on reflection coefficient in micropolar thermoelastic with and without energy dissipation media. Sharma [15] presented reflection at free surface in micropolar thermoelastic solid with two temperatures. The existence of couple-stress in materials was originally postulated by Voigt [16]. However, Cosserat and Cosserat [17] were the first to develop a mathematical model to analyze materials with couple stresses. The idea was revived and generalized much later by Toupin [18], Mindlin and Tiersten [19], Green and Rivlin [20], Mindlin [21] and others. In these developments, the gradient of the rotation vector, as a curvature tensor, has been recognized as the effect of the second gradient of deformation in materials. Unfortunately, there are some difficulties with the present formulations. Perhaps the most disturbing troubles are the indeterminacy of the spherical part of the couple-stress tensor and the appearance of the body couple in the constitutive relation for the force-stress tensor

(Mindlin and Tiersten [19]. Nowaki [22] investigated the couple-stresses in the theory of thermoelasticity. Zhao and Li [23] investigated the Influence of couple-stresses on stress concentrations around the cavity.

Kumar et al. [24] studied propagation of SH-waves in couple stress elastic half space underlying an elastic layer. Kumar et al. [25] discussed plane waves and fundamental solution in couple stress generalized thermoelastic solid. [26] Kalkal at el. discussed reflection and transmission between thermoelastic and initially stressed fibre-reinforced thermoelastic half-spaces under dual-phase-lag model. Singh at el.[27] investigated reflection and transmision of thermoelastic waves at the corrugated interface of crystalline structure. Abo-Dahab and Mahmoud [28] presented problem of P- and SV-waves reflection and transmission during two media under three thermoelastic theories and electromagnetic field with and without gravity. Hou at el. [29] presented reflection and transmission of thermoelastic waves in multilayered media.

In the present paper, the reflection phenomenon at the free surface of a functionally graded micropolar couple stress solid medium has been studied. In couple stress thermoelastic solid medium, potential functions are introduced to represent two longitudinal waves and two transverse waves. The amplitude ratios of various reflected waves to that of incident wave are derived numerically and depicted graphically.

2. BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

On the basis of Parfitt and Eringen (1969), the constitutive relations are given by

$$(1-\nabla^2)t_{mn} = \bar{\lambda}\delta_{mn}\epsilon_{rr} + (\bar{\mu}+\bar{K})\epsilon_{mn} + \bar{\mu}\epsilon_{nm}$$

$$(1-\nabla^2)m_{mn} = \bar{\alpha}\delta_{mn}\gamma_{rr} + \bar{\beta}\gamma_{mn} + \bar{\gamma}\gamma_{nm}$$

where $\gamma_{mn}(=\phi_{m,n})$ is the curvature tensor and $\epsilon_{mn}(=u_{n,m}-\epsilon_{mnl}\phi_l)$ is the relative distortion tensor where $\mathbf{u}$ denotes displacement vector and $\boldsymbol{\phi}$ denotes microrotation vector. $\bar{\lambda}$ and $\bar{\mu}$ are Lame's constant; $\bar{K}, \bar{\alpha}, \bar{\beta}$ and $\bar{\gamma}$, are constitutive coefficients; ∇^2 is the laplacian operator.

Following Parfitt and Eringen (1969), the basic equations for micropolar couple stress elastic medium given by

$$(\bar{\lambda}+\bar{\mu})\nabla(\nabla.\mathbf{u}) + (\bar{\mu}+\bar{K})\nabla^2\mathbf{u} + \bar{K}\times\boldsymbol{\phi} = \bar{\rho}\ddot{\mathbf{u}} \quad (1)$$

$$(\bar{\alpha}+\bar{\beta})\nabla(\nabla.\boldsymbol{\phi}) + \bar{\gamma}\nabla^2\boldsymbol{\phi} + \bar{K}\nabla\times\mathbf{u} - 2\bar{K}\boldsymbol{\phi} = \bar{\rho}\bar{j}\ddot{\boldsymbol{\phi}} \quad (2)$$

where $\bar{\rho}$ is the density of the medium and $\bar{j}$ denotes the coefficients of equilibrated inertia.

3. DYNAMICS OF EXPONENTIALLY GRADED COUPLE STRESS MEDIUM

The exponentially gradedness present in the media are supposed to be in the form of exponential function with respect to x_2-direction as

$$\bar{\lambda} = \lambda^{(1)}e^{l_1x_2},\quad \bar{\mu} = \mu^{(1)}e^{l_1x_2},\quad \bar{K} = K^{(1)}e^{l_1x_2},\quad \bar{\rho} = \rho^{(1)}e^{l_1x_2},$$

$$\bar{\gamma} = \gamma^{(1)}e^{l_1x_2},\quad \bar{\alpha} = \alpha^{(1)}e^{l_1x_2},\quad \bar{\beta} = \beta^{(1)}e^{l_1x_2},\quad \bar{j} = j^{(1)}e^{l_1x_2}, \quad (3)$$

where $\lambda^{(1)}, \mu^{(1)}, K^{(1)}, \alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}, \rho^{(1)}$ and $j^{(1)}$ are the value of the corresponding elastic constants $\bar{\lambda}, \bar{\mu}, \bar{K}, \bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\rho}$ and $\bar{j}$ respectively associated with the media at $x_2 = 0$. l_1 represents the exponential gradient parameter of the medium.

Using Helmholtz decomposition theorem on vectors, the displacement component $\mathbf{u}$ related to the potential $\sigma(x_1,x_3,t)$ and $Q(x_1,x_3,t)$ and $\boldsymbol{\phi}$ related to the potential $\vartheta(x_1,x_3,t)$ and $V(x_1,x_3,t)$ are as

$$\mathbf{u} = \nabla\sigma + \nabla\times\mathbf{Q},\qquad \nabla\cdot\mathbf{Q} = 0$$

$$\boldsymbol{\phi} = \nabla\vartheta + \nabla\times\mathbf{V},\qquad \nabla\cdot\mathbf{V} = 0 \quad (4)$$

Using equations (3)-(4) in equations (1)-(2), we obtain

$$(\lambda^{(1)} + 2\mu^{(1)} + K^{(1)})\nabla^2\sigma - \rho^{(1)}\ddot{\sigma} = 0 \quad (5)$$

$$(\mu^{(1)} + K^{(1)})\nabla^2\mathbf{Q} + K^{(1)}\nabla\times\mathbf{V} - \rho^{(1)}\ddot{\mathbf{V}} = 0 \quad (6)$$

$$(\alpha^{(1)} + \beta^{(1)} + \gamma^{(1)})\nabla^2\vartheta - 2K^{(1)}\vartheta - \rho^{(1)}j^{(1)}\ddot{\vartheta} = 0 \quad (7)$$

$$\gamma^{(1)}\nabla^2\mathbf{V} + K^{(1)}\nabla\times\mathbf{Q} - 2K^{(1)}\mathbf{V} - \rho^{(1)}j^{(1)}\ddot{\mathbf{V}} = 0 \quad (8)$$

Equations (6) and (8) show that $\mathbf{Q}$ and $\mathbf{V}$ are coupled and equations (5) and (7) show that σ and ϑ are independent. We consider a plane wave propagation in a homogeneous isotropic couple stress micropolar elastic medium. For this we assume the solution of the form

$$\{\sigma,\vartheta,\mathbf{Q},\mathbf{V}\}(a,b,\mathbf{A},\mathbf{B})\exp\{il(n\cdot\mathbf{r}-ct)\} \quad (9)$$

where $\omega = lc$ is the frequency, l is the wave number and c is the phase velocity, a, b, $\mathbf{A}$ and $\mathbf{B}$ are undetermined amplitudes that are dependent on time and coordinate $r = x_m(m = 1,3)$, n is the unit vector.

Using equation (9) in equation (5), we obtain

$$C_1^2 = \frac{(\lambda^{(1)}+\mu^{(1)}+K^{(1)})}{\rho^{(1)}} \tag{10}$$

Using equation (9) in equation (7), we obtain

$$C_2^2 = \left[\frac{(\alpha^{(1)}+\beta^{(1)}+\gamma^{(1)})}{\rho^{(1)}j^{(1)}}\right]\left[-\frac{2K^{(1)}}{\rho^{(1)}j^{(1)}}\right] \tag{11}$$

Using equation (9) in equations (6) and (8), we obtain

$$\left[(\mu^{(1)}+K^{(1)})l^2 - \rho^{(1)}\omega^2\right]\mathbf{A} - il\ K^{(1)}\ \mathbf{B} = 0 \tag{12}$$

$$il\ K^{(1)}\ \mathbf{A} - \left[\gamma^{(1)}l^2 + 2K^{(1)} - \rho^{(1)}j^{(1)}\omega^2\right]\mathbf{B} = 0 \tag{13}$$

which yield the following polynomial equation

$$A_1C^4 + A_2C^2 + A_3 = 0 \tag{14}$$

where

$$A_1 = 1-\Omega;\quad A_2 = -b_4^2 - \frac{1}{2}b_3^2\Omega + (1-\Omega)(-b_2^2b_3^2);\quad A_3 = b_2^2b_3^2b_4^2;$$

$$\Omega = \frac{2\omega_0^2}{\omega^2};\quad \omega_0^2 = \frac{K^{(1)}}{\rho^{(1)}j^{(1)}};\quad b_2^2 = \frac{\mu^{(1)}}{\rho^{(1)}};\quad b_4^2 = \frac{\gamma^{(1)}}{\rho^{(1)}j^{(1)}};\quad b_3^2 = \frac{K^{(1)}}{\rho^{(1)}}$$

The roots of the equation (14) are given by

$$C_3^2 = \frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1};\quad C_4^2 = \frac{-A_2-\sqrt{A_2^2-4A_1A_3}}{2A_1}$$

Equations (14) is cubic in C^2 with complex coefficients, whose roots will provide us the speed of the propagation waves. In order to investigate the nature of these waves, inserting equation (9) in equation (4), we observed that the particle motion associated with potential Q is normal to the direction of wave propagation $\mathbf{n}$ and the wave associated with $\mathbf{Q}$ is transverse in nature. Also, the wave associated with $\mathbf{V}$ is transverse in nature (by using equations (6) and (8)). Since, both the waves are coupled and hence, known as coupled transverse waves. Thus, there exists two sets of coupled transverse waves, namely, transverse displacement wave and transverse microrotational wave.

4. REFLECTION PHENOMENON

The plane interface coincides with x_1x_3- plane in the co-ordinate system $Ox_1x_2x_3$ and x_1 axis is taken along the interface $x_3 = 0$. We consider region H_1 occupying $x_3 \geq 0$ respectively as shown in figure [1].

Consider 2-D problem in the x_1, x_3- plane

$$\mathbf{u} = (u_1, 0, u_3),\qquad \boldsymbol{\phi} = (0, \phi_2, 0)$$

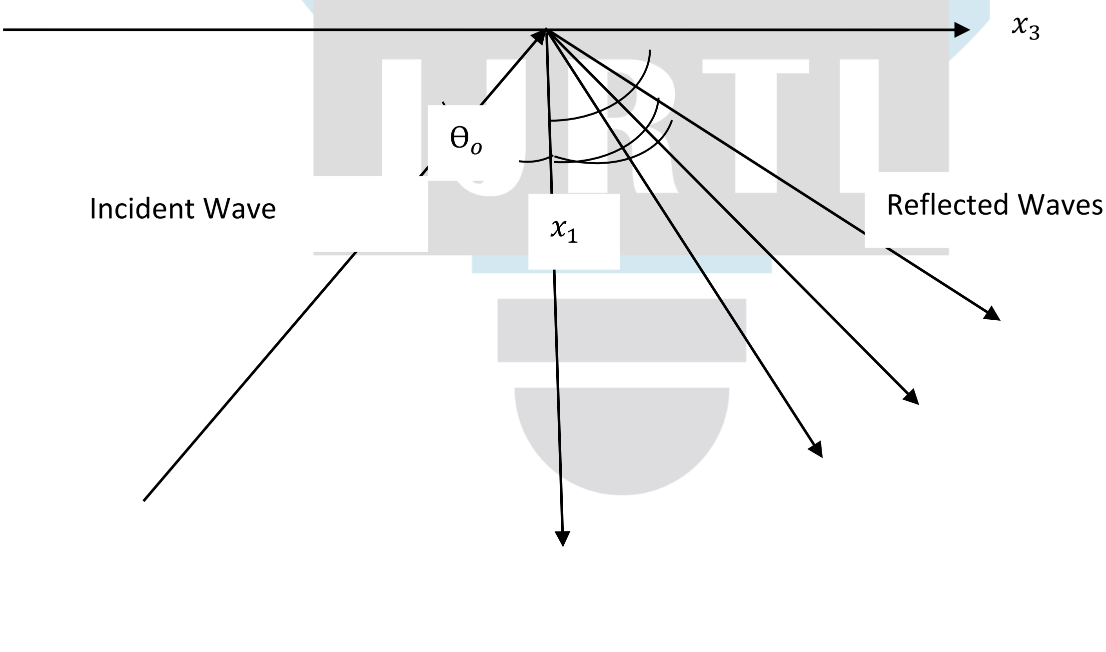


Fig. 1 Geometry of the Problem

4.1 Incidence of coupled longitudinal waves

Let us consider that the longitudinal wave are incident at $x_3 = 0$ making an angle θ_0 with M_0, k_1 and C_1 as amplitude, wave number and velocity respectively. The incident wave will generate three reflected waves with M_s, l_s θ_s and C_s as amplitude, wave number, angle and speed respectively, where $s = 1,3,4$.

Hence, the total wave field is given by

$$\sigma = M_0 \exp\{ik_1(\sin\theta_0 x_1 - \cos\theta_0 x_3) - i\omega_1 t\} + M_1 \exp\{ik_1(\sin\theta_r x_1 + \cos\theta_r x_3) - i\omega_1 t\} \quad (15)$$

$$\mathbf{Q} = \sum_{r=3,4} M_r \exp\{ik_r(\sin\theta_r x_1 + \cos\theta_r x_3) - i\omega_r t\} \quad (16)$$

$$\mathbf{V} = \sum_{r=3,4} \chi_r M_r \exp\{ik_r(\sin\theta_r x_1 + \cos\theta_r x_3) - i\omega_r t\} \quad (17)$$

where $\chi_{3,4}$ are the coupling parameters determined by equations (12) and (13) as

$$\chi_{3,4} = \frac{1}{ik_{3,4}b_3^2}[(b_2^2 + b_3^2)k_{3,4}^2 - \omega^2] \quad (18)$$

4.2 Boundary conditions

The boundary conditions for couple stress micropolar thermoelastic solid half-spaces are

$$t_{33} = t_{31} = m_{32} = 0, \quad at \quad x_3 = 0 \quad (19)$$

where the components for microrotational vector, displacement vector, couple stress tensor and stress tensor are given by

$$u_1 = \sigma_{,1} - Q_{2,3}, \quad u_3 = \sigma_{,3} + Q_{2,1}, \quad \phi_2 = V_{1,3} - V_{3,1}$$
$$t_{33} = \lambda^{(1)}\sigma_{,11} + (\lambda^{(1)} + 2\mu^{(1)} + K^{(1)})\sigma_{,33} + (2\mu^{(1)} + K^{(1)})Q_{2,13}$$
$$t_{13} = (2\mu^{(1)} + K^{(1)})\sigma_{,13} + \mu^{(1)}Q_{2,11} - (\mu^{(1)} + K^{(1)})Q_{2,33} - K^{(1)}\phi_2 \quad (20)$$

Here, comma(,) denotes partial derivative.

We assume that $\omega_1 = \omega_3 = \omega_4 = \omega$ and Snell's law holds, which gives

$$k_1\sin\theta_0 = k_1\sin\theta_1 = k_3\sin\theta_3 = k_4\sin\theta_4 = l_0(say)$$

The potential given in Eqns $(15)-(17)$ satisfy the boundary conditions (19) at $x_3 = 0$, therefore,

$$\sum_{r=0,1} [\lambda^{(1)} + (2\mu^{(1)} + K^{(1)})\cos^2\theta_r]k_r^2 M_r + \sum_{r=3,4} (2\mu^{(1)} + K^{(1)})\sin\theta_r\cos\theta_r k_r^2 M_r = 0 \quad (21)$$

$$(2\mu^{(1)} + K^{(1)})\sin\theta_0\cos\theta_0 k_1^2 M_0 - (2\mu^{(1)} + K^{(1)})\sin\theta_1\cos\theta_1 k_1^2 M_1 + \sum_{3,4} [\mu^{(1)}\cos 2\theta_r + K^{(1)}\cos^2\theta_r - \frac{K^{(1)}\chi_r}{k_r^2}]k_r^2 M_r = 0 \quad (22)$$

$$\gamma^{(1)}\chi_3\cos\theta_3 k_3 M_3 + \gamma^{(1)}\chi_4\cos\theta_4 k_4 M_4 = 0 \quad (23)$$

Eqns $(21)-(23)$ can be written in the matrix form as

$$[A][Y] = [Z] \quad (24)$$

where $A = [a_{ij}]_{3\times 3}$ and $Y = \begin{pmatrix} y_1 \\ y_3 \\ y_4 \end{pmatrix}_{3\times 1}$.

The reflection coefficients are represented by $Y_r = \frac{M_r}{M_0}$ $(r = 1,3,4)$. The entries of $[A]$ and $[Z]$ are given by

$$a_{11} = 1, \quad a_{12} = [\lambda^{(1)} + (2\mu^{(1)} + K^{(1)})(1 - c_{31}^2\sin^2\theta_0)]/P_1 c_{31}^2$$

$$a_{13} = (2\mu^{(1)} + K^{(1)})\sin\theta_0\sqrt{(1 - c_{41}^2\sin^2\theta_0)}/P_1 c_{41}; \quad a_{21} = \sin\theta_0\cos\theta_0,$$

$$a_{22} = \frac{1}{(2\mu^{(1)} + K^{(1)})c_{31}^2}\left[\mu^{(1)}(1 - 2c_{31}^2\sin^2\theta_0) + K^{(1)}(1 - c_{31}^2\sin^2\theta_0) - \frac{K^{(1)}\chi_3}{k_3^2}\right]$$

$$a_{23} = \frac{1}{(2\mu^{(1)} + K^{(1)})c_{41}^2}\left[\mu^{(1)}(1 - 2c_{41}^2\sin^2\theta_0) + K^{(1)}(1 - c_{41}^2\sin^2\theta_0) - \frac{K^{(1)}\chi_4}{k_4^2}\right]$$

$$a_{31} = 0, \quad a_{32} = \sqrt{1 - c_{31}^2\sin^2\theta_0}/c_{31}, \quad a_{33} = \chi_4\sqrt{1 - c_{41}^2\sin^2\theta_0}/\chi_3 c_{41}$$

here

$$c_{p1} = \frac{c_p}{c_1}, (p = 3,4); \quad Z_1 = 1, \quad Z_2 = \sin\theta_0\cos\theta_0, \quad Z_3 = 0, \quad P_1 = -[\lambda^{(1)} + (2\mu^{(1)} + K^{(1)})\cos^2\theta_0 k_1^3]$$

5. NUMERICAL ANALYSIS

With the aim to discuss the impact of gradient parameter on the reflection and energy coefficients of reflected waves is studied numerically with following physical constants taken from Tomar and Khurana [32]

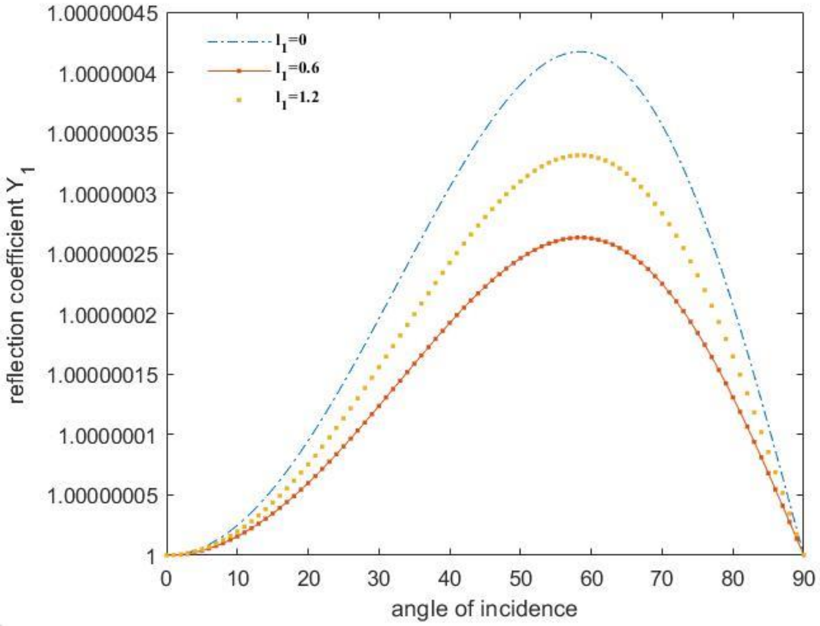
$$\bar{K} = 0.0139 \times 10^{10} dynecm^{-2}; \quad \bar{\mu} = 1.98 \times 10^{10} dynecm^{-2};$$
$$\bar{\lambda} = 7.85 \times 10^1 1 dynecm^{-2}; \quad \bar{\rho} = 2.7 \times 10^3 gcm^{-3};$$

$$\bar{\gamma} = 5.0 \times 10^{11} dyne; \quad j = 0.2 \times 10^{-19} cm^2; \quad \bar{\alpha} = 3.0 \times 10^{11} dyne;$$
$$\bar{\beta} = 4.0 \times 10^{11} dyne; \quad e_0 = 0.39$$

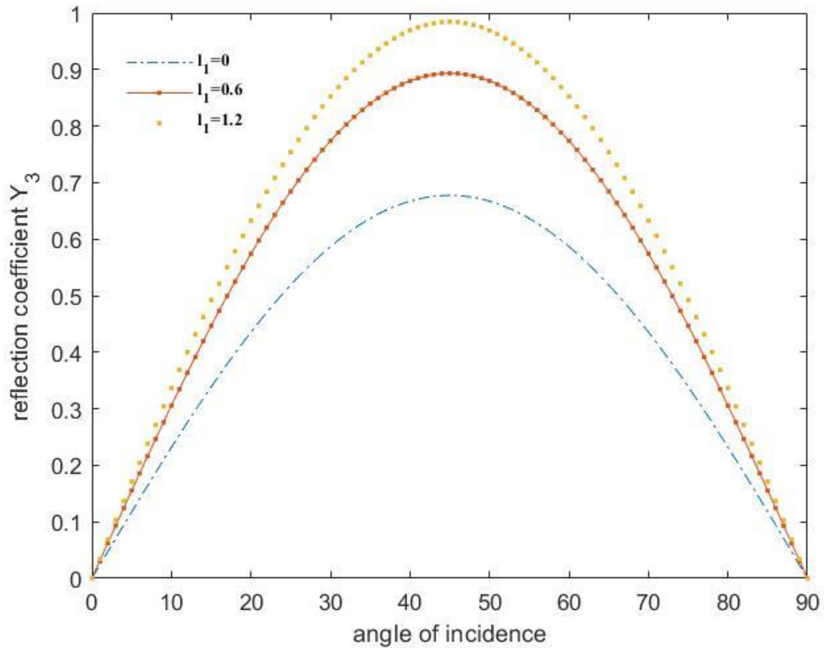
The remaining parameters which are taken into account are

$$l_1 = 0,\ 0.6,\ 1.2$$

The computer generated result are shown graphically in figures 2-4.



$Figure 2$: Impact of gradient parameter on Y_1 with θ_0



$Figure 3$: Impact of gradient parameter on Y_3 with θ_0

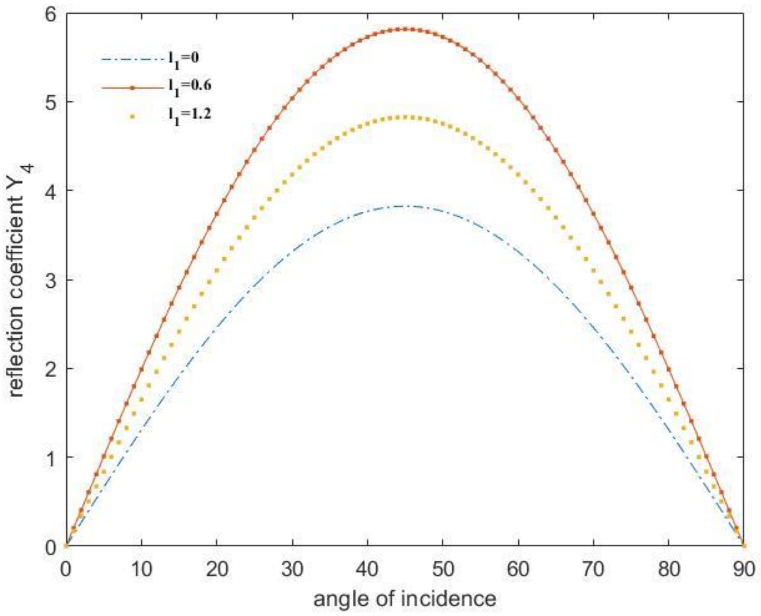


*Figure*4: Impact of gradient parameter on Y_4 with θ_0

Figure 2 depicts that the impact of gradient parameter l_1 on the reflection coefficient Y_1 with the angle of incidence θ_0. The reflection coefficient first increase slightly for range $0° \leq \theta_0 \leq 10°$; thereafter increase sharply for the range $10° \leq \theta_0 \leq 50°$ and after making a crest, decreasing sharply for the range $70° \leq \theta_0 \leq 90°$.
On comparing the curves of reflection coefficient Y_1, we conclude that
1. The reflection coefficient Y_1 without gradient parameter i.e. $l_1 = 0$ dominates the reflection coefficient Y_1 with gradient parameter i.e. $l_1 \neq 0$.
2. The reflection coefficient Y_1 with gradient parameter $l_1 = 1.2$ dominates the reflection coefficient Y_1 with gradient parameter $l_1 = 0.6$.
3. The reflection coefficient Y_1 has a maximal value equal to unity at the moderate range of angle of incidence.
4. The pattern of all the curves of the reflection coefficient Y_1 is similar.

Figures 3 and 4 depict the impact of gradient parameter l_1 on the reflection coefficients Y_3 and Y_4 with the angle of incidence θ_0. The curves of the reflection coefficients increases sharply eith the angle of incidence for the range $0° \leq \theta_0 \leq 40°$; and thereafter making a crest, a decrease is reported for the range $50° \leq \theta_0 \leq 90°$.
On comparing the curves of reflection coefficients Y_3 and Y_4, we conclude that
1. The curves of reflection coefficients Y_3 and Y_4 without gradient parameter i.e. $l_1 \neq 0$ dominates the curves of reflection coefficient Y_3 and Y_4 with gradient parameter i.e. $l_1 = 0$ respectively.
2. The reflection coefficient Y_3 with gradient parameter $l_1 = 1.2$ dominates the reflection coefficient Y_3 with gradient parameter $l_1 = 0.6$ while the reflection coefficient Y_4 with gradient parameter $l_1 = 0.6$ dominates the reflection coefficient Y_1 with gradient parameter $l_1 = 1.2$.
3. The maximum value of reflection coefficients Y_3 and Y_4 is at the moderate range of angle of incidence.
4. The pattern of all the curves of the reflection coefficients Y_3 and Y_4 is similar.

6 Conclusion

In current research paper, the study of wave propagation due to incidence of longitudinal displacement wave in functionally graded micropolar couple stress elastic medium has been undertaken. The impact of gradient parameter has been examined. Further, we found there are four waves, viz. a longitudinal displacement wave, a longitudinal microrotational wave and a set of two coupled waves propagating with different speeds.
The major consequences are as follows:
1. The phase speeds, reflection coefficients with respect to angle of incidence are calculated numerically and graphically.
2. The impact of gradient parameter on the reflection coefficient are not seen significantly at normal and grazing incidence.
3. The impact of gradient parameter on the reflection coefficient are maximum at intermediate angle of incidence.
4.The reflection coefficients with gradient parameter $l_1 = 1.2$ dominates the reflection coefficients with gradient parameter $l_1 = 0.6$ except Y_4. While the Y_4 with gradient parameter $l_1 = 0.6$ dominates the reflection coefficient Y_1

with gradient parameter $l_1 = 1.2$.
5. The curves of all reflection coefficients without gradient parameter i.e. $l_1 = 0$ dominates the curves of all reflection coefficients with gradient parameter i.e. $l_1 \neq 0$ respectively except Y_3 and Y_4.

ACKNOWLEDGEMENT
This work is financially supported by CSIR, New Delhi unde Junior Research Fellowship CSIR-Award letter no. 09/1279(11483)/2021-EMR-I through second author.

REFERENCES:

[1] A. C. Eringen and E. S. Suhubi, Non-linear theory of simple micro elastic solids-I", International Journal of Engineering Sciences, vol. 2 ,pp.189-203, 1964.
[2] E. S, Suhubi and A.C. Eringen, "Non-linear theory of micro elastic solid-II",International Journal of Engineering Sciences ,vol.2, pp. 389-404, 1964.
[3]A. C. Eringen, "Simple microfluids" International Journal of Engineering Sciences, vol. 2, pp. 205-217, 1964.
[4] A. C., Eringen, "Mechanics of Micromorphic Continua" Proc. Eleventh Intl. Congr. Appl. Mech., Springer-Verlag, 1965.
[5] A. C. Erinqen, "Theory of Micropolar Fluids", Journal of Mathematics and Mechanics
Vol. 16(1), pp. 1-18, 1966.
[6] V. R. Parfitt and A. C. Eringen: "Reflection of plane waves from the flat boundary of a micropolar solid half space" J. Acoust. Soc. Am., vol.45, pp.1258-1272, 1969.
[7] T. Ariman, "Wave propagation in a micropolar elastic half-space" Acta Mech., vol. 13, pp. 11–20, 1972.
[8] E. Boschi, "Lamb and Love wave-propagation in an infinite micropolar thermoelasticity" Meccanica, vol. 7, pp. 154–157, 1973.
[9] A. C. Eringen, "Plane waves in nonlocal micropolar elasticity" International Journal of Engineering Sciences, vol. 22, pp.1113–1121, 1984.
[10] S. Hassanpour and G. L. Heppler (2015), "Micropolar elasticity theory: a survey of linear isotropic equations, representative notations, and experimental investigations" Math. Mech. Solids, vol.22, pp. 224–248, 2015.
[11] M. Mirzajani, N. Khaji and M. Hori "Stress wave propagation analysis in one dimensional micropolar rods with variable cross section using micropolar wave finite method", Int. J. Appl. Mech., vol. 10: 1850039, 2018
[12] A. Gharahi, and P. Schiavone(2019). "Uniqueness of solution for plane deformations of a micropolar elastic solid with surface effects" Continuum Mech. Therm., vol. 32, pp.9–22, 2019
[13] R. Kumar and R. R. Gupta, "Plane waves reflection in micropolar transversely isotropic generalized thermoelastic half-space", Mathematical Sciences, vol. 6(6), DOI: 10.1186/2251-7456-6-6, 2012
[14] R. Kumar, K. D. Sharma and S. K. Garg, "Effect of two temperatures on reflection coefficient in micropolar thermoelastic with and without energy dissipation media", Advances in Acoustics and Vibration, vol. 2014,11 pages,2014 http://dx.doi.org/10.1155/2014/846721
[15] K. Sharma, "Reflection at free surface in micropolar thermoelastic solid with two temperatures", Int. J. of Applied Mechanics and Engineering, vol.18, No.1, pp.217-234, 2013
[16] W. Voigt, "Theoretische studien uber die elastizitatsverhaltnisse der kristalle", Abh. Ges. Wiss. Gottingen, vol. 34, pp. 3-51, 1887
[17] E. Cosserat and F. Cosserat, "Theory des corps deformables" A. Herman Et. Fils, Paris, 1909.
[18] R. A. Toupin, "Elastic materials with couple-stresses", Arch. Ration. Mechan. Anal., vol. 11(1), pp. 385–414, 1962
[19] R. D. Mindlin, and H. F. Tiersten, "Effects of couple-stresses in linear elasticity", Arch. Ration. Mech. Anal., vol.11, pp. 415–448, 1962
[20] A. E.Green and R. S. Rivlin, "Simple force and stress multiples", Arch. Rat. Mech. Analysis, vol. 16,pp. 325-354, 1964
[21] R. D. Mindlin, "Influence of couple-stresses on stress-concentrations", Exp. Mech., vol. 3, pp. 1–7, 1963
[22] W. Nowaki, "Couple-stresses in the theory of thermoelasticity: Irreversible aspects of continuum mechanics and transfer of physical characteristics in moving fluids", IUTAM Symposia, pp. 259-278, 1968
[23] Z. Jian-bin and L. Zi-guo, "Influence of couple-stresses on stress concentrations around the cavity", Applied Mathematics and Mechanics, vol. 21(8), pp. 893-900, 2000
[24] R. Kumar, K. Kumar and R. C. Nautiyal, "Propagation of SH-waves in couple stress elastic half space underlying an elastic layer", Afrika Matematika, vol. 24, pp. 477-485, 2013
[25] R. Kumar, K. Kumar and R. C. Nautiyal, "Plane waves and fundamental solution in couple stress generalized thermoelastic solid", Afrika Matematika, vol. 25(3), pp. 591-603, 2014

[26] K. K. Kalkal, S. K. Sheokand, and S. Deswal, “Reflection and transmission between thermoelastic and initially stressed fibre-reinforced thermoelastic half-spaces under dual-phase-lag model,” Acta Mech., vol. 230**, pp.** 87–104, 2019.

[27] P. Singh, A. K. Singh, and A. Chattopadhyay, “Reflection and transmision of thermoelastic waves at the corrugated interface of crystalline structure,” J. Therm. Stresses vol. 44, pp. 1–44, 2021.

[28] S. M. Abo-Dahab and E. E. Mahmoud, “Problem of P- and SV-waves reflection and transmission during two media under three thermoelastic theories and electromagnetic field with and without gravity,” Waves Rand. Compl. Med. vol 31, pp. 1–24, 2021.

[29] W. Hou, L. Fu, and J. M. Carcione, “Reflection and transmission of thermoelastic waves in multilayered media,” Geophys. vol. 87, PP. 117–128, 2022.

[30] S. K. Tomar and A. Khurana, “Transmission of longitudinal wave at plane interface between micropolar elastic and chiral solid half-spaces incident from micropolar half-spaces” J. Sound Vib., vol.311, pp.973–990, 2008