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An Efficient Generalized Logarithmic Type Estimator Using Probability Sampling

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Abstract **- In this research paper, we have proposed logarithmic type estimator to estimate the population parameter using auxiliary information under simple random sampling. To determine the bias and mean square error of the suggested estimator have been presented up to the first degree of approximation. The estimated categories that have been suggested perform more effectively than comparable estimates for each unit when compared to other existing estimates. We have conducted an empirical demonstration to support the usefulness of the suggested estimators.**

Keywords **-** Bias and Mean Square Error, Auxiliary Information, Simple Random Sampling.

1. Introduction

The fact is generally accepted, when using studies with large populations, the effectiveness of the estimators is increased when multiple auxiliary variables are used. The following technique can be used to select some samples from the population or at any point during the estimator creation process. In this work, the auxiliary variables were effectively used. It is well known that when the research variable and choice possibilities are considerably related, employing simple random sampling estimators of the population mean for equal probability sampling yields a large increase in efficiency when compared to the usual estimator simple mean. To estimate the population characteristics of the primary variable under investigation, a straightforward random sampling strategy is frequently employed. In sampling theory, it is typically beneficial to incorporate supplementary data in order to get precise estimates of various population parameters. However, the most important component is that there should be a correlation between the variables being studied and the other components. The use of auxiliary data in the form of product type and exponential ratio estimators was initially presented by Bahl and Malik [5]., Singh et al. [3] and some significant current research also Murthy [2] include who recommended a log type estimator using the knowledge of auxiliary information presented in a set of variables. Practically and empirically, authors evaluated the intended types of estimators to a few previous estimators, and when supplementary data existed, they claimed that the new estimators performed better. They also came to the conclusion from their investigation that the updated estimators performed better than the current estimators. According to a study, the suggested types of estimators outperformed standard estimators of regression, ratios, and product types.

Methods of Sampling and Notations

Consider a population of N units, where Ụ= M_1, M_2,……….., M_N. Let X and Y used for the auxiliary variables and the research variables, respectively. Let (y_i, x_i) represent the N pairs of sample observations for the auxiliary variables and study variables, respectively, for the ith unit that were acquired using simple random sampling without replacement from the population size of N. Let the population means of the auxiliary and study variables be represented by $\bar{X}$ and $\bar{Y}$ respectively and the sample means by $\bar{x}$ and $\bar{y}$.

Let's clarify,

$s_y^2 = S_y^2(e_y + 1)$, $\quad s_x^2 = S_x^2(e_x + 1)$, $\quad s_z^2 = S_z^2(e_z + 1)$

$$E(e_x) = E(e_y) = E(e_z) = 0$$

$$E(e_y^2) = \frac{1}{n}(\Delta_{400} - 1),\ E(e_x^2) = \frac{1}{n}(\Delta_{040} - 1),\ E(e_z^2) = \frac{1}{n}(\Delta_{004} - 1)$$

$$E(e_x e_y) = \frac{1}{n}(\Delta_{220} - 1),\ E(e_x e_z) = \frac{1}{n}(\Delta_{022} - 1), E(e_z e_y) = \frac{1}{n}(\Delta_{202} - 1)$$

$$s_{tx}^2 = (1+g)S_x^2 - gs_x^2,\ \ s_{ty}^2 = (1+g)S_y^2 - gs_y^2,\ \ s_{tz}^2 = (1+g)S_z^2 - gs_z^2$$

Where $\Delta_{abc} = \frac{u_{abc}}{u_{200}^{a/2} u_{020}^{b/2} u_{002}^{c/2}}$, $g = \frac{n}{N-n}$

$u_{abc} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^a (x_i - \bar{X})^b (z_i - \bar{Z})^c$, where a, b and c being non -negative integers

Table1: The current estimators in this field and MSE

Sr. No.	Estimator	MSE
1.	mean per unit estimator S_y^2	$\frac{S_y^4}{n}(\Delta_{400}-1)$
2.	Ratio estimator $\bar{S}_R = s_y^2 \frac{S_x^2}{s_x^2}$	$\frac{S_y^4}{n}[\Delta_{040}+\Delta_{400}-2\Delta_{220}]$
3.	Product estimator $\bar{S}_P = s_y^2 \frac{s_x^2}{S_x^2}$	$\frac{S_y^4}{n}[\Delta_{040}+\Delta_{400}+2\Delta_{220}]$

The present study offers a logarithmic type estimator based on the previously determined sample's logarithmic type function with advantages of variation. When trying to find a similarly optimum scale, we look at logarithmic type estimations.

2. Proposed Estimator

$\mathrm{P}_0 = \mathrm{n}_1 \mathrm{s}_y^2 + (1-\mathrm{n}_1)\mathrm{s}_y^2\left(\frac{\mathrm{s}_{tx}^2}{\mathrm{S}_x^2}\right)\log\left(\frac{\mathrm{s}_{tz}^2}{\mathrm{S}_z^2}\right)^{\beta}$, where $\mathrm{n}_1$is suitably chosen constant i.e. $\mathrm{n}_1 = 2$.

$= \left(\mathrm{e}_y+1\right)\mathrm{S}_y^2\left[\mathrm{n}_1 + (1-\mathrm{n}_1 - \mathrm{g}\mathrm{e}_x + \mathrm{n}_1\mathrm{g}\mathrm{e}_x)\beta\left(-\mathrm{g}\mathrm{e}_z - \frac{\mathrm{g}^2\mathrm{e}_z^2}{4}\right)\right]$

$\mathrm{E}\left(\mathrm{P}_0 - \mathrm{S}_y^2\right) = \mathrm{S}_y^2(1-\mathrm{n}_1)\left\{\frac{\mathrm{g}\beta}{4\mathrm{n}}[4\mathrm{g}(\Delta_{022}-1) - \mathrm{g}(\Delta_{004}-1) - 4(\Delta_{202}-1)]\right\}$

$$E\left(P_0 - S_y^2\right)^2 = \mathrm{MSE} = \frac{1}{\mathrm{n}}\mathrm{S}_y^4\begin{bmatrix}(1-\mathrm{n}_1)^2\mathrm{g}^2\beta^2(\Delta_{004}-1) + \mathrm{n}_1^2(\Delta_{400}-1) - \\ 2(1-\mathrm{n}_1)\mathrm{g}\beta\mathrm{n}_1(\Delta_{202}-1) + \mathrm{n}(1-\mathrm{n}_1)^2\end{bmatrix} \quad --- (\mathbf{1})$$

3. Optimum Variance

Differentiating the equation (1) partially with respect to β and find the optimum values of β

$$\beta_{opt} = \frac{\mathrm{n}_1(\Delta_{202}-1)}{\mathrm{g}(1-\mathrm{n}_1)(\Delta_{004}-1)}$$

The minimized optimum values of mean square errors/variance for β

$$\mathrm{MSE}_{min} = \frac{1}{\mathrm{n}}\mathrm{S}_y^4\left[\mathrm{n}(1-\mathrm{n}_1)^2 + \mathrm{n}_1^2(\Delta_{400}-1) - \frac{{\mathrm{n}_1}^2(\Delta_{202}-1)^2}{(\Delta_{004}-1)}\right]$$

4. Efficiency Comparison:

4.11 Comparison of P_0 with mean per unit estimator:

In this section, firstly we compare the MSE of P_0 with variance of mean per unit estimator is given in table 1. P_0 will be more efficient than mean per unit estimator if it satisfies the following condition:

$$\mathrm{V}(S_y^2) - \mathrm{MSE}(\mathrm{P}_0) \geq \frac{S_y^4}{n}(\Delta_{400}-1) - \frac{S_y^4}{n}\begin{bmatrix}(1-\mathrm{n}_1)^2\mathrm{g}^2\beta^2(\Delta_{004}-1) + \mathrm{n}_1^2(\Delta_{400}-1) - \\ 2(1-\mathrm{n}_1)\mathrm{g}\beta\mathrm{n}_1(\Delta_{202}-1) + \mathrm{n}(1-\mathrm{n}_1)^2\end{bmatrix}$$

$$\geq 0$$

4.12 Comparison of P_0 with ratio estimator:

Secondly, we compare the MSE of P_0 with MSE of ratio estimator is given in table 1. P_0 will be more efficient than ratio estimator if it satisfies the following condition:

$$\mathrm{MSE}(\bar{S}_R) - \mathrm{MSE}(\mathrm{P}_0) \geq \frac{S_y^4}{n}[\Delta_{040}+\Delta_{400}-2\Delta_{220}] - \frac{S_y^4}{n}\begin{bmatrix}(1-\mathrm{n}_1)^2\mathrm{g}^2\beta^2(\Delta_{004}-1) + \mathrm{n}_1^2(\Delta_{400}-1) - \\ 2(1-\mathrm{n}_1)\mathrm{g}\beta\mathrm{n}_1(\Delta_{202}-1) + \mathrm{n}(1-\mathrm{n}_1)^2\end{bmatrix}$$

$$\geq 0$$

4.13 Comparison of P_0 with product estimator:

Thirdly, we compare the MSE of P_0 with MSE of product estimator is given in table 1. P_0 will be more efficient than product estimator if it satisfies the following condition:

$$\mathrm{MSE}(\bar{S}_P) - \mathrm{MSE}(P_0) \geq \frac{S_y^4}{n}[\Delta_{040} + \Delta_{400} + 2\Delta_{220}] - \frac{S_y^4}{n}\begin{bmatrix}(1-n_1)^2 g^2 \beta^2 (\Delta_{004} - 1) + n_1^2(\Delta_{400} - 1) - \\ 2(1-n_1) g\beta n_1 (\Delta_{202} - 1) + n(1-n_1)^2\end{bmatrix} \geq 0$$

5. Empirical study

We have calculated the effectiveness of the proposed estimator P_0 in comparison to the existing estimator. To calculate the percentage relative efficiency of estimator, we use the formula as follows:

$$\mathrm{PRE} = \left[\frac{MSE(\delta)}{MSE(S_y^2)}\right] \times 100$$

Where δ is some estimator of population mean S_y^2.

Population I [Source: Sarjinder Singh (2003), p.1116][6]. y: Fish caught in year 1995,

x: Fish caught in year 1993

$N = 69$, $n = 25$, $\Delta_{400} = 7.7685$, $\Delta_{040} = 9.9860$, $\Delta_{004} = 9.9851$, $\Delta_{220} = 8.3107$, $\Delta_{202} = 8.1715$, $\Delta_{022} = 9.6631$, $S_y^2 = 37199578$.

Population II[Source: Murthy (1967), p.399][1]. y: Area under wheat in 1964,

x: Area under wheat in 1963

$N=34$, $n=15$, $\Delta_{400} = 3.7879$, $\Delta_{040} = 2.9123$, $\Delta_{004} = 2.8082$, $\Delta_{220} = 3.1046$, $\Delta_{202} = 2.9790$, $\Delta_{022} = 2.7379$, $S_y^2 = 2256455704$.

Table 2. Percentage Relative Efficiency of "proposed estimator" with the corresponding existing estimator based on population I

Estimators	Percentage Relative Efficiency			
	β = 0.3	β = 0.35	β = 0.4	β = 0.45
S_y^2	100	100	100	100
$\bar{S}_R$	16.74078	16.74078	16.74078	16.74078
$\bar{S}_P$	507.8806	507.8806	507.8806	507.8806
P_0	845.4564	858.8895	872.5368	886.3984

Table 3. Percentage Relative Efficiency of "proposed estimator" with the corresponding existing estimator based on population II

Estimators	Percentage Relative Efficiency			
	β = 0.3	β = 0.35	β = 0.4	β = 0.45
S_y^2	100	100	100	100
$\bar{S}_R$	17.61182	17.61182	17.61182	17.61182
$\bar{S}_P$	463.051	463.051	463.051	463.051
P_0	1008.927	1021.449	1034.173	1047.099

6. Conclusion

In the present study we compare percentage relative efficiency as manifested in table 2 and 3, it has been noticed that the proposed logarithmic type estimator P_0 has better performance than the existing estimator. We conclude that our proposed estimator is more efficient than the mean per unit estimator, ratio estimator and product estimator.

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