New approach in two-level trade credit EOQ model with default risk and reminder cost

Inventory model with new approach

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***Abstract*—This investigation proposes a novel approach in trade credit model for deteriorating items that incorporates default risk and reminder costs—drawing inspiration from rural Indian practices—to reduce customer defaults and enhance retailer profitability. The model optimizes decision variables using the generalized reduced gradient method, demonstrating that reminder costs can significantly boost profits**

***Index Terms*—Inventory Model, Two-level trade-credit, Default risk, Reminder cost, New approach**

I. INTRODUCTION

To stimulate demand in today's highly competitive environment, entities across the supply chain—such as producers, wholesalers, and sellers—commonly provide trade credit to their downstream counterparts. This credit strategy effectively increases buyers' financial flexibility and drives higher sales volumes. For instance, in some European nations, nearly 80% of commercial dealings utilize trade credit [1]. In contrast, the traditional EOQ framework assumes that buyers must pay the full cost upon delivery, which can create financial pressure and restrict their purchasing ability.

To ease this financial burden, upstream suppliers provide a credit period (CP), allowing buyers to postpone payment without any initial liability. The full amount is paid at the end of this grace period. If the buyer defaults beyond the allowed time, the supplier typically applies an extra interest charge on the overdue balance. Notably, during this interval, retailers can earn interest on the sales revenue, making trade credit even more appealing [2].

Despite its benefits in boosting demand, trade credit carries the serious drawback of customer default. Some buyers misuse the deferred payment option and fail to repay, leading to financial losses for sellers—a risk that grows with extended credit periods. To counter this, especially in rural India, merchants often hire individuals to follow up with defaulters and recover dues. This results in an extra operational cost, known as *reminder cost*, aimed at minimizing default risks. While this approach improves recovery, it comes with a trade-off: increased expenses and potential damage to the seller's reputation, which could negatively influence future demand.

This study formulates an EOQ model for deteriorating items under a two-level trade credit structure, where the wholesaler provides credit to the retailer, who then partially extends it to end customers. The paper introduces two innovative concepts to enhance the traditional trade credit inventory framework.

1. We propose "*reminder cost*" as a strategic tool to mitigate default risk by reducing the number of customers who fail to repay within the credit period. While effective in dues recovery, this approach may deter some customers, leading to reduced demand and impacting overall profitability. Findings indicate that retailer profit initially rises with reminder cost but declines beyond an optimal point, highlighting a trade-off. The analysis also confirms that profit is concave with respect to reminder cost.

2. Alongside reminder costs, the study introduces a new payment policy for retailers based on the prevailing bank loan system. Under this approach, the retailer takes a bank loan—at a lower interest rate than that of the wholesaler—to pay the supplier and cover ordering costs at the end of the credit period. The loan is repaid in fixed instalments using sales revenue and interest earned during the period. By optimizing the repayment duration, the retailer aims to maximize profit. After full repayment, the remaining revenue and accrued interest constitute the retailer's net income for profit evaluation.

Key features of some investigations are presented in Table 1

Table 1: Key features of some trade credit inventory models

References	Trade-credit	Default risk	Reminder cost	Formulation approach
[3]	One-level	✓	✗	Conventional
[4]	Two-level	✗	✗	Conventional
[5]	------------	✗	✗	Conventional
[6]	Two-level	✗	✗	Conventional
[7]	------------	✗	✗	Conventional
[8]	Two-level	✗	✗	Conventional
[9]	------------	✗	✗	Conventional
[10]	Two-level	✗	✗	Conventional
[11]	One-level	✗	✗	Conventional
This investigation	Two-level	✓	✓	New

II. Notations

C_o: Setup cost per order incurred by the retailer (including interest) in $

C_p: Unitary cost for the retailer's purchase in $

C_s: Unit-wise selling price for the retailer in $

h: Annual retailer's unit storage cost, payable at the wrap-up the business cycle in $

I_e: Retailer's annual earned interest rate in $

I_c: Interest charge of the wholesaler per year in $; with $I_c > I_e$

I_b: Bank interest rate per year in $ ($I_e < I_b < I_c$)

M: Wholesaler to the retailer CP

λ: Constant deterioration rate

Q; The quantity of orders placed by the retailer

n: Number of installments in which retailer's pays loan amount with compound interest

$D(N,r_c)$: Annual consumption rate, function of r_c and N

a_1: Annual base demand without N and r_c

I(t): Stock level at time t

PC: Total purchasing price of retailer

HC: Total holding cost

r_c: Reminder level (decision variable)

N: Retailer's trade credit duration for the customer in years (decision variable)

T: Optimal cycle duration in years (decision variable)

wrt means with respect to

TLTC means two-level trade credit

CP means credit period

III. Assumptions

The mathematical model is formulated under the assumptions stated as follows:

1. This study considers a case with one dedicated wholesaler and one dedicated retailer, centered around a perishable item. Owing to its perishability, the model is examined within a one-time period setting.
2. The rate of deterioration is constant.
3. The mathematical formulation is based on the TLTC inventory model proposed, where the wholesaler and retailer offer credit periods M and N to the retailer and customers, respectively.
4. If the buyer does not settle the payment within the allowed credit time MMM, the seller imposes interest on the unpaid amount until full repayment. Simultaneously, the buyer accrues interest on the income received from selling the goods.
5. Offering credit to buyers introduces the possibility of non-payment by some. To mitigate this risk, the seller uses measures like hiring staff to follow up with defaulters. This leads to an extra expense, referred to as the reminder cost (RC), which is represented as:

$$RC = \rho r_c^2$$

where r_c is the reminder level and ρ is a cost coefficient. Clearly, if $r_c = 0$, then RC = 0.

6. The reminder strategy reduces the number of defaulters. As defaults increase with longer credit periods, the default risk is modeled as a function of r_c and N, given by:

$$d = d(N, r_c) = 1 - e^{(\beta N - a_1 r_c)}$$

where $a_1 \geq 0$ is a constant and $\beta N - a_1 r_c > 0$ to ensure $d(N, r_c) > 0$. This expression shows that default risk increases with N and decreases with r_c.

7. Some customers may find the reminder mechanism undesirable, leading to a negative impact on demand. Therefore, demand D (r_c, N) is modeled as a linear function of r_c and N:
$$D = D(r_c, N) = a_1(1 + a_2 N - \alpha r_c)$$
where a_1, a_2, and α are positive constants.
8. In the proposed system, the entire revenue from sales is deposited into an interest-accruing account. Initial costs, including ordering, operations, and other payments, are met using local loans. At time M, the retailer uses the accumulated revenue and interest to partly pay the wholesaler and repay local loans. Since bank loans come at a lower interest rate than wholesalers or private lenders, the retailer then takes a bank loan to clear any remaining dues. This loan is repaid in fixed installments at compound interest, using the sales revenue and accrued interest. The optimal repayment period is determined to maximize profit. After the loan is repaid, the remaining revenue and interest constitute the retailer's net income. This structured approach minimizes borrowing costs and strategically utilizes interest income to maximize profit.

IV. MATHEMATICAL FORMULATIONS

A wholesaler supplies a perishable item to a retailer, granting a trade CP M for payment settlement. If the retailer delays payment beyond this duration, the wholesaler levies interest on the remaining balance at a rate I_c. The retailer, upon acquiring the product, extends a CP N to end consumers. The length of this period influences customer purchase behaviour.

Yet, some customers misuse the prolonged payment window and fail to pay. The proportion of such defaults tends to rise with an increase in the retailer's credit duration N. To curb these defaults, the retailer assigns staff to follow up with credit customers, incurring a cost modelled as ρr_c^2, where r_c reflects the intensity of reminder efforts. Increasing r_c effectively lowers the number of defaulters.

However, frequent reminders may be viewed as bothersome by some buyers, discouraging them from future purchases and thereby suppressing demand. Thus, although reminders reduce the likelihood of payment defaults, they may simultaneously have an adverse impact on customer demand. Considering all these dynamics, we proceed with the following model.

The stock level of EOQ model at time t is $\frac{dI(t)}{dt} = -D(r_c, N) - \lambda I(t)$, subject to $I(T) = 0$ and $I(0) = Q$. On solving this, we have $I(t) = \frac{D(r_c,N)}{\lambda}(e^{\lambda(T-t)} - 1)$ and total order quantity $Q = \frac{D(r_c,N)}{\lambda}(e^{\lambda T} - 1)$. Therefore, total purchasing price $PC = CpQ$ and total holding cost $HC = \frac{hD(r_c,N)}{\lambda^2}(e^{\lambda T} - \lambda T - 1)$.

Based on numerical value of N, M *and* T. There are two potential cases arises $T + N \geq M$ *and* $T + N \leq M$ for retailer's profit under new approach. Here, we shall formulate the mathematical model under the case of $T + N \geq M$ and for the other case, formulation can be obtained similarly

In this setting, the retailer uses the sales proceeds and the interest earned during the period $[0, M - N]$ to pay the wholesaler at time M. To settle the remaining dues—such as purchasing and additional operational expenses—at time M, the retailer opts for a bank loan since the bank offers a lower interest rate than the wholesaler. The borrowed sum, along with accrued compound interest, is repaid in equal installments, ideally by a chosen time t' where $M \leq t' \leq N + T$, to optimize profit. The total earnings from product sales in $[0, M - N]$ and the interest accrued on that revenue from N *to* M is given as

$$SR_1 = C_s D(1 - d)(M - N)\left[1 + I_e \frac{M - N}{2}\right]$$

After paying SR_1 at M, remaining dues is $DA_1\ (say) = PC + OC - SR_1$. To cover this amount, the retailer secures a bank loan at a compound interest rate I_b and repays it over the period up to time t' (to be determined), through n equal payments. Each of these payments, referred to as the Equal Payment Installment (EPI), (cf. http://www.paisabazaar.com/emi-calculator) is given by

$$EPI = DA_1 \frac{I'_b(I'_b + 1)^n}{(I'_b + 1)^n - 1}, \qquad where\ \ I'_b = \frac{I'_b(t' - M)}{n}$$

Sum of revenue from $M - N$ *to* $t' - N$ earned interest on it

$$SR_2 = C_s D(1 - d)(t' - M)\left[1 + I_e \frac{t' - M}{2n}\right]$$

Now t' satisfied the following equation

$$n \times EPI = SR_2$$

Sum of the sales revenue from $t' - N$ to T and earned interest on it from t' to $N + T$

$$SR_3 = C_s D(1-d)(N+T-t')\left[1 + I_e \frac{N+T-t'}{2n}\right]$$

Hence total profit amount

$$TPN = SR_4 - HC - RC$$

V. NUMERICAL EXPERIMENT

Let's consider an inventory model with following parameter's: $a_1 = 3800, a_2 = 2, C_p = 1, C_s = 1.5, C_o = 245, I_e = 8\%, I_b = 13\%, I_c = 17\%, \beta = 0.3, M = 0.25, \alpha = 0.0144, \alpha_1 = 0.03, h = 0.2, \lambda = 10\% \; and \; \rho = 185.$

Optimizing TPN using LINGO 21.0 software, we obtain the optimal solution is $T^* = 0.507, t' = 0.509. r_c{}^* = 0.260, N^* = 0.118 \; and \; TPN^* = \$\,1217.32$. The optimal profit without reminder level i.e., $r_c = 0$ is $TPN^* = \$\,1193.13$, mins profit is decreasing.

Now, the optimal profit with and without reminder level with different number of installments

Number of installments	**2**	**3**	**4**	**5**	**6**	**7**
Profit with reminder level	1228.076	1231.801	1233.691	1234.834	1235.600	1236.149
Profit without reminder level	1208.854	1212.494	1214.340	1215.457	1216.205	1216.741

It is clear from the table; profit is more with reminder level and also increasing with installments increases within time.

VI. NATURE OF THE PROFIT FUNCTION

Here the Hessian matrix of the profit function in wrt all decision variable is

$$\begin{bmatrix} \frac{\partial^2 TPN}{\partial N^2} & \frac{\partial^2 TPN}{\partial N \partial T} & \frac{\partial^2 TPN}{\partial N \partial r_c} \\ \frac{\partial^2 TPN}{\partial N \partial T} & \frac{\partial^2 TPN}{\partial N^2} & \frac{\partial^2 TPN}{\partial T \partial r_c} \\ \frac{\partial^2 TPN}{\partial N \partial r_c} & \frac{\partial^2 TPN}{\partial T \partial r_c} & \frac{\partial^2 TPN}{\partial r_c^2} \end{bmatrix} = \begin{bmatrix} -8723.18 & 3452.43 & 299.60 \\ 3452.43 & -4500.07 & 407.36 \\ 299.60 & 407.36 & -763.13 \end{bmatrix}$$

It is clear from the Hessian matrix, $\frac{\partial^2 TPN}{\partial N^2}, \frac{\partial^2 TPN}{\partial N^2} \; and \; \frac{\partial^2 TPN}{\partial r_c^2}$ all are negative. Hence the profit function is concave wrt single decision variable $N, T \; and \; r_c$.

Again $\begin{vmatrix} -8723.18 & 3452.43 \\ 3452.43 & 3452.43 \end{vmatrix} = 2.73 \times 10^7 > 0$ and $\begin{vmatrix} -8723.18 & 3452.43 & 299.60 \\ 3452.43 & -4500.07 & 407.36 \\ 299.60 & 407.36 & -763.13 \end{vmatrix} = -3.63 \times 10^{10} < 0$,.

Hence the profit function is concave in all decisions variable simultaneously.

VII. CONCLUSION

Here the mathematical formulation under a two-level trade credit EOQ (Economic Order Quantity) model. One of the key pitfalls of trade credit is the risk of customer default, where some customers fail to pay after the credit period ends. To mitigate this risk, retailers adopt reminder strategies to prompt customers to pay the outstanding dues. Although sending reminders incurs additional costs, it has a positive impact on overall profit. For the first time, the concept of a *reminder level* is introduced in an inventory model, aiming to reduce default risk and enhance profitability. Likewise, a novel solution approach is proposed for the first time. Numerical experiments clearly show that increasing the number of installments within a fixed time period leads to higher profits—an interesting and valuable insight for inventory models under trade credit scenarios.

VIII. FUTURE EXTENSION

The inclusion of reminder cost represents a significant advancement that can be seamlessly integrated into all types of trade-credit-based inventory models, irrespective of their underlying frameworks or assumptions. While trade-credit strategies are commonly employed to boost customer demand and offer financial leeway, the persistent issue of payment default poses a major challenge for both suppliers and retailers. Introducing reminder costs acts as a preventive mechanism, encouraging timely repayments and thereby minimizing the risk of financial loss. Although this strategy may slightly dampen demand, its overall effectiveness in curbing defaults makes it a beneficial component. Furthermore, this innovative approach can be applied across various model types—such as EOQ, EPQ, and two-warehouse systems—whether they operate under full or partial credit terms.

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