

A Novel Approach Solving Sequence Problem

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ABSTRACT -- The present work deals with sequencing problems for 'n' jobs on two machines and 'n' jobs on 'm' machines. Minmin Maximax Method are provided for obtaining an optimal sequence, for determining the minimum duration taken to complete all the jobs and also for finding the minimum idle time of the given machines. The steps used for obtaining the job sequence by the Minmin Maximax Method in comparison to the Johnson's method are very simple and based on arithmetic reasoning. Also, the procedure adopted for solving the sequencing problems is easiest and involves the minimum numbers of iterations to obtain the sequence of jobs. Also, we have solved several numerical examples to show that the solutions obtained by the Minmin Maximax Method are consistent and efficient.

KEYWORDS: Sequencing Problems, Optimal Sequence, Idle Time, Total Elapsed Time.

1 INTRODUCTION

In many Industrial and manufacturing situations, there may arise conditions in which the available jobs to be assigned in a set of given machines. Sequencing is nothing but the appropriate order of a finite number of machines which has to perform a finite number of different jobs. In a sequencing problem, our aim is to find the optimal order or the sequence in which the jobs have to be performed in such a way that the total time or cost is minimized. The selection of an appropriate order for finite number of different jobs to be done on a finite number of machines is called sequencing problem. By sequencing we assign a particular time for completing a particular job. The main objective of sequencing is to arrive at a position where we will get minimum processing time. There are different types of sequencing which are followed in industries such as first in first out basis, priority basis, job size basis and processing time basis.

Suppose n jobs are to be processed on m machines for successful completion of a project.

2 REVIEW OF LITERATURE

A review of earlier works is very significant and it helps to find and analyze Minmin Maximax Method. In the present chapter an attempt has been made to review the past literature pertaining to establish a Minmin Maximax Method to find optimal assignment of the jobs with minimum elapsed time and idle time of the machines Operations Research is relatively a new discipline that has its origin during World War II, and later became very popular throughout the world. It is widely used in almost all the fields and has become successful in all the fields. Operations Research helps us to make better decisions in complex scenarios. It also includes the application of scientific tools for finding the optimum solution to a problem involving the operations of a system. Sequencing problem is considered to be one of the most important applications of Operations research. A series, in which a few jobs or tasks are to be performed following an

order, is called sequencing. The aim of sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the total amount of time required to complete the full process of all the jobs. The optimality of the sequencing problem can be known by the minimized costs, maximized profits, minimized elapsed time and meeting the due dates etc. Many researchers in the past have shown interest in sequencing problems because of its significance in the production field.

In 1954, an algorithm was proposed by Johnson for scheduling jobs in two machines. Its primary objective is to find an optimal sequence of jobs and to reduce the total amount of time it takes to complete all the jobs. It also reduces the amount of idle time between the two machines. Furthermore, Johnson's method has been extended to 'm' machines problem with an objective to complete all the jobs in a minimum duration. Generally, in sequencing problems, the processing times are valued precisely. But in reality, it is perceived that the processing times during the performance of the job are imprecise and uncertain. In order to handle these uncertainties, we use fuzzy interval and fuzzy numbers. Here, we consider intervals. Interval computation was first suggested by Dwyer [1951]. The concept of fuzzy sets was proposed by Zadeh [1965]. Following Johnson, Smith and Dudek [1967] developed a general algorithm for solving n- jobs on m- machines sequencing problems of the flow shop whenever no passing is allowed.

3 METHODOLOGY

Table 3.1: Processing of n jobs on 2 machines

Step 1: List the jobs along with their processing times in a table as shown below:

Machines	Jobs						
	x_1	x_2	x_3	x_k	x_n
M_1	a_{11}	a_{12}	a_{13}	a_{1k}	a_{1n}
M_2	a_{21}	a_{22}	a_{23}	a_{2k}	a_{2n}

Step 2:

Examine the rows for processing times on machines M_1 and M_2 and find the minimum processing time in each row. Similarly, select the maximum processing time for each column.

Go to step 3.

Step 3:

Select the maximum of all the maximum processing time of n jobs. Let the maximum of maximum processing time be occurred at k th job on n th machine. Mathematically, we can say,

$$\text{Max}\{\text{Max}\{(a_{11}, a_{12}, \dots, a_{1n}), (a_{21}, a_{22}, \dots, a_{2n})\}\} = a_{2k}. \text{Proceed to next step.}$$

Step 4:

Select the minimum of all the minimum processing time of 2 machines. Let the minimum of minimum processing time occurred at n th machine for the k th job. Mathematically, we can say

$\text{Min}\{\text{Min}\{(a_{11}, a_{21}), (a_{12}, a_{22}), \dots, (a_{1n}, a_{2n})\}\} = a_{2k}$. Move on to next step.

Step 5:

The following cases may arise:

(i) If maximin value equals the minimax value, that is

$$\text{Max}\{\text{Max}\{(a_{11}, a_{12}, \dots, a_{1n}), (a_{21}, a_{22}, \dots, a_{2n})\}\} = a_{2k} = \text{Min}\{\text{Min}\{(a_{11}, a_{21}), (a_{12}, a_{22}), \dots, (a_{1n}, a_{2n})\}\},$$

Then, along that column choose the minimum value and assign the jobs as done in method

1. Proceed to next step.

(ii) If maximin value not equals the minimax value, that is

$$\text{Max}\{\text{Max}\{(a_{11}, a_{12}, \dots, a_{1n}), (a_{21}, a_{22}, \dots, a_{2n})\}\} = a_{1k} \text{ and}$$

$$\text{Min}\{\text{Min}\{(a_{11}, a_{21}), (a_{12}, a_{22}), \dots, (a_{1n}, a_{2n})\}\} = a_{2n}, \text{ then, find the Minimum of Maximin and Minimax, that is, find Mini.}\{a_{1k}, a_{2n}\}.$$

(a) If the minimum is a_{1k} , then choose the minimum from the column which corresponds to a_{1k} and assign the job as done in method 1.

(b) If not, choose the minimum along the row which corresponds to a_{2n} and assign the job as done in method 1. Proceed to next step.

Step 6:

If there occurs more than one Maximin or Minimax value, then choose the Next minimum of Maximin and Minimax. Then, proceed the steps from 4.

Step 7:

Cross off the jobs already assigned and repeat steps 2 to 4, placing the remaining jobs first or next to last, until all the jobs have been assigned.

Step 8:

Calculate idle time for machines M_1 and M_2 as follows:

Idle time for M_1 = Total elapsed time – (time when the last job in a sequence finishes on M_1)

Idle time for M_2 = Time at which first job in a sequence finishes on M_1

$$+\sum(\text{time when the } j \text{ th job in a sequence starts on } M_2)$$

$$-(\text{time when the } (j-1) \text{ th job in a sequence finishes on } M_2)$$

Step 9:

The total elapsed time to process all jobs through two machines is calculated as follows: Total

elapsed time = Time taken when the n^{th} job in a sequence is finished on machine M_2

$$= \sum_{j=1}^n t_{2j} + \sum_{j=1}^n IT_{2j}$$

Where t_{2j} is the time required for processing j^{th} job on machine M_2 and IT_{2j} is the time for which machine M_2 remains idle before starting the work on j^{th} job but after processing $(j-1)^{\text{th}}$ job.

Table 3.2: Processing of n Jobs on m Machines

The following are the steps for sequencing problems of n jobs and m machines:

Step 1: The processing time of n jobs $(1,2,\dots,n)$ on m machines $(1,2,\dots,m)$ is given table 1:

Machines	Jobs						
	x_1	x_2	x_3	x_k	x_n
M_1	a_{11}	a_{12}	a_{13}	a_{1k}	a_{1n}
M_2	a_{21}	a_{22}	a_{23}	a_{2k}	a_{2n}
M_3	a_{31}	a_{32}	a_{33}	a_{3k}	a_{3n}
.....,
.....
M_m	a_{m1}	a_{m2}	a_{m3}	a_{mk}	a_{mn}

Step 2: Find $\min a_{1j}$, $\min a_{kj}$ and maximum of each of $a_{2j}, a_{3j}, \dots, a_{k-1j}$, for all $j=1,2,\dots,n$.

Step 3: Check whether any one of the following condition is satisfied:

$\min a_{1j} \geq \max a_{ij}$, for $i = 2, 3, \dots, k-1$ (or)

$\min a_{kj} \geq \max a_{ij}$, for $i = 2, 3, \dots, k-1$

Step 4: If the inequalities of Step 3 are not satisfied, method fails. Otherwise go to next step.

Step 5: Convert the k machine problems into 2 machine problems by introducing two fictitious machine G and H , such that,

$$a_{Gj} = a_{1j} + a_{2j} + \dots + a_{k-1j},$$

$$a_{Hj} = a_{2j} + a_{3j} + \dots + a_{kj}.$$

Step 6: Repeat all the Steps from step 3 to step 8 of n jobs

on 2 machines procedure.

4 NUMERICAL EXZAMPLE

Let us consider the following problem in which 6 jobs must be performed by machines M_1 and M_2 : Processing Time is given below in the following table:

Table 4.1: Processing 6 jobs in 2 machines

Machines	Jobs					
	A	B	C	D	E	F
M_1	1	3	8	5	6	3
M_2	5	6	3	2	2	10

SOLUTION

Table 4.2:

Machines	Jobs					
	A	B	C	D	E	F
M_1	1	3	8	5	6	3
M_2	5	6	3	2	2	10

Step1: Examine the rows for processing times on machines M_1 and M_2 and find the minimum processing time in each row. Similarly, select the maximum processing time for each column.

Table 4.3:

Machines	Jobs						
	A	B	C	D	E	F	Minimum
M_1	1	3	8	5	6	3	1
M_2	5	6	3	2	2	10	2
Maximum	5	6	8	5	6	10	

Step2: Select the maximum of all the maximum processing times and similarly select the minimum of all the minimum processing time. Maximax value is 10 and Minimin value is 1. Both the values are represented in rectangles.

Table 4.4:

Machines	Jobs						
	A	B	C	D	E	F	Minimum
M_1	1	3	8	5	6	3	1
M_2	5	6	3	2	2	10	2
Maximum	5	6	8	5	6	10	

Step3: Now Select the minimum of the Minimin and Maximax values given in rectangles of the above table. Therefore, we have, $\text{Min}\{10,1\}=1$.

Here, the Minimum value is 1 which corresponds to job A, of machine M_1 . Hence, place job A on the leftmost, that is the first cell, and hence we get,

A					
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After assigning the job, cancel the assigned column job.

Table 4.5:

Machines	Jobs					
	A	B	C	D	E	F
M_1	1	3	8	5	6	3
M_2	5	6	3	2	2	10

Step4: Again, repeat step 2. Hence, repeating step 2, we get, Maximax value as 10 and Minimin value as 2.

Table 4.6:

Machines	Jobs					
	B	C	D	E	F	Minimum
M_1	3	8	5	6	3	3
M_2	6	3	2	2	10	2
Maximum	6	8	5	6	10	

Step5: Now, select the minimum of the minimin and maximax values which are given in the rectangles of the above table. Therefore, we have, $\text{Min}\{10,2\}=2$.

Now, the Minimum value is 2, which corresponds to both jobs D and E, of machine M_2 . Since, the minimum value corresponds to machine M_2 . Hence, giving preference to the rightmost job E and placing the job E on the Rightmost, that is the last cell, we get,

A					E
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After assigning the job, cancel the assigned column job.

Table 4.7:

Machines	Jobs					
	B	C	D	E	F	
M_1	3	8	5	6		3
M_2	6	3	2	2	10	

Step6: Again, select the maximum of all the maximum processing times and similarly select the minimum of all the minimum processing time. Maximax value is 10 and Minimin value is 2.

Table 4.8:

Machines	Jobs					
	B	C	D	F	Minimum	
M_1	3	8	5	3	3	
M_2	6	3	2	10		2
Maximum	6	8	5	10		

Step7:Now, select the minimum of the minimin and maximax values which are given in the rectangles of the above table. Therefore, we have, $\text{Min}\{10,2\}=2$.

Now, the Minimum value is 2 which corresponds to job D, of machine M_2 . Hence, place job D on the second Rightmost, that is the first cell, and hence we get,

A				D	E
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After assigning the job, cancel the assigned column job

Table 4.9:

Machines	Jobs			
	B	C	D	F
M_1	3	8	5	3
M_2	6	3	2	10

Step8:Again, select the maximum of all the maximum processing times and similarly select the minimum of all the minimum processing time. Maximum value is 10 and Minimum value is 3.

Table 4.10:

Machines	Jobs			
	B	C	F	Minimum
M_1	3	8	3	3
M_2	6	3	10	3
Maximum	6	8	10	

Step9:Now, select the minimum of the minimin and maximax values which are given in the rectangles of the above table. Therefore, we have, $\text{Min}\{10, 3\} = 3$.

Here, the Minimum value is 3 corresponds to job B, of machine M_1 . Hence, place job B on the second Leftmost, that is the second cell, and hence we get,

A	B			D	E
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After assigning the job, cancel the assigned column job.

Table 4.11:

Machines	Jobs		
	B	C	F
M_1	3	8	3
M_2	6	3	10

Step10:

Table 4.12:

Machines	Jobs		
	C	F	Minimum
M ₁	8	3	3
M ₂	3	10	3
Maximum	8	10	

Continue the process until all the jobs are assigned. Here, Maximax value is 10 and Minimin value is 3.

Step11: Now, select the minimum of the values given in rectangles of the above table as done in step 9. Therefore, we have, $\text{Min}\{10,3\}=3$.

Now, the Minimum value is 3 corresponds to job F, of machine M₁. Hence, place job F on the Third Leftmost, that is the third cell and hence we get,

A	B	F		D	E
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After assigning the job, cancel the assigned column job.

Table 4.13:

Machines	Jobs	
	C	F
M ₁	8	3
M ₂	3	10

A	B	C	F	D	E
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The minimum total elapsed time for the optimal sequence is calculated in the following.

Table 4.14:

Seq.of jobs	MachineM ₁				MachineM ₂			
	Time in	Processing time	Time out	Idle time	Time in	Processing time	Time out	Idle time
A	0	1	1	-	1	5	6	1
B	1	3	4	-	6	6	12	-
C	4	8	12	-	12	3	15	-
F	12	3	15	-	15	10	25	-
D	15	5	20	-	25	2	27	-
E	20	6	26	3	27	2	29	-

5 RESULT

Minmin Maximax Method is the refinement of Johnson’s method, because the processing time of all the machines for each job is considered at the same time. We consider all minimum and maximum processing time at the same time. And we take minimum values and the maximum values from them. Here, Minmin Maximax Method pessimistic and optimistic techniques are taken at the same time, which will be more

appropriate to find the optimal solution.

Table 5.1: Comparison of Total elapsed Time and Idle time for Johnson’s method and the above method.

Methods of sequencing	Optimal sequence	Idle time for Machine M ₁	Idle time for Machine M ₂	Total Elapsed Time
Johnson’s Method	$A \rightarrow F \rightarrow B \rightarrow C \rightarrow E \rightarrow D$	3timeunits	1timeunits	29timeunits
Minmin Method	$A \rightarrow B \rightarrow C \rightarrow F \rightarrow D \rightarrow E$	3timeunits	1timeunits	29timeunits

CONCLUSION

In this present work, we have proposed Minmin Maximax Method for solving job sequence of ‘n’ jobs on two machines in minimum number of iterations. The following are the advantages of the proposed Minmin Maximax Method: The computational steps followed to obtain the optimal solution are very easy. The solutions obtained by the proposed method are optimal. The final optimal solution is obtained in a short period of time. Minmin Maximax Method can be easily applied for small number of jobs as well as complex jobs of sequencing problems. It is simple to use and thus any one can adopt it easily. The numerical illustrations are more efficient to obtain an optimal sequence, completion time of jobs and total time elapsed time to process all jobs through machines. It helps to formulate uncertainty in actual environment and also serves as application for the decision makers in real life situation.

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