A Neutrosophic Set Theory Based Approach for Optimal Time Series Forecasting

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***Abstract*— This article introduces a new model to forecast the time series data set, which is exclusively based on the neutrosophic set (NS) theory. The main objective of employing the NS theory in time series data set is that it can represent the uncertainty associated with them into three different memberships, as: truth, indeterminacy and falsity. In this study, this representation of time series data set is named as a *neutrosophic time series (NTS)*. The proposed NTS approach is applied in forecasting the time series data set. The proposed approach has been verified and validated with benchmark data sets. Various comparative studies demonstrate the adequacy of the proposed model in forecasting the time series data set.**

***Index Terms*— Uncertainty, Neutrosophic set, Fuzzy set, Time series forecasting .**

I. Introduction

In 1965, Zadeh [1] made his first endeavor to represent the uncertainty, which is found at various events. This theory of uncertainty representation is called as a fuzzy set theory. It is based on the non-probabilistic representation of uncertainty inherited in the set of events, where the degree of membership of each event must lies between 0 and 1. This concept of uncertainty representation is appropriate for the events, where the information about their occurrence is completely available. Based on fuzzy set theory, Song and Chissom [2] initially introduced a model, which is referred as the "Fuzzy Time Series (FTS)". In FTS modeling approach, each time series value is represented by the fuzzified linguistic variable. Then, fuzzy logical relationships (i.e., decision rules) between the fuzzified linguistic variables are established to obtain the forecasting results [3]). Later, various time series forecasting models have been proposed based on the approach of Song and Chissom [2]. For example, researchers have illustrated the applications of FTS modelling approach in forecasting the university enrollment data set of Alabama [4-13]. In the recent year, many researchers gave their attention towards the application of FTS modelling approach in financial forecasting. This template provides authors with most of the formatting specifications needed for preparing electronic versions of their papers. Margins, column widths, line spacing, and type styles are built-in; examples of the type styles are provided throughout this document and are identified in italic type, within parentheses, following the example. PLEASE DO NOT RE-ADJUST THESE MARGINS. Some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow.

Many real time problems contain often inadequate or incomplete information. Atanassov proposed the concept of Intuitionistic fuzzy sets (IFS), as an optional way to manage uncertainty in situations, where available information isn't adequate for defining the uncertainty. In the IFS, this loss of information is represented in terms of both membership and non-membership degree functions. In view of the IFS concept, Joshi and Kumar [14] introduced a new model for time series forecasting.

Smarandache [15] introduced a new theory to represent the uncertainty based on the knowledge of neutral thought. This theory is termed as a neutrosophic set (NS), which provides the clear distinction between fuzzy set theory and Intuitionistic fuzzy sets (IFS). In the NS theory, it is considered that an event can have three degree of memberships, viz., truth-membership (T_f), indeterminacy-membership (I_f) and falsity membership (F_f). However, values of these memberships must lie between [0, 1]. In literature, several works have been reported showing the studies and applications of the NS in various domains.

The fuzzy set [1] only considers the truth-membership to represent the uncertainties inherited in time series data; while the IFS considers both truth-membership and false-membership to describe the uncertainties associated with time series data. On the other hand, the NS theory has the capability to describe any uncertainty in terms of three memberships, viz., truth-membership, indeterminacy-membership and falsity membership together [16]. In spite of having such capability, there is no any work is accounted in literature using the NS theory for time series forecasting. Therefore, in this study, a new model has been presented to improve the forecasting accuracy of time series data set by employing the NS theory.

Initially, this study provides a mathematical way to represent time series data set in terms of three properties, viz., truth-membership, indeterminacy-membership and falsity membership. This representation of time series data is termed as a *neutrosophic time series (NTS)*. Further, a formula is introduced to compute the information inherited in the NTS using the three memberships (i.e., truth-membership, indeterminacy-membership and falsity membership). During the modelling process, each time series datum in the NTS is represented by its corresponding entropy value. Then, *neutrosophic entropy relationships (NERs)* are prepared among the entropy values. From these NERs, various *neutrosophic entropy relationship groups (NERGs)* are formed, which are further used to obtain the forecasting results.

The model, which is presented in this study, is referred as a "*NTS based time series forecasting model*". To validate the proposed model, experiments are conducted on two different benchmark time series data sets, viz., university enrolments data set and TAIFEX index data set. The proposed model exhibits edge over the various existing FTS based models in terms of forecasting accuracy.

The remainder of this article is organized as follows. **Section 2** provides the basics of NS theory and its application in time series forecasting. The proposed NTS based time series forecasting model is presented in **Section 3**. Empirical analyses are presented in **Section 4**. Finally, conclusion is discussed in **Section 5**.

II. BACKGROUND FOR THE STUDY

In this section, we provide the basics of neutrosophic set (NS) theory. This section also provides the basic definitions of terminologies that are used throughout the article [17-20].

Definition 1: (Neutrosophic Set (NS) [16]. Assume that U be a universe of discourse. A NS ℕ in the U can be represented by a truth-membership function T_f, an indeterminacy-membership function I_f, and a falsity-membership function F_f, where T_f, I_f, F_f :

$$U \rightarrow [0, 1] \text{ and } \forall u \in U,\ u \equiv (T_f(u_i), I_f(u_i), F_f(u_i)) \in \mathbb{N}.$$

An instance of the NS is considered as a single valued neutrosophic set (SVNS). The NS ℕ can be represented as a SVNS on the universe of discourse $U = \{u_1, u_2, u_3, \ldots, u_n\}$, as follows:

$$\mathbb{N} = \sum_{i=1}^{n} \frac{u_i}{<T_f(u_i), I_f(u_i), F_f(u_i)>} \tag{1}$$

Example 1: Let us consider that a time series data set of daily temperature, whose universe of discourse U consists of n-different values, as: $U = \{u_1, u_2, u_3, \ldots, u_n\}$. Here, daily temperature conditions are considered as "Low", "Medium", "High", and so on. Now, these daily temperature conditions can be characterized using the three membership functions, viz., $T_f(u_i)$, $I_f(u_i)$, and $F_f(u_i)$. Now, a SVNS can be defined for the daily temperature conditions on the U, as:

$$\mathbb{N} = \frac{u_1}{<T_f(u_1), I_f(u_1), F_f(u_1)>} + \frac{u_2}{<T_f(u_2), I_f(u_2), F_f(u_2)>} + \ldots\ldots\ldots + \frac{u_n}{<T_f(u_n), I_f(u_n), F_f(u_n)>} \tag{2}$$

In the following definition, formulae are provided to determine the values of T_f, I_f and F_f for the elements of $U = \{u_1, u_2, u_3, \ldots, u_n\}$, as follows:

Definition 2: (Determination of T_f, I_f, and F_f) [16]. The values of truth-membership function T_f, an indeterminacy-membership function I_f and a falsity-membership function F_f for the elements of $U = \{u_1, u_2, u_3, \ldots, u_n\}$ can be determined using the following equations, as follows:

$$T_f(u_i) = \frac{u_i - min(U)}{max(U) - min(U)} \tag{3}$$

$$F_f(u_i) = 1 - T_f(u_i) \tag{4}$$

$$I_f(u_i) = \sqrt{T_f(u_i)^2 - F_f(u_i)^2} \tag{5}$$

In Eq. 3, *min* and *max* represent the minimum and maximum values, which return the minimum and maximum values from the U, respectively.

Definition 3: (Complement of a SVNS) [16]. The complement of a SVNS ℕ is denoted by f^c, and can be defined, as: $T_f^c(u) = F_f(u)$, $I_f^c(u) = 1 - I_f(u)$ and $F_f^c(u) = T_f(u)$, such that $\forall u \in U$.

Definition 4: (Neutrosophication) [16]. The operation of neutrosophication transforms a crisp set into a neutrosophic set. Thus, a *neutrosifier* Ñ is applied to a neutrosophic subset *i* of the universe of discourse U yields a neutrosophic subset Ñ(i : ℕ), which can be expressed, as:

$$\tilde{N}(i{:}\,\mathbb{N}) = \int_U (T_f(u_i), I_f(u_i), F_f(u_i)\mathbb{N}(u) \tag{6}$$

Here, $(T_f(u), I_f(u), F_f(u))$ ℕ (u) represents the product of a scalar $(T_f(u), I_f(u), F_f(u))$ and the NS ℕ (u), and R is the union of the family of NS $(T_f(u), I_f(u), F_f(u))$ ℕ (u), $u \in U$.

Definition 5: (Deneutrosophication) [16]. The operation of *deneutrosophication* transforms a NS into a crisp set.

A time series data set can be represented in terms the NS theory, which is referred as a *neutrosophic time series* (NTS). In the following, we provide the definition for the NTS, as:

Definition 6: (Neutrosophic Time Series (NTS)). Let *F(u)* be a subset of the universe of discourse $U = \{u_1, u_2, u_3, \ldots, u_n\}$, on which truth-membership function $T_f(u_i)$, an indeterminacy-membership function $I_f(u_i)$ and a falsity-membership function $F_f(u_i)$ are defined, and let ℕ (u) be a collection of $T_f(u_i)$, $I_f(u_i)$, and $F_f(u_i)$. Then, ℕ (u) is called a NTS on *F(u)*.

The SVNS can deal with the uncertainty associated with time series events. Entropy provides the measure of uncertainty represented by such set of events. It can be defined for the NTS value based on the following proposed formula, as follows:

***Definition 7*:** (Entropy of NTS value). The entropy of a NTS value ℕ(u) is denoted as a function $E_{\mathbb{N}}(u)$, where $E_{\mathbb{N}}(u)$: ℕ(u) → [0, 1], which can be defined, as follows:

$$E_{\mathbb{N}}(u) = 1 - \frac{1}{n}\sum_{u_i \in U} \left(T_f(u_i) + I_f(u_i) + F_f(u_i)\right) \times E_1 E_2 E_3 \tag{7}$$

Here, $E_1 = |T_f(u_i) - T_f^c(u_i)|$, $E_2 = |F_f(u_i) - F_f^c(u_i)|$, and $E_3 = |I_f(u_i) - I_f^c(u_i)|$.

***Definition 8*:** (Neutrosophic Entropy Relationship (NER)). Assume that *F(u-1)* = $E_{\mathbb{N}}(u_i)$ and F(u) = $E_{\mathbb{N}}(u_j)$. The relationship between *F(u-1)* and *F(u)* is referred as a NER, which can be represented, as:

$$E_{\mathbb{N}}(u_i) \rightarrow E_{\mathbb{N}}(u_j), \tag{8}$$

where $E_{\mathbb{N}}(u_i)$ and $E_{\mathbb{N}}(u_j)$ are referred as *previous state* and *current state* of the NER, respectively.

Definition 9: (Neutrosophic Entropy Relationship Group (NERG)). Assume the following NERs, as:

$$E_{\mathbb{N}}(u_i) \rightarrow E_{\mathbb{N}}(u_{k1}),\ E_{\mathbb{N}}(u_i) \rightarrow E_{\mathbb{N}}(u_{k2}), \ldots\ldots\ldots, E_{\mathbb{N}}(u_i) \rightarrow E_{\mathbb{N}}(u_{km}) \tag{9}$$

In this study, it is considered that the NERs having the same previous state are grouped together into a same NERG. It can be represented, as follows:

$$E_{\mathbb{N}}(u_I) \rightarrow E_{\mathbb{N}}(u_{K1}), E_{\mathbb{N}}(u_{K2}), \ldots\ldots, E_{\mathbb{N}}(u_{KM}) \tag{10}$$

III. The Proposed Time Series Forecasting Model

This section introduces each phase of the proposed model step-by-step.

Step 1: *Define the universe discourse for the historical time series data set.* Assume that E_{min} and E_{max} are the minimum and maximum values of the historical time series data set. Based on E_{min} and E_{max}, the universe of discourse U can be defined, as: U = [E_{min} - A_N, E_{max} + A_P], where A_N and A_P be the two adjustment factors.

Explanation: Assume that E_{min} and E_{max} be the minimum and maximum enrollments of the historical time series data set, as listed in Table 1. Based on E_{min} and E_{max}, the universe of discourse U can be defined, as: U = [E_{min}-A_N, E_{max}+A_P], where A_N and A_P be the two adjustment factors. From the historical time series data set, as shown in Table 1, we have E_{min} =13055 and E_{max} =19337. Hence, initially, we can assume that A_N =3055 and A_P =5663. Therefore, the universe of discourse U is obtained, as: U = [10000; 25000], where, min(U)=10000 and max(U)=25000.

Table 1: The university enrollments data set of Alabama

Year	Actual Enrollments	Year	Actual Enrollments	Year	Actual Enrollments
1971	13055	1979	16807	1987	16859
1972	13563	1980	16919	1988	18150
1973	13867	1981	16388	1989	18970
1974	14696	1982	15433	1990	19328
1975	15460	1983	15497	1991	19337
1976	15311	1984	15145	1992	18876
1977	15603	1985	15163		
1978	15861	1986	15984		

Step 2: *Apply the neutrosophication process to represent the historical time series data set into NS.* Each value of the historical time series data set is represented using the NS, as:

$$\mathbb{N}_{t_i} = \frac{t_i}{<T_f(t_i), I_f(t_i), F_f(t_i)>} \tag{11}$$

In Eq. 12, T_f, I_f and F_f represent the truth-membership, indeterminacy-membership and falsity-membership functions for the historical time series value t_i, respectively, where T_f, I_f, F_f : U → [0, 1] and ∀t_i ∈ U, $t_i \equiv t_i(T_f(t_i), I_f(t_i), F_f(t_i)) \in \mathbb{N}_{t_i}$. Here, the values of $T_f(t_i)$, $I_f(t_i)$, and $F_f(t_i)$ can be computed in terms of the universe of discourse U = [min (U); max(U)], as:

$$T_f(t_i) = \frac{t_i - \min(U)}{U_{Bi}(new) - L_{Bi}} \tag{12}$$

$$F_f(t_i) = 1 - T_f(t_i) \tag{13}$$

$$I_f(t_i) = \sqrt{T_f(t_i)^2 - F_f(t_i)^2} \tag{14}$$

Explanation: The enrollment value for the year 1971 is 13055. We can represent the enrollment value 13055 as a NS using Eq. 12, as:

$$\mathbb{N}_{13055} = \frac{13055}{< T_f(13055), I_f(13055), F_f(13055) >}$$

Here, $T_f(13055)$, $I_f(13055)$ and $F_f(13055)$ can be obtained in terms of the universe of discourse U = [10000; 25000] (where, min(U)=10000 and max(U)=25000) using Eqs. 13-14, respectively, as:

$$T_f(13055) = \frac{13055 - \min(U)}{\max(U) - \min(U)}$$

$$= \frac{13055-10000}{25000-10000} = 0.20$$

$$F_f(13055) = 1 - T_f(13055) = 1 - 0.20 = 0.80$$

$$I_f(13055) = \sqrt{T_f(13055)^2 + F_f(13055)^2}$$

$$= \sqrt{(0.20)^2 + (0.80)^2} = 0.82$$

Hence, Eq. 16 can be written to represent the enrollment value 13055 as a NS, as:

$$\mathbb{N}_{13055} = \frac{13055}{<0.20,0.80,082>} \qquad (15)$$

In this way, the historical time series data set can be represented in terms of the NS using the neutrosophication process. This representation is called as the NTS (refer to Definition 6). The NTS representation of the university enrollments data set of Alabama is shown in Table 2.

Step 3: *Compute the entropy for the neutrosophic time series (NTS) data set.* The entropy value of a particular NTS $N(t_i)$ *w.r.t.* the universe of discourse U can be obtained (refer to Eq. 7), as:

$$E_{\mathbb{N}}(t_i) = 1 - \frac{1}{n}\sum_{t_i \in U}(T_f(t_i) + I_f(t_i) + F_f(t_i)) \times E_1E_2E_3$$

Here, $E_1 = |T_f(t_i) - T_f^c(t_i)|$, $E_2 = |I_f(t_i) - I_f^c(t_i)|$, and $E_3 = |F_f(t_i) - F_f^c(t_i)|$.

Explanation: The enrollment value of year 1971 is 13055, whose NS representation is given above. Hence, by following Eq. 7, entropy for the enrollment value 13055, can be obtained, as:

$$E_{\mathbb{N}}(t_i) = 1 - \frac{1}{3}(0.20 + 0.80 + 0.82) \times 0.60 \times 0.60 \times 0.64 = 0.86$$

Here, $E_1 = |T_f(t_i) - T_f^c(t_i)| = |0.20 - 0.80| = 0.60$, $E_2 = |I_f(t_i) - I_f^c(t_i)| = |0.82 - 0.18| = 0.64$, and $E_3 = |F_f(t_i) - F_f^c(t_i)| = |0.80 - 0.20| = 0.60$. In this way, entropy values for each of the NSs are computed using Eq. 7. Entropy value corresponding to each enrollment value is shown in Table 2.

Table 2*:* The NTS representation of the university enrollments data set of Alabama.

Year	Actual Enrollments	NTS Representation	Entropy Value
1971	13055	13055/<0.20, 0.80 ,0.82>	0.86
1972	13563	13563/<0.24, 0.76, 0.80>	0.90
1973	13867	13867/<0.26, 0.74, 0.79>	0.92
1974	14696	14696/<0.31, 0.69, 0.75>	0.96
1975	15460	15460/<0.36, 0.64, 0.73>	0.98
1976	15311	15311/<0.35, 0.65, 0.74>	0.98
1977	15603	15603/<0.37, 0.63, 0.73>	0.98
1978	15861	15861/<0.39, 0.61, 0.72>	0.99
1979	16807	16807/<0.45, 0.55, 0.71>	1.00
1980	16919	16919/<0.46, 0.54, 0.71>	1.00
1981	16388	16388/<0.43, 0.57, 0.71>	0.99
1982	15433	15433/<0.36, 0.64, 0.73>	0.98
1983	15497	15497/<0.37, 0.63, 0.73>	0.98
1984	15145	15145/<0.34, 0.66, 0.74>	0.97
1985	15163	15163/<0.34, 0.66, 0.74>	0.97
1986	15984	15984/<0.40, 0.60, 0.72>	0.99
1987	16859	16859/<0.46, 0.54, 0.71>	1.00
1988	18150	18150/<0.54, 0.46, 0.71>	1.00
1989	18970	18970/<0.60, 0.40, 0.72>	0.99
1990	19328	19328/<0.62, 0.38, 0.73>	0.98
1991	19337	19337/<0.62, 0.38, 0.73>	0.98
1992	18876	18876/<0.59, 0.41, 0.72>	0.99

***Step 4:** Establish the neutrosophic entropy relationships (NERs) among the entropy values.* Based on Definition 8, NER can be established between two continuous entropy values of a historical time series data set. For example, the $E_{\mathbb{N}}(t_i)$, $E_{\mathbb{N}}(t_{i+1})$ be two consequent entropy values, then the NER can be given, as: $NER(E_{\mathbb{N}}(t_i), E_{\mathbb{N}}(t_{i+1})) = E_{\mathbb{N}}(t_i) \rightarrow E_{\mathbb{N}}(t_{i+1})$.

Explanation: In Table 2, the entropy values for Years 1973 and 1974 are 0.92 and 0.96, respectively. So, we can establish a NER between 0.92 and 0.96, as: 0.92 → 0.96. In this way, we have obtained the NERs for the enrollments data set, which are presented in Table 3.

Table 3: The NERS for the enrollments data set

NERs	NERs	NERs	NERs
0.86 → 0.90	0.98 → 0.99	0.98 → 0.97	0.99 → 0.98
0.90 → 0.92	0.99 → 1.00	0.97 → 0.97	0.98 → 0.98
0.92 → 0.96	1.00 → 1.00	0.97 → 0.99	0.98 → 0.99
0.96 →0.98	1.00 → 0.99	0.99 → 1.00	
0.98 → 0.98	0.99 → 0.98	1.00 → 1.00	
0.98 → 0.98	0.98 → 0.98	1.00 → 0.99	

***Step 5:** Create the neutrosophic entropy relationship groups (NERGs) among the neutrosophic entropy relationships (NERs).* Based on Definition 9, the NERs can be grouped into a NERG. In the set of neutrosophic entropy relationships, the relationship having same previous state can be grouped together. This particular group of NERs is called as the neutrosophic entropy relationship groups (NERGs).

Explanation: From Table 3**,** consider the seven NERs with same previous state, as: 0.98 → 0.98, 0.98 → 0.98, 0.98 → 0.99, 0.98 → 0.98, 0.98 → 0.97, 0.98 → 0.98, and 0.98 → 0.99. Therefore, these NERs can be grouped into the NERG, as: 0.98 → 0.98, 0.98, 0.99, 0.98, 0.97, 0.98, 0.99. In this way, the NERGs can be prepared from the NERs. A list of NERGs for the enrollments data set is presented in Table 4.

Table 4: The NERGS of the enrollments data set

NERGs	NERGs
0.86 → 0.90	0.97 → 0.97, 0.99
0.90 → 0.92	0.98 → 0.98, 0.98, 0.99, 0.98, 0.97, 0.98, 0.99
0.92 → 0.96	0.99 → 1.00, 0.98, 1.00, 0.98
0.96 →0.98	1.00 → 1.00, 0.99, 1.00, 0.99

***Step 6:** Obtain the forecasted values from the NTS data set.* Deneutrosophication process is applied to obtain the forecasted values from the NTS data set. In the following, a deneutrosophication process is proposed, which is explained, as:

a) For a particular day/year t_i, obtain the corresponding entropy value, as: $E_{\mathbb{N}}(t_i)$.
b) Find the NERG for the corresponding $E_{\mathbb{N}}(t_i)$, which can be represented in the following form, as:

$$E_{\mathbb{N}}(t_i) \rightarrow E_{\mathbb{N}}(t_{k1}), E_{\mathbb{N}}(t_{k2}), \ldots\ldots\ldots, E_{\mathbb{N}}(t_{km}) \quad (16)$$

Here, $E_{\mathbb{N}}(t_{k1}), E_{\mathbb{N}}(t_{k2}), \ldots, E_{\mathbb{N}}(t_{km})$ are the current state's neutrosophic time series values from day/year t_{k1}, t_{k2}, . . ., t_{km}, respectively.
c) Obtain the average entropy available in the current state of the NERG, as:

$$E_{\mathbb{N}}(\text{average}) = \frac{1}{m}\sum_{i=1}^{m} E_{\mathbb{N}}(t_{km}) \quad (17)$$

d) Apply the following deneutrosophication formula to calculate the forecasted value for the day/ year t_{i+1}, as:

$$Forecast(t_{i+1}) = \frac{t_i \times E_{\mathbb{N}}(average)}{E_{\mathbb{N}}(t_i)} \quad (18)$$

Explanation: Suppose, we want to forecast the enrollment for the year F(1978). For this year, the corresponding entropy value can be obtained from Table 2, which is 0.99. Then, obtain the NERG for the entropy value 0.99 from Table 4, which is 0.99 → 1.00, 0.98, 1.00, 0.98. In this NERG, 0.99 is the called the previous state, and 1.00, 0.98, 1.00, 0.98 are called the current state of the NERG. Now, by using Eq. 17, compute the average entropy available in current state of the NERG, as:

$$E_{\mathbb{N}}(\text{average}) = \frac{(1.00+0.98+1.00+0.98)}{4} = 0.99$$

Now, based on Eq. 18, the forecasted enrollment for the year F(1978), can be computed, as:

$$Forecast(1978) = \frac{15861 \times 0.99}{0.99} = 15861$$

Hence, the forecasted enrolment value for the year 1978 is 15861.

In this study, Average Forecasting Error Rate (AFER) is used to evaluate the performance of the proposed method. It can be defined as follows:

$$\text{AFER} = \frac{1}{n}\sum_{i=1}^{n} \frac{|actual\_value_i - forecasted\_value_i|}{actual\_value_i} \times 100\ (\%) \qquad (19)$$

In Eq. 19, each $forecasted\_value_i$ and $actual\_value_i$ represent the forecasted and actual value of year/day i, respectively; and n is the total number of years/days to be forecasted. A smaller value of AFER indicates a good forecasting accuracy.

IV. Empirical Analysis

In this section, experimental results are presented, which are carried on the university enrollments data set and TAIFEX index data set.

A. Forecasting the university enrollments data set

In this section, initially the empirical analysis is represented for the results acquired by forecasting the university enrollments data set. To evaluate the performance of the proposed model, its forecasting accuracy is compared with various existing benchmark models [2, 5-14]. These comparison results are depicted in Table 5 in terms of the AFER. From Table 5, it is obvious that the proposed model acquires a very minimum AFER as compared to the competing models.

B. Forecasting the TAIFEX index

In this case, the proposed model is applied to predict the TAIFEX index data set for the period 8/3/1998 – 9/30/1998 (mm/dd/yyyy). Forecasted results are depicted in Table 6. In this table, the proposed model is compared with previous existing models [4, 20, 21, 26] based on the AFER. It has been observed that the proposed model gives better performance than the previous existing models [4, 20, 21, 26].

Table 5: Comparison of the proposed model with various existing models (for university enrollments data set)

Year	**Actual Enrollment**	**Model [2]**	**Model [4]**	**Model [9]**	**Model [6] (MEPA)**	**Model [6] (TFA)**	**Model [11]**	**Model [7]**	**Model [13]**	**Model [5]**	**Model [10]**	**Model [8]**	**Model [26]**	**Model [14]**	**Proposed Model**
1971	13055.00	-	-	-	-	-	-	-	-	-	-	-	-	-	13510.4
1972	13563.00	14000.00	14000.00	14025.00	15430.00	14230.00	14195.00	14242.00	-	13512.00	13500.00	-	13563.00	14250.00	13867.78
1973	13867.00	14000.00	14000.00	14568.00	15430.00	14230.00	14424.00	14242.00	13500.00	13998.00	13800.00	13500.00	13867.00	14246.00	14476.53
1974	14696.00	14000.00	14000.00	14568.00	15430.00	14230.00	14593.00	14242.00	14500.00	14658.00	14700.00	14500.00	14696.00	14246.00	15005.38
1975	15460.00	15500.00	15500.00	15654.00	15430.00	15541.00	15589.00	15474.30	15500.00	15341.00	15600.00	15500.00	15425.00	15491.00	15460
1976	15311.00	16000.00	16000.00	15654.00	15430.00	15541.00	15645.00	15474.30	15466.00	15501.00	15400.00	15500.00	15420.00	15491.00	15311
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
1988	18150.00	16813.00	16833.00	17283.00	16871.00	17507.00	17090.00	16988.30	18500.00	17159.00	17100.00	18500.00	17192.00	17950.00	17966.66
1989	18970.00	19000.00	19000.00	18369.00	19333.00	18872.00	18325.00	19144.00	18534.00	18832.00	18900.00	18500.00	18923.00	18961.00	18778.38
1990	19328.00	19000.00	19000.00	19454.00	19333.00	18872.00	19000.00	19144.00	19345.00	19333.00	19200.00	19337.00	19333.00	18961.00	19328
1991	19337.00	19000.00	19000.00	19454.00	19333.00	18872.00	19000.00	19144.00	19423.00	19083.00	19050.00	19500.00	19136.00	18961.00	19337
1992	18876.00	19000.00	19333.00	18872.00	19000.00	19144.00	18752.00	19083.00	19050.00	18704.00	19136.00	18961.00	18907.00	-	18685.33
AFER (%)	-	**3.10**	**3.19**	**2.55**	**2.67**	**2.73**	**2.66**	**2.38**	**1.53**	**1.53**	**1.35**	**1.24**	**1.13**	**6.98**	**0.81**

Table 6: Comparison of the proposed model with various existing models (for TAIFEX index data set)

Date (mm/dd/yyyy)	Actual Data	Model [4]	Model [20]	Model [21]	Model [26]	Proposed Model
08/03/1998	7552	-	-	-	-	7530.21
08/04/1998	7560	7450	7450	7450	-	7536.98
08/05/1998	7487	7450	7450	7450	-	7475.37
08/06/1998	7462	7500	7450	7500	7450	7454.32
08/07/1998	7515	7500	7500	7500	7550	7498.97
08/10/1998	7365	7450	7450	7450	7350	7372.88
08/11/1998	7360	7300	7350	7300	7350	7368.69
08/12/1998	7330	7300	7300	7300	7350	7343.58
08/13/1998	7291	7300	7350	7300	7250	7310.99
08/14/1998	7320	7183.33	7100	7188.33	7350	7335.22
08/15/1998	7300	7300	7350	7300	7350	7318.51
08/17/1998	7219	7300	7300	7300	7250	7235.68
… … … … … … …						
09/29/1998	6806	6850	6750	6850	6850	6815.24
09/30/1998	6787	6850	6750	6750	6750	6753.05
AFER (%)	**-**	**1.15**	**1.03**	**0.89**	**0.43**	**0.30**

V. Conclusion

In this article, a novel method is presented to forecast the time series data set based on the NS theory. In this study, time series data set is represented in terms of the NTS. The proposed method uses the concept of entropy to compute corresponding information inherited in each of the NSs in the NTS. Based on the corresponding entropy values, NERGs are established to obtain the forecasting results. In this study, the complete process of computing the entropy value from the NS is explained along with the processes of neutrosophication and deneutrosophication. The evaluation of forecasting accuracy of the proposed model is performed by comparing forecasting results with various existing models. Various comparative analyses indicate the strong effectiveness of the proposed model. In future, the performance of the proposed model will be compared with various existing models [20-32].

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