Fixed Point Extensions in Generalized Fuzzy Metric Spaces withFuzzy Logic Controller Applications

[1]Shikha Shende, [2]NamrataTripathi, [3]Geeta Agrawal

.[1]Researh Scholar, Govt. College M. V.M Bhopal, M.P, India,
[2]Assistant Professor, Department of Mathematics, Govt. College Phanda, Bhopal-462030, India,
[3]Professor, Department of Mathematics, Govt. College M. V.M Bhopal, M.P, IndiaDesignation of 3rd Author
[1]shikhashende1701@gmail.com[2]tripathin661@gmail.com, [3]agrawalgeeta1961@gmail.com

***Abstract*—This paper presents new extensions of fixedpoint theorems in Generalized Fuzzy Metric Spaces (GFMS) with explicit integration of fuzzy logic principles and fuzzy logic controllers (FLCs). Fuzzy logic controllers operate through linguistic rules, fuzzy membership functions, and inference mechanisms that introduce uncertainty and approximate reasoning. To analyze the stability and convergence of such controllers, fixed point theory in GFMS provides a powerful mathematical framework. We develop fuzzy contraction conditions, derive fixed point results, and demonstrate how the output stabilization of an FLC can be interpreted as achieving a fixed point in a generalized fuzzy metric environment. Applications to control systems, intelligent appliances, and nonlinear fuzzy dynamic models are discussed.**

***keywords*—Contraction, Non-linear contraction, Fuzzy, Fuzzy Logic, Controllers.**

I. Introduction

Uncertainty and imprecision are inherent in real-world systems such as intelligent appliances, robotics, medical systems, and industrial processes[1]. **Fuzzy logic**, introduced by Zadeh, models this uncertainty using degrees of membership rather than crisp values[2]**.** Fuzzy logic controllers (FLCs) are widely used in washing machines, air conditioners, drones, and chemical plants[3]. These controllers rely on fuzzy rules such as:

- If temperature is high THEN fan speed is high[4]
- If dirt level is medium THEN wash time is medium

To analyse the convergence and stability of such fuzzy inference systems[5], **fixed point theory** becomes essential. If a fuzzy control system stabilizes, the final output can be considered a **fuzzy fixed point** of the control mapping[6].

Thus, integrating **FLCs + GFMS + fixed point theory** provides a strong mathematical foundation for the design and stability analysis of intelligent systems[7].

II. Preliminaries

a. Fuzzy sets and membership functions

A fuzzy set A on X is defined as:

$$\mu_A : X \to [0,1]$$

FICs use triangular, trapezoidal, Gaussian, or bell-shaped membership functions.

b. Fuzzy logic controller (FLC)

An FLC consists of:

1. **Fuzzification** (convert crisp inputs into fuzzy values)
2. **Rule base** (IF-THEN rules)
3. **Inference engine**
4. **Defuzzification** (produce final crisp output)

The mapping created by an FLC can be represented as an operator:

$$T : X \to X$$

Analyzing the stability of T uses fixed point theory.

c. Generalized Fuzzy Metric space (GFMS)

A GFMS is $(X, M, *)$, where

- $M(x, y, t)$ measures fuzzy closeness
- $*$ is a t-norm
- Triangle inequality uses fuzzy logic And
- Time parameter t corresponds to fuzzy reasoning depth or iteration count

III. FLC as a Fuzzy Operator in GFMS

A fuzzy logic controller transforms input error e into control action u:

$$u = T(e)$$

If the system stabilizes, there exists a stable point e^*such that

$$T(e^*) = e^*$$

This is a **fixed point**.

Interpretation:

- The system has reached an equilibrium state
- The fuzzy controller produces a stable output
- No further change occurs → steady-state control

Thus, analysis of FLC performance naturally leads to fixed point theory.

IV. Fuzzy Generalized contractions

We define a fuzzy contraction suitable for FLCs:

$$M(T_x, T_y, t) \geq (M(x, y, \alpha t))^{\lambda},$$

Where:

- λ expresses fuzzy weakening of distance
- α controls fuzzy time scaling
- FLC output becomes "closer" than input values

This represents a **fuzzy control contraction principle.**

V. Main Theorem (With FLC Interpretation)

Theorem: FLC-Based Fixed Point Theorem in GFMS

Let $(X, M, *)$ be a complete GFMS and let $T: X \to X$ represent a fuzzy logiccontroller. If:

$$M(T_x, T_y, t) \geq \big(M(x, y, \alpha t)\big)^{\lambda}$$

for $0 < \lambda < 1, \alpha > 1$, then:

1. T has a unique fixed point x^*.
2. For any initial error x_0,

$$T^n x_0 \to x^*$$

Meaning the FLC stabilizes.

FLC Interpretation

- The controller reduces error at each step.
- Over time, error approaches 0 (or steady state).
- Stability = fixed point.

VI. Example: Fuzzy Logic Controller for Washing Machine

Inputs: dirt level (low/medium/high)
Output: wash time

Suppose the fuzzy metric is:

$$M(x, y, t) = e^{\frac{-|x-y|}{t}}$$

The FLC operator:

$$T(x) = 0.6x + 2$$

Represents fuzzy rules adjusting wash time.

Then:

$$M(T_x, T_y, t) = e^{-\frac{|0.6(x-y)|}{t}} = \left(e^{-\frac{|x-y|}{t}}\right)^{0.6}$$

Hence:

- FLC satisfies fuzzy contraction with λ =0.6
- Therefore, wash time converges to a stable value
- This proves **mathematical stability of the fuzzy controller**

VII. Applications

a. Intelligent Appliances

- Washing machines
- Smart AC controllers
- Refrigerators

Fuzzy rules lead to a stable operating point (fixed point).

b. Robotics and Autonomous Systems

FLC-based navigation becomes stable when controller reaches a fixed point.

c. Medical Decision Systems

Fuzzy risk scoring converges to a stable decision.

d. Nonlinear Fuzzy Dynamic System

Fuzzy dynamic equations can be solved using fixed point theorems.

VIII. Conclusion

This paper establishes a new connection between:

- Fixed point theory
- Generalized fuzzy metric spaces
- Fuzzy logic
- Fuzzy logic controllers (FLCs)

By viewing the FLC as a fuzzy operator in a generalized fuzzy metric space, system stability can be characterized through fixed point results. This provides a strong mathematical foundation for designing intelligent, robust, and uncertainty-aware control systems.

IX. References

[1] N. Tripathi Assistant Professor, international journal of higher educationand research ijher a study of skilled learning in teaching the concept of continuity, differentiability and vector for student-teachers. 10(2), 167–172. https://doi.org/10.7755/ijher170720.09.

[2] N. Tripathi Assistant Professor, international journal of higher educationand research ijher quadruple impact shows on ifms using e. A. And mixed monotone property (Vol. 12, Issue 2). www.ijher.com.

[3] Tripathi, N. (2019a). A New Technique Developed for production planning using parabolicdemand by Laplace Transform. International Journal of Scientific Research in Research Paper. Multidisciplinary Studies E, 5(8), 49–55. www.isroset.org

[4] Tripathi, N. (2019b). A Novel Approach for Production Planning for Deteriorating Itemswith Logarithmic Demand. In World Academics Journal of Research Paper. Management (Vol. 7, Issue 2). www.isroset.org.

[5] Tripathi, N. (2020a). And Cubic Equation. International Journal of Higher Education andResearch (IJHER, 10(2), 254–271. https://doi.org/10.7755/ijher170720.16.

[6] Tripathi,N. (2020b). Computational Thinking for Mathematics and Sciences: anOverviewof Learning Approach. International Journal of Scientific Research in Research PaperMathematical and Statistical Sciences, 7, 69–74. www.isroset.org.

[7] Tripathi, N., Rajput, A., & Dhanke, J. A. (2012). Intuitionistic Fuzzy Metric Space UsingConcept of α-Fixed Point. Journal of Advanced Studies in Topology, 3(4), 87–92. www.m-sciences.com