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The Examining examples and mathematics

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Abstract-

The Examining examples can be help students understand definitions.While a square may be defined as a quadrilateral with four equal sides and one right angle,seeing concrete examples of squares of various sizes, as well as considering rectangular non-examples, can help children clarify the notion of square. When we teach linear algebra and introduce the concept of subspace, we often provide examples and non-examples for students. We may point out that the polynomials of degree less than or equal to two form a subspace of the space of all polynomials,whereas the polynomials of degree two do not. Is the provision of such examples always desirable? Would it perhaps be better to ask undergraduate students to provide their own examples and non-examples? Would they be able to? Given a false conjecture, would students be able to come up with counterexamples? Several studies shed light on these questions.

In the successful Math majors Generate Their Own Examples- In upper-division courses like abstract algebra and real analysis, students often encounter a host of formal definitions, many new to them. After presenting a few examples and non-examples along with a few proofs of theorems, we hope they will use these definitions to tackle problems, examine conjectures, and construct their own proofs. Is this the best way to proceed? How do such students deal with new definitions To answer this question, Randall P.Dahlberg and David L. Housman of Allegheny College conducted an in-depth study of eleven undergraduate students - ten seniors and one junior. All but one, who was in computer science, were math majors. The students had successfully completed introductory real analysis and algebra, as well as courses in linear algebra and foundations and a seminar covering set theory and the foundations of analysis. In individually conducted audio-taped interviews, the authors presented the students with a written definition of a "fine function," which they had made up to see how the students would deal with a formally defined concept. A function was called fine if it had a root (zero) at each integer. When interviewed, students were first asked to study this definition for five to ten minutes, saying or writing as much as possible of what they were thinking, after which they were asked to generate The Examining examples and mathematics examples and non-examples of "fine functions." Subsequently, they were given functions, such as and asked to determine whether these were examples and, if so, why. Next, they were asked to determine the truth of four conjectures, such as "No polynomial is a fine function." Finally they were asked about their perceptions of the interview. Four basic learning strategies were used by the students on being presented with this new definition - example generation, reformulation, decomposition and synthesis, and memorization. Examples generated included the constant zero function and a sinusoidal graph with integer x-intercepts. Reformulations included Decomposition and synthesis included underlining parts of the definition and asking about the meaning of "root." Two students simply read the definition – they could not provide examples without interviewer help and were the ones who most often misinterpreted the definition. They found the interview quite different from their usual mathematics classes, where examples and explanations were provided.Of these four strategies, example generation (together with reflection) elicited the most powerful "learning events," i.e., instances where the authors thought students made real progress in understanding the newly introduced concept. Students who initially employed example generation as their learning strategy came up with a variety of discontinuous, periodic continuous, and non-periodic continuous examples and were able to use these in their explanations. Those who employed memorization or decomposition and synthesis as their learning strategies often misinterpreted the definition, e.g., interpreting the phrase "root at each integer" to mean a fine function must vanish at each integer in its domain, but that need not include all integers. Students who employed reformulation as their learning strategy developed algorithms to decide whether functions given them were fine,but had difficulty providing counterexamples to false conjectures Finally, Dahlberg and Housman note the relative ineffectualness of their attempted interventions.One student agreed, after a question and answer period with the interviewer, that the zero function was indeed a fine function, but immediately switched herattention to other ideas, not returning until much later when, through self-discovery, she actually realized the zero function was a fine function. Dahlberg and Housman suggest it might be beneficial to introduce students to new concepts by having them generate their own examples or having them decide whether teacher-provided candidates are examples or non-examples, before providing students examples and explanations. However, some of their students were reluctant to engage in either example generation or usage a not uncommon phenomenon in such circumstances.

Examples Can Be Disconcerting Asked For

Coming up with examples requires different cognitive skills from carrying out algorithms - one needs to look at mathematical objects in terms of their properties.To be asked for an example, whether of a "fine function" or something else, can be disconcerting. Students have no prelearned algorithms to show the "correct way."This is what Orit Hazzan and Rina Zazkis, of the Technion - Israel Institute of Technology, found when they asked three groups of preservice elementary teachers to provide examples of

(1) a 6-digit number divisible first by 9, then by 17,

(2) a function whose value at $x = 3$ is -2, and

(3) a sample space and an event that has probability 2/7 in that space. In addition, they asked the students to explain how they generated their examples and to provide five additional examples. The students used a variety of approaches to generate examples, beginning with trial and error, e.g., some simply picked a number at random and checked whether it was divisible by 9. Others picked a number N, and upon dividing by 17 and getting a remainder of 2, would use N-2 for their next trial. Students often found constructing examples and making the necessary choices difficult, e.g., they inquired of the interviewers whether the elements of the sample space were to be numbers, letters, or other objects. Some students designed their own algorithms for generating functions, one focused on

$y = ax + b$, plugged in (3, -2) to get –

$2 = a*3 + b$, chose $a = 2$ and solved for $b = -8$, finally declaring her function to be $y = 2x - 8$.

Interestingly, very few students produced "trivial examples," such as 170,000 for a

6-digit number divisible by 17 or y = -2 as their function. Hazzan and Zazkis

conjecture that these examples might not be seen as prototypical - a function is expected to involve x and a 6-digit number is seen as having a wider variety of digits. There was also a strong tendency to (directly) check the correctness of examples, some students who had created a number divisible by 17 by choosing a multiplier and performing the multiplication, verified the correctness of their example by division. Quite a number of students had difficulty dealing with"degrees of freedom," . in order to find a number divisible by 9, one student who knew the sum of the digits needed to be divisible by 9, first chose 18, noted that 8 and 2 make 10, then broke 8 into the sum of 4, 3, and 1, and declared that 82431 should be divisible by 9. When asked for another strategy, she suggested something very similar making the initial sum 27, instead of 18.Constructing examples proved to be more difficult for these students than checking the divisibility of a number, calculating the value of a function, or finding the probability of an event. They were often uncertain how to proceed and were especially troubled by having to make choices in mathematics. The authors suggest that teachers at all levels assign more "give an example" problems. Further more,when students are allowed to discuss mathematical ideas and propose conjecturesin class, teachers need to be able to evaluate student-generated examples, as well as to be able to propose counterexamples for their students' consideration. Students quite often fail to see a single counterexample as disproving a conjecture. This can happen when a counterexample is perceived as "the only" one that exists, rather than being seen as generic, e.g., sometimes the square root of 2 is considered the only irrational or |x| is perceived as the only continuous, nondifferentiable function.

TheGenerating versus examples That Are allegorical-

Perhaps not surprisingly, experienced secondary mathematics teachers are better at generating explanatory counterexamples than preservice teachers. Irit Peled, University of Haifa, and Orit Zaslavsky, the Technion, asked some of each to generate at least one counterexample for each of the two following unfamiliar, false geometry statements supposedly given by a secondary student.

(1) Two rectangles, having congruent diagonals, are congruent.

(2) Two parallelograms,

having one congruent side and one congruent diagonal, are congruent. They were also asked to explain how they came up with their counterexamples. None generated more than one counterexample for each task.Two groups participated in the study -- 38 inservice teachers, most of whom had more than five years of teaching experience and a B.Sc. in mathematics and 45 third year student-teachers who had completed several advanced undergraduate mathematics courses. For the first conjecture (Task 1), 97% of the inservice teachers gave adequate counterexamples, i.e., ones that refuted the claim, but only 53% of the student-teachers did so. For the second conjecture (Task 2), 76% of the teachers and 42% of the student-teachers gave adequate counterexamples.The counterexamples were analyzed for their explanatory power as specific, semi-general, and general. A specific counterexample is one which contradicts the claim,but gives no indication as to how one might construct similar or related counterexamples. For example, for Task 1 one subject carefully drew two rectangles of different dimensions, but with congruent diagonals. A counterexample was called semi-general if it provided some idea how one might generate similar or related counterexamples, but did not tell "the whole story" or did not cover "the whole space" of counterexamples. For instance, on Task 1, one subject drew two rectangles with congruent diagonals, but the angle between the two diagonals of second rectangle was indicated as twice that of the first rectangle.(Here it should be noted that, while some conjectures might not lend themselves to the generation of numerous counterexamples, i.e., they might be correct except for a small number of special "pathological" cases, these two conjectures were chosen to be far from "almost correct.") A general counterexample provides insight as to why a conjecture is false and suggests a way to generate an entire counterexample space. In response to Task 1, one subject specified that the angle between the diagonals could be arbitrary, rather than merely double that of the first rectangle.Both teachers and student-teachers produced counterexamples of all the above types, but the former produced more semi-general and general counterexamples(92% vs. 38% on Task 1, and 61% vs. 33% on Task 2).Both of these types were labeled explanatory by the authors. The difficulty in suggesting only a specific counterexample lies in its potential for misleading students, whereas the pedagogical value of explanatory counterexamples lies in their ability to provide insight into why a conjecture fails. The authors suggest that both prospective and in-service mathematics teachers could benefit from an analysis and discussion of the pedagogical aspects of counterexamples

I Don't Know that What It Says, How Can I Find an Example -

This hypothetical quote, illustrates the chicken-and-egg quandary some students might typically face when encountering a formal definition, whether of "fine function" or quotient group. A definition asserts the existence of something having certain properties. However, the student has often never seen or considered such a thing. To give an example or non-example, he/she would need at least some understanding of the concept. But how can he/she obtain such understanding? A good, and possibly the best, way seems to be through an examination of examples. Thus, the student is faced with an epistemological dilemma: Mathematical definitions, by themselves, supply few (psychological) meanings. Meanings derive from properties. Properties, in turn, depend on definitions. [This is a paraphrase from Richard Noss' plenary address to the September 1996 Research in Collegiate Mathematics Education Conference, as reported in Focus 17(1), 1&3, February 1997.] For mathematicians, this does not seem to be a dilemma. We suspect they view definitions differently than students - this allows them to search for examples in order to gain understandings of formal definitions.Not only does such circularity play a role in students' failure to construct examples,so does their limited knowledge of concepts involved in a formal definition. When Zaslavsky and Peled asked 67 preservice and 36 inservice secondary teachers to provide examples of binary operations which were commutative and nonassociative, their subjects had great difficulty. Only 33% of the experienced teachers and 4% of the third-year undergraduate students came up with complete,correct, and well-justified examples. Just 56% of the experienced teachers and 31% of the student teachers were able to provide any kind of example (correct or incorrect). Upon investigating why this might be so, the authors found their subjects' underlying mathematical knowledge was deficient. For example, one subject defined a * b = | a + b |

and claimed this was nonassociative because

| a + b | + | c |

does not equal

| a | + | b + c |

Another proposed the operation of subtraction claiming it was commutative

because

-2 - 3 = -3 - 2,

rather than 3 - (-2). Yet another proposed the unary operation and tried to check commutativity using The authors suggest their subjects tended to conflate commutativity and associativity due to the way the "issue of order" is treated in schools. For example,when a child is asked to calculate

6 + 7 + 4,

he is usually encourage to do it more efficiently as (6 + 4) + 7 and told order

doesn't matter.Dahlberg and Housman also noted that their undergraduate subjects had trouble with the underlying concepts, e.g., function and root, making it hard to generate examples and non-examples of "fine functions." One student identified "root" with"continuity," three others initially thought the graph of the zero function was a point, and one did not believe the zero function was periodic. In addition, most students' initially thought in terms of functions which were nonconstant polynomials or continuous.

Dividing code method-

Since success in mathematics, especially at the advanced undergraduate and graduate levels appears to be associated with the ability to generate examples and counterexamples, what is the best way to develop this ability? One suggestion, given above, is to ask students at all levels to "give me an example of "Granted the inherent epistemological difficulties of finding examples for oneself, are we, in a well-intentioned attempt to help students understand newly defined concepts, ultimately hobbling them, by providing them with predigested examples of our own? Are we inadvertently denying students the opportunity to learn to generate examples for themselves? Difficulties with the strikingly simple idea of "fine function" suggest some students may be excessively dependent upon explicit instruction. Another in-between suggestion, given above, is to provide students with a list of potential examples (or counterexamples) and ask them to decide whether they are indeed examples (or counterexamples) and why. Are there other ways we might help students become example generators? Finally, a tendency to generate examples is not the same as an ability to do so it would be interesting to know how each of these relates to understanding and doing mathematics.

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