Vertex Antimagic Edge Slither Labeling Of Protracted Leaf Caterpillar Graphs

R. SREENIVASAN

ASSISTANT PROFESSOR, PG AND RESEARCH DEPARTMENT OF MATHEMATICS,

AGURCHAND MANMULL JAIN COLLEGE, MEENAMBAKKAM, CHENNAI, INDIA

sreenivasan.raghavan@gmail.com

ABSTRACT: Graphs in the modern scientific world is a boon as different complicated scenarios can be best explained using dots and lines. The applications of graph theory extend far and wide that any complex real time situation can be easily elucidated using graphs. In this paper, an attempt has been made to prove that the protracted leaf caterpillar graph admits vertex antimagic edge slither labeling.

KEYWORDS: Vertex Antimagic Edge Slither Labeling, Caterpillar graph, protracted leaf caterpillar graph

INTRODUCTION: In today's modern world the advancement of science and technology has a telling effect on lives of everyone. Right from linking of places to human DNA structure, every complex structure can be best described using graphs. The concept of graphs, hence therefore has paramount importance in representing structures into pictorial representations. With much more in its repertoire, the concept of graphs has stamped its importance remote and widespread.

VERTEX ANTIMAGIC EDGE SLITHER LABELING: The acuity of labeling in graph theory parlance has been far and wide, measured as the most fluent form of demonstrating different real life situations. The concept of labeling of graphs first initiated from the reflexion of magic labeling given by the graph theoretic scholar J. Sedlacek [8] who first proposed this concept. His work formed a benchmark for the origin of a variety of types of labeling in graphs. J.A. Macdougall, M.Miller, Slamin and W.D.Wallis [5] were the first to define the concept of vertex antimagic total labeling. The notion of antimagic labeling was first given by N. Hartsfield and J. Ringel [3]. The idea of vertex antimagic edge labeling was first introduced by R. Bodendiek and G. Walther [1]. It was Joseph A. Gallian [2] who first compiled all these in the book titled "A dynamic survey of graph labeling". It was Martin Baca and Mirka Miller [4], who were the first scholars to propose the definition for vertex antimagic edge labeling in graphs. The definition as given by them goes as follows; the graph G to an (a, d) antimagic edge labelled graph if it is connected and for a positive integer a and a non-negative integer d, there exists a bijective mapping given by h: $E \rightarrow \{1,2,\ldots,|E(G)|\}$ such that the induced mapping given by f: $V(G) \rightarrow W$, where $W = \{a, a+d, a+2d, \ldots, a+|V-1|d\}$ is also a bijection. The concept of slither labeling is a manipulation of the vertex antimagic edge labeling such that the labeling of the edges of the graphs follow a slithering pattern.

CATERPILLAR GRAPH: The concept of caterpillar graphs being taken in this paper was first defined by P. Packiavathi, S. Balamurugan, nd R. B. Gnanajothi[7], is a graph that is constructed from a path on k vertices by adding x_i pendant vertices to the i^{th} vertex of the path P_k. A caterpillar graph is a manipulation tree having the property that the removal of the leaves and incident edges renders a path P_k that is defined as the spine of the caterpillar. A caterpillar graph is said to be a complete graph if every vertex on the spine is adjacent to at least one leaf of the caterpillar. R. Sreenivasan [9] has already proved the admittance of vertex antimagic edge slither labeling to P_n^m, m = 2,3. The protracted caterpillar graph is a graph having the same number of leaves emanating from a vertex on either side of the path and is denoted by $P_r^{m,n}$. A typical protracted leaf caterpillar graph is given as follows;

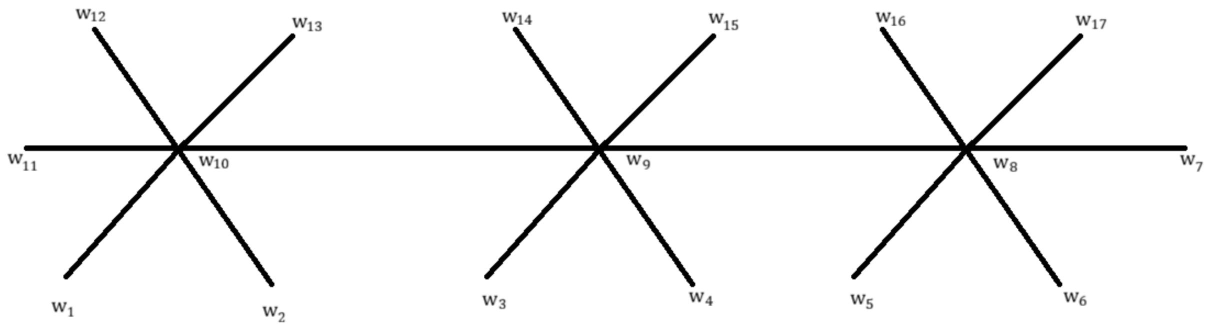

Fig 1. Protracted Caterpillar Graph $P_5^{2,2}$

MAIN RESULT:

Theorem 1: The protracted caterpillar graph $P_r^{m,n}$ defined on a path on n vertices admits vertex antimagic edge slither labeling.

PROOF:

CASE – 1: Consider the caterpillar graph $P_r^{m,n}$ formed from a path of n vertices. There are m,n leaves that originate from each side of the vertices of the path. Label the edges of the graph by slither pattern. Let the number of leaves in the graph be m,n = 3,3. Consider the graph $P_5^{3,3}$**.** The slither labeling in $P_5^{3,3}$ is given as follows;

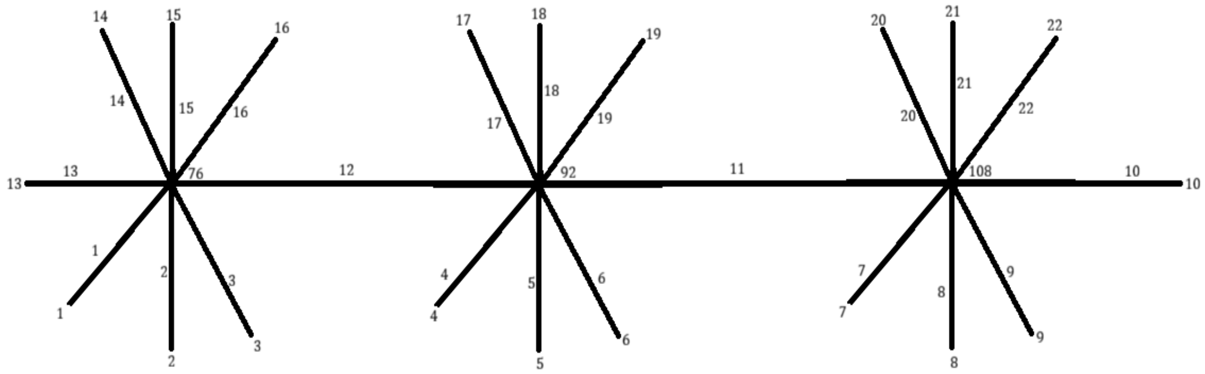

Fig 2. Protracted Caterpillar Graph $P_5^{3,3}$

The edges of the graph are labelled with positive integers in a slither pattern. The label of a vertex is the sum of labels of the edges that are incident with the vertex. In the graph $P_5^{3,3}$, the vertex labels are all distinct and hence the graph admits vertex antimagic edge slither labeling.

CASE – 2: Consider the caterpillar graph $P_6^{4,4}$, the path on 6 vertices and four leaves originate from each interior vertex of the path. The slither labeling is given as follows;

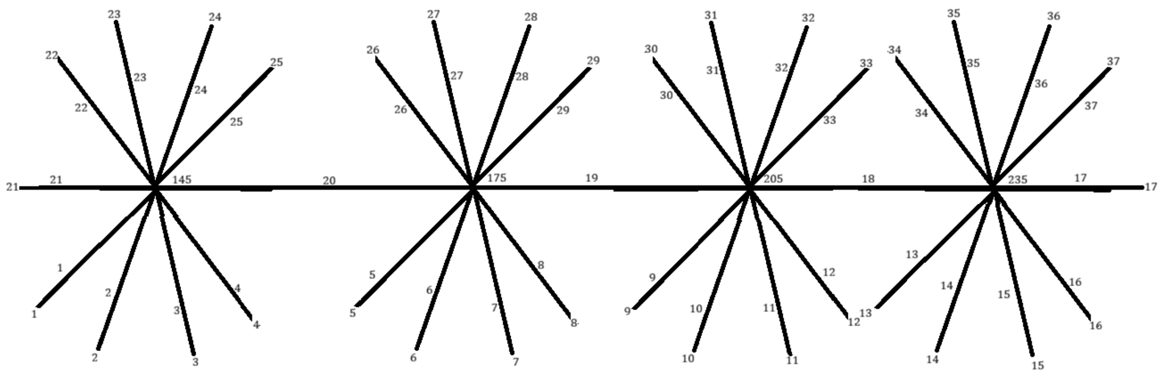

Fig 3. Protracted Caterpillar Graph $P_6^{4,4}$

In the given graph $P_6^{4,4}$, the labels of the vertices are all distinct and hence the graph admits vertex antimagic edge slither labeling.

CASE – 3: Consider the caterpillar graph $P_5^{3,2}$, the path on five vertices and two leaves originate from each interior vertex of the path. The slither labeling is given as follows;

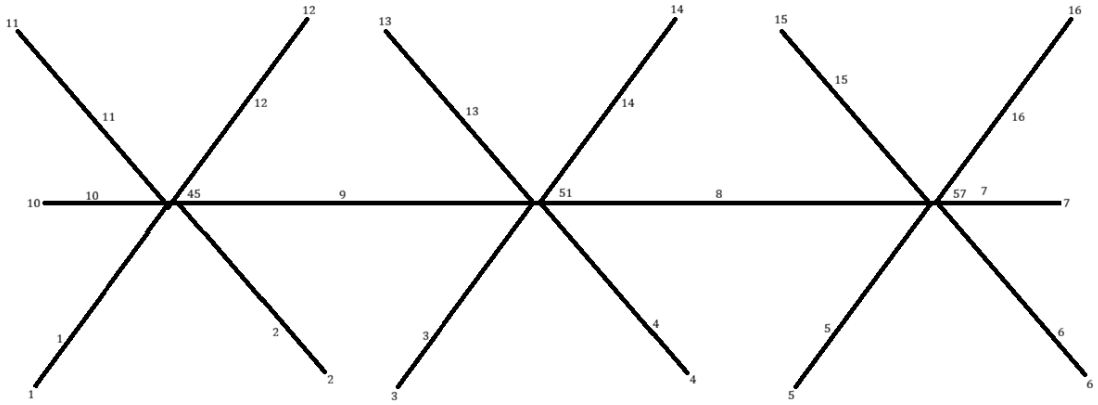

Fig 4. Protracted Caterpillar Graph $P_5^{3,2}$

In the given graph $P_5^{3,2}$, the labels of the vertices are all distinct and hence the graph admits vertex antimagic edge slither labeling.

Thus it follows that the Protracted Caterpillar graph $P_r^{m,n}$ accepts vertex antimagic edge slither labeling for all values of r > 1and m, n $\geq$ 2.

CONCLUSION: In this paper, the protracted caterpillar graph $P_r^{m,n}$ with m, n leaves are taken and the applicability of vertex antimagic edge slither labeling has been proved. The same concept of slither labeling pattern can be tried for different caterpillar graphs.

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