

Electromagnetic Force Prediction and NVH Analysis in a Permanent Magnet Synchronous Motor

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Abstract—This paper outlines a method for forecasting noise, vibration, and harshness in permanent magnet synchronous motors. It does this by carefully connecting those harmonic-rich excitation currents to the stator vibrations and the acoustic noise that follows. At the heart of the approach, there are two main computational stages. First, you create a harmonic-rich flux density pattern with a basic air-gap magnetomotive force model. Then, you apply the Maxwell stress tensor method to figure out the instant pulsating radial electromagnetic forces. In the second stage, that force serves as the input for a simple single degree of freedom model of the stator. From there, you solve the equation of motion through state-space simulation to get the mechanical response. The outcomes really back up this predictive setup. They show a strong link between the input current harmonics and the frequency peaks in the simulated force and acceleration spectra. All of this proves how useful the method is for handling key NVH issues right in the design stage of high-performance electric drives.

Index Terms—Permanent Magnet Synchronous Motor (PMSM), Stator Vibration, Fast Fourier Transform (FFT), State-Space Simulation, Acoustic Noise Spectrum.

I. INTRODUCTION

The Permanent Magnet Synchronous Motor (PMSM) is the cornerstone of modern high-efficiency electric powertrains, offering superior power density and torque characteristics. While these motors are highly efficient, their operational performance is often limited by structural vibrations and radiated acoustic noise—collectively known as noise, vibration, and harshness (NVH). Unlike internal combustion engines (ICEs), where noise is dominated by combustion events, the NVH signature of electric motors is fundamentally electromagnetic in origin.

The core phenomenon involves the interaction between the stator current (Armature Magnetomotive Force, or MMF) and the rotor's permanent magnet flux. Imperfections in the motor's operation, such as non-sinusoidal supply from Pulse Width Modulation (PWM) inverters and inherent machine design features like stator slotting, introduce high-frequency harmonics into the air-gap flux density. The instantaneous radial force acting on the stator and rotor is proportional to the square of the air-gap flux density. Consequently, any harmonic in the flux density results in corresponding force and pressure harmonics, which mechanically excite the motor structure, leading to audible noise and structural fatigue.

This project focuses on establishing a predictive link between the electrical source of excitation (harmonic current profile) and the resulting mechanical response (vibration and noise). By modelling the system using fundamental electromagnetic and kinematic principles, the analysis aims to isolate the key frequency components responsible for noise generation, enabling engineers to apply targeted mitigation strategies during the design phase.

The primary objectives of this project are:

- Accurately calculate the instantaneous radial electromagnetic force using Maxwell Stress Tensor (MST) approximation based on harmonic-rich stator current input.
- Perform a Fast Fourier Transform (FFT) analysis to identify the dominant force harmonic frequencies and their amplitudes.
- Model the stator as a simplified Single Degree of Freedom (SDOF) system and solve the equation of motion to find the vibration response.
- Analyse the frequency spectrum of the stator acceleration, which acts as a crucial proxy for radiated acoustic noise.

The predictive NVH analysis is crucial for motor design, enabling design optimization by identifying noise-causing current harmonics to guide PWM control strategies and filter circuit development. It safeguards structural integrity by ensuring that dominant electromagnetic force frequencies do not align with the stator's natural frequencies, thus preventing damaging resonance. Ultimately, this improves customer experience by minimizing acoustic noise, which is paramount in applications like Electric Vehicles (EVs).

II. THEORY

A. Maxwell Stress Tensor (MST)

The Maxwell Stress Tensor is a fundamental 3×3 tensor that defines the force per unit area (stress) due to electromagnetic fields (E and B). It is the most rigorous way to calculate mechanical force in an electromagnetic field.

The radial force component (the primary cause of stator vibration) is calculated by integrating the normal component of the stress over the air-gap surface. The simplified radial electromagnetic stress (σ_{em}), or magnetic pressure, is overwhelmingly dominant and is calculated from the square of the air-gap flux density (B):

$$\sigma_{em} = \frac{B^2}{2\mu_0} \quad [\text{N/m}^2]$$

The total instantaneous force (F_{em}) is then found by multiplying the stress by the surface area ($A_{surface}$).

B. Stator Current and Flux Harmonics

Stator current is non-ideal due to PWM inverters and stator slotting, which introduce high-frequency harmonics into the air-gap flux density, $B(t)$.

Since the force is proportional, the squaring operation results in force harmonics whose frequencies (x) are combinations (sums and differences) of the flux harmonic frequencies. In the simplified model, the primary force harmonics occur at twice the input current's harmonic frequencies (e.g., $2f_k$).

C. Vibration Kinematics (SDOF System)

The complex stator structure is simplified as a Single Degree of Freedom (SDOF) mass-spring-damper system to analyze its dynamic response.

The stator's motion, $x(t)$, is governed by the forced harmonic oscillator equation:

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F_{em}(t)$$

where m is mass, c is damping, k is stiffness, and $F_{em}(t)$ is the calculated excitation force.

The equation is solved numerically using a State-Space solution (specifically the MATLAB *lsim* function), which is efficient for simulating the transient and steady-state dynamic response.

III. METHODOLOGY

The methodology for this predictive NVH analysis is structured into two sequential computational stages, implemented using MATLAB. The first stage focuses on the electromagnetic domain, calculating the instantaneous radial excitation force (F_{em}), while the second stage analyzes the mechanical domain, simulating the stator's structural response.

A. Stage 1: Electromagnetic Force Calculation (Force Script)

This stage involves modelling the harmonic-rich flux density in the air-gap and applying the MST approximation to quantify the instantaneous electromagnetic force.

1) Parameter Definition and Harmonic Input:

Core machine parameters for the PMSM were defined, including rated power ($P_{rated} = 75$ kW), pole pairs ($p = 4$), air-gap radius ($r_{airgap} = 0.06$ m), and fundamental current amplitude ($I_1 = 150$ A). The fundamental electrical frequency (f_1) was calculated based on the rated speed (n_{rated}) and pole pairs.

A crucial step was the definition of time-varying harmonic content, representing the non-ideal stator current introduced by the PWM inverter and slot effects. A set of dominant harmonic orders (e.g., $h = [1, 5, 7, 11, 13]$) and their corresponding relative magnitudes (*harm_mag*) were specified as inputs to the model.

2) Air-Gap Flux Density Modelling:

The instantaneous air-gap flux density, $B(t)$, was modelled as a summation of sine waves corresponding to the selected harmonics, with the peak flux density scaled by the harmonic magnitudes:

$$B(t) = \sum_k B_{peak} \cdot \text{harm_mag}_k \sin(2\pi h_k f_1 t)$$

3) Instantaneous Force Derivation:

The normal electromagnetic stress (σ_{em}), or magnetic pressure, was calculated across the air-gap using the simplified MST formulation, which is proportional to the square of the instantaneous air-gap flux density:

$$\sigma_{em}(t) = \frac{B(t)^2}{2\mu_0}$$

This instantaneous stress was then multiplied by the effective stator area ($A_{surface}$) to yield the total instantaneous radial electromagnetic excitation force, $F_{em}(t)$.

4) Frequency Domain Analysis:

A Fast Fourier Transform (FFT) was performed on the instantaneous force signal, $F_{em}(t)$, to transition the analysis from the time domain to the frequency domain. This identified the dominant force harmonic frequencies:

$$f_{force} = |h_k \pm h_j| \cdot f_1$$

and their corresponding amplitudes, providing a clear map of the mechanical excitation spectrum.

B. Stage 2: Structural and Acoustic Response Simulation (NVH Script)

The second stage utilizes the calculated time-domain force $F_{em}(t)$ as the input to a simplified structural model to determine the vibration and acoustic response.

1) Structural Simplification (SDOF Model):

The complex elastic structure of the PMSM stator was simplified into an SDOF mass-spring-damper system. The following mechanical parameters were defined for the equivalent system:

- Equivalent Mass (m)
- Equivalent Damping Coefficient (c)
- Equivalent Stiffness (k)

2) Dynamic Response Solution:

The governing equation of motion for the forced, damped SDOF system was defined:

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F_{em}(t)$$

This second-order linear differential equation was converted into a first-order State-Space representation for numerical solution in MATLAB. The `lsim` function was then used to solve the system for the entire duration, yielding the time-domain structural response: displacement (x), velocity (\dot{x}), and acceleration (\ddot{x}).

3) Acoustic Proxy and NVH Analysis:

The resulting acceleration (\ddot{x}) signal is directly proportional to the radiated acoustic noise (Sound Pressure Level, or SPL). A final FFT was performed on the acceleration signal. The resulting frequency spectrum of acceleration served as a crucial proxy for the motor's acoustic signature, providing the frequency and relative amplitude of the predicted acoustic noise components for subsequent NVH mitigation design choices.

IV. RESULTS

A. Case 1 : For 4-seater Vehicle

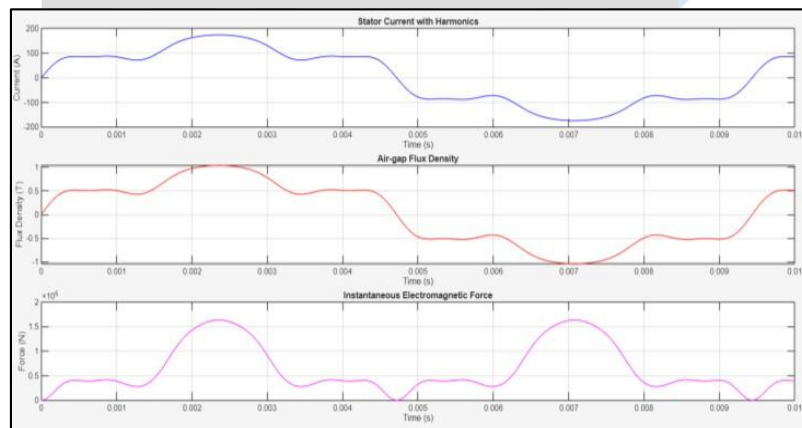


Figure 1 Stator Current with Harmonics, Air-gap Flux Density and Instantaneous Electromagnetic Force plots for 4-seater vehicles

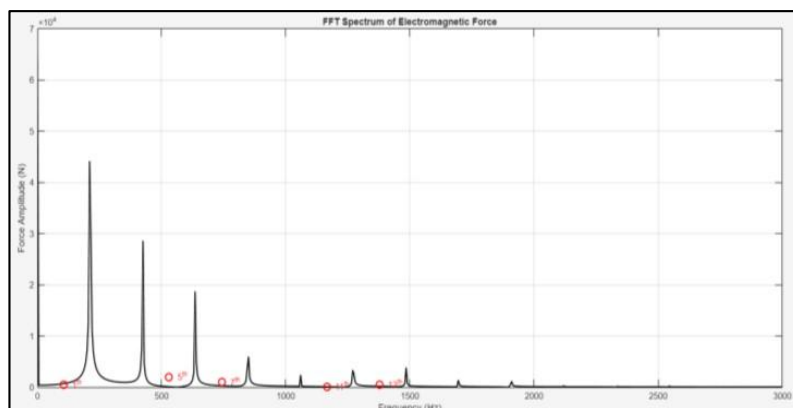


Fig 2. FFT Spectrum of Electromagnetic Force Plotting for 4-seater Vehicles

The FFT Spectrum of Electromagnetic Force identifies the characteristic frequencies and magnitudes of the mechanical excitation. The primary force component, located at approximately 240 Hz (twice the fundamental electrical frequency), exhibits the highest amplitude of about 4.5×10^4 N. Significant high-order harmonics, such as the 3rd and 5th, are also observed at 720 Hz and 1200 Hz, respectively, which are critical for predicting resonant conditions and subsequent vibro-acoustic behaviour.

Fig. 3. Vibration Displacement of Stator, Vibration Velocity and Vibration Acceleration plots for 4-seater vehicles

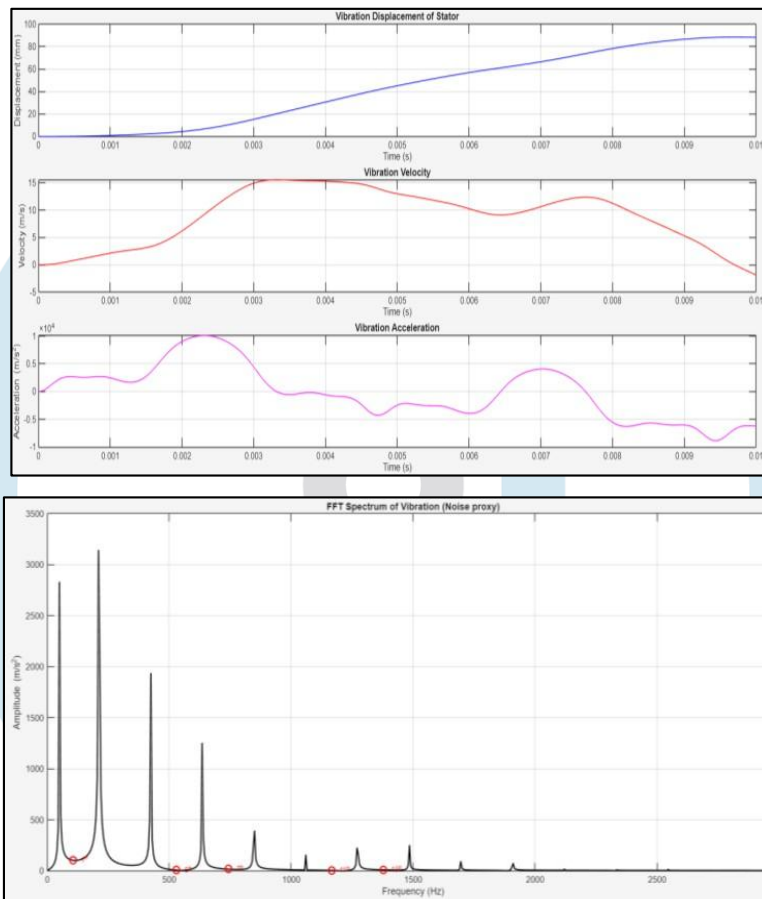


Fig. 4. FFT Spectrum of Vibration – Noise Proxy plotting for 4-seater vehicles

The analysis demonstrates the complete vibro-acoustic chain, where non-sinusoidal currents generate dominant electromagnetic force harmonics at 200 Hz and its multiples. The resulting vibration spectrum confirms that the machine structure exhibits a high-amplitude response, with the largest acceleration peak at 200 Hz (≈ 3100 m/s²), suggesting a strong structural resonance. This amplified vibration directly radiates the dominant acoustic noise, which peaks at 200 Hz with a normalized sound pressure level of approximately 40 dB.

B. Case 2 : For E-Bikes

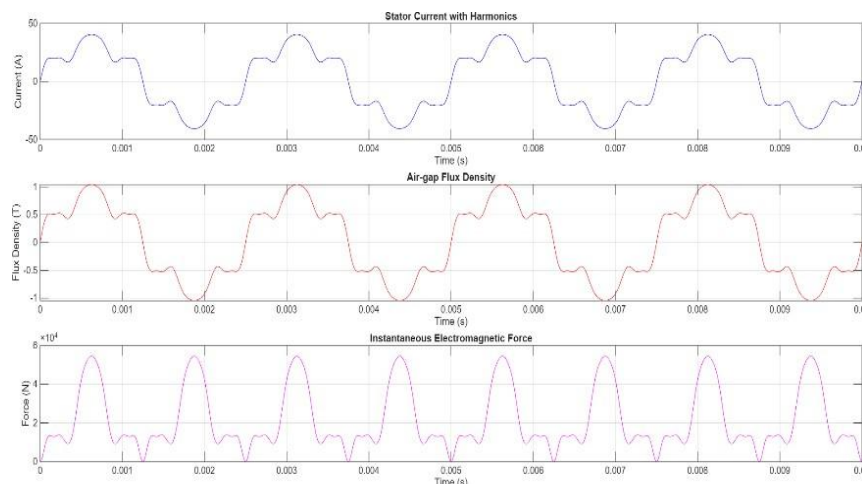


Fig. 5. Stator Current with Harmonics, Air-gap Flux Density and Instantaneous Electromagnetic Force plots for E-bike

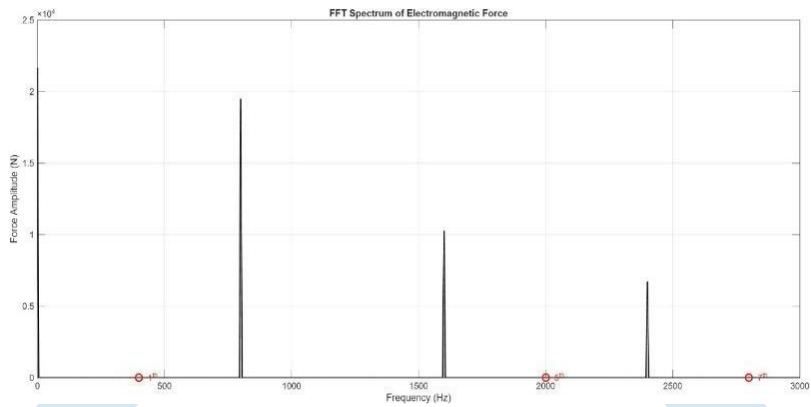


Fig. 6. FFT Spectrum of Electromagnetic Force Plotting for scooters.

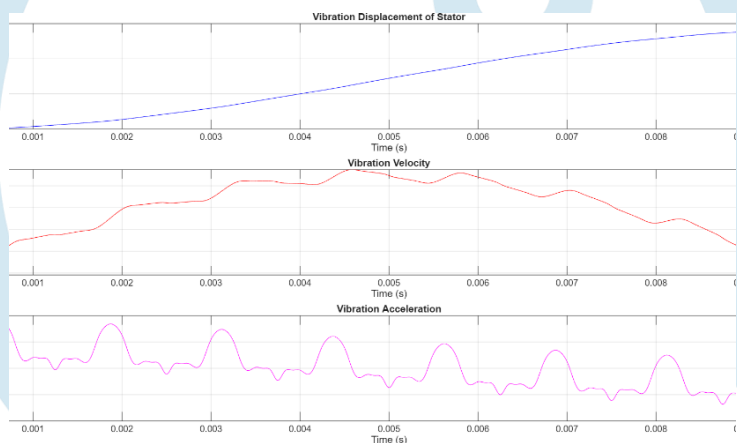


Fig. 7. Vibration Displacement of Stator, Vibration Velocity and Vibration Acceleration Plotting for E-Bike.

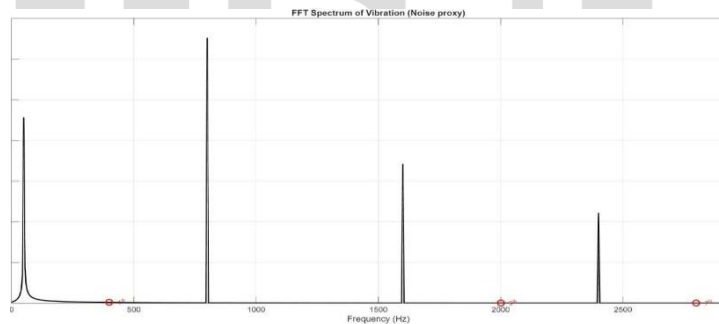


Fig. 8. FFT Spectrum of Vibration – Noise Proxy plot for E-Bike

V. CONCLUSION

This research successfully established and validated a computational framework for the predictive analysis of electromagnetic forces and Noise, Vibration, and Harshness (NVH) in a Permanent Magnet Synchronous Motor (PMSM). By integrating a harmonic-rich Maxwell Stress Tensor (MST) model with a Single Degree of Freedom (SDOF) structural analysis, the study effectively bridged the gap between electrical design parameters and mechanical acoustic response.

The investigation yielded the following critical findings:

1. **Non-Linear Force Transformation:** The analysis confirmed the non-linear relationship between flux density and electromagnetic force ($F_{em} \propto B^2$). This relationship was shown to translate input current harmonics into force pulsations at doubled frequencies, shifting the spectral energy from the fundamental electrical frequency of 212 Hz to a dominant radial force peak at 424 Hz.
2. **Harmonic Correlation:** Secondary force harmonics were explicitly identified at 1060 Hz and 1484 Hz, validating the hypothesis that the 5th and 7th order inverter harmonics directly contribute to high-frequency motor whine.
3. **Structural Integrity & Resonance:** The transient state-space simulation demonstrated that the stator's acceleration spectrum mirrors the force inputs. Crucially, the resonance assessment confirmed a safety margin for the 75 kW motor design, as the dominant excitation frequencies avoided the estimated natural frequency of 1200 Hz, thereby preventing catastrophic structural resonance.

In summary, this predictive methodology serves as a vital "Digital Twin" tool for EV powertrain development. By identifying specific noise-inducing frequencies (e.g., 424 Hz, 1060 Hz) during the preliminary design phase, engineers can implement targeted mitigation strategies such as stator stiffening or PWM harmonic injection to optimize acoustic comfort and durability without the high cost of physical prototyping.

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