Radial Dependence of Gravitational Time Dilation: A Comparative Boundary Analysis of Stellar Remnants

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Abstract **-** When massive stars reach the end of their lifecycles, they leave behind dense remnants characterised by intense gravitational pull. This study examines the boundaries of gravitational time dilation surrounding these theoretical stellar cores. By applying the exterior Schwarzschild metric, we express time dilation solely as a function of radial distance from the center of mass. This approach yields an analytical and graphical demonstration of the unique spatial properties that differentiate neutron stars from black holes. Specifically, we show that the time dilation curve for a neutron star abruptly ends at its physical surface, determined by the nuclear equation of state, while the curve for a black hole extends indefinitely, approaching the event horizon where a distant observer would perceive time as coming to a standstill. AI has been used for the better framing of content.

I. INTRODUCTION

Albert Einstein's 1915 formulation of General Relativity fundamentally redefined gravity, shifting the paradigm from a classical Newtonian force to the geometric curvature of spacetime. One of the most profound predictions of this framework is gravitational time dilation, the phenomenon wherein clocks positioned deeper within a gravitational potential well tick more slowly relative to those in flat, uncurved space. While this effect is famously subtle in weak gravitational fields like our own solar system, it becomes dramatically pronounced, and ultimately absolute, in the vicinity of ultra-compact masses.

The terminal stages of massive stellar evolution provide the universe's ultimate laboratories for observing these extreme relativistic phenomena. When a massive star exhausts its nuclear fuel, its core succumbs to a catastrophic implosion. The fate of this collapsing core is dictated by a delicate thermodynamic balance. If the core mass remains below a critical threshold, the collapse is violently halted by neutron degeneracy pressure, forging a neutron star, a macroscopic object with a rigid, ultra-dense physical crust. However, if the mass exceeds the Tolman-Oppenheimer-Volkoff (TOV) limit, no known outward pressure can counteract gravity. The core collapses indefinitely, piercing its own Schwarzschild radius and leaving behind a black hole, a pure vacuum spacetime solution defined solely by its event horizon.

For decades, the comparative study of the spacetime geometries surrounding these extreme remnants was largely confined to theoretical mathematics. Today, however, the advent of high-precision observational astronomy has transformed these abstract concepts into measurable realities. Instruments like the Neutron Star Interior Composition Explorer (NICER) and advanced spectroscopic surveys have provided unprecedented, verified data regarding the exact masses and physical boundaries of specific celestial objects.

Leveraging this verified observational data, this paper maps the distinct spatial boundaries of these extreme remnants. By isolating the time dilation factor exclusively as a function of radial distance, we construct a comparative visual framework. This static, spatial model serves to bridge the gap between abstract tensor calculus and observable astrophysics, rigorously contrasting a remnant that possesses a tangible physical boundary against one characterised by an asymptotic spatial plunge into an event horizon.

II. LITERATURE REVIEW

The base framework for modelling gravitational time dilation begins with Einstein's (1915) general theory of relativity, and then it is further developed for non-rotating, spherical masses by Schwarzschild (1916). After that the idea of what happens when stellar cores collapse inside this theory was altered, mostly by Oppenheimer and Volkoff (1939), who set down the theoretical mass ceiling, the one that separates steady neutron stars from black holes. More recently, progress in the nuclear equation of state (Lattimer & Prakash, 2001) along with sharp observational astronomy, for example, NICER telescope readings of PSR J0030+0451 (Miller et al., 2019) and the near-infrared spectroscopic mass checks for V404 Cygni (Khargharia et al., 2010), made it possible to anchor those theoretical borders in empirical measurements.

Lately, methodological steps have pointed out how useful it is, pedagogically and analytically, to model these relativistic effects in graphic form. Foundational work by Abheek Panigrahi (2025, 2026) used a kind of direct, visual inspection to map the Schwarzschild metric, and it showed the scale invariance of gravitational time dilation for different black hole masses, in both the near-horizon setting and the far-field region. Starting from that graphical structure, this paper extends the story into a comparative boundary analysis. By bringing in observationally confirmed remnants that sit on opposite sides of the TOV limit, this study wants to contrast, visually and mathematically, the gradual spatial plunge near an event horizon with the rigid physical cut-off created by a neutron star's crust.

III. THEORY

Rather than using theoretical estimates, we research two different, observationally verified remnants with the objective to determine the spatial boundaries of time dilation in physical reality:

● Neutron Star (PSR J0030+0451): An isolated pulsar whose mass and radius have been determined with great precision by the NICER (Neutron Star Interior Composition Explorer) telescope. It has a known mass of approximately 2.86×10^{30} kg and a physical radius of 13,020 m.

● Black Hole (V404 Cygni): An extensively researched stellar-mass black hole in a low-mass X-ray binary system. Near-infrared spectroscopic and broad kinematic analyses verify its mass at approximately 1.79×10^{31} kg and its Schwarzschild radius is approximately 26,580 m.

III.1. GRAVITATIONAL TIME DILATION

The ratio of proper time t' experienced by an observer at a radial distance R relative to the coordinate time t of a distant, stationary observer is calculated using the exterior Schwarzschild metric:

$$t'/t = \sqrt{1 - \frac{2GM}{rc^2}}$$

· t' = time near the massive object as observed by a distant observer

· t = time experienced in space; unaffected by mass

· G = Universal Gravitational Constant

· R = Radial distance

· M = Mass of the object

· c = Speed of Light

The Schwarzschild radius formula is the following:

$$r_s = \frac{2GM}{c^2}$$

· G = Universal Gravitational Constant

· M = Mass of the object

· c = Speed of Light

· r_s = Schwarzschild Radius At this radius occurs the Event horizon, beyond which not even light can escape.

III.2. The Tolman-Oppenheimer-Volkoff (TOV) Limit.

The structural divergence between these two verified remnants is dictated by the Tolman-Oppenheimer-Volkoff (TOV) limit. This limit defines the maximum theoretical mass at which neutron degeneracy pressure can successfully counteract the inward crush of gravity. Modern astrophysical constraints place the TOV limit between 4.18×10^{30} kg and 4.57×10^{30} kg.

PSR J0030+0451 (2.86×10^{30} kg) sits safely below the TOV limit, guaranteeing thermodynamic stability and the survival of its 13,020 m physical surface. Conversely, V404 Cygni (1.79×10^{31} kg) strictly exceeds the upper bounds of the TOV limit. For this mass, no known physical mechanism can halt gravitational collapse, mandating that the remnant shrinks beyond its own Schwarzschild radius.

III.3. Radial Boundary Conditions

The validity of the exterior Schwarzschild metric relies on the observer being situated outside the mass distribution of the object. Therefore, the function $D(R, r_s)$ is subjected to specific radial domain limits driven by observational data:

- PSR J0030+0451: Evaluated from $R = \infty$ inward to the verified physical surface limit R = 13,020 m.

- V404 Cygni: Evaluated from $R = \infty$ inward to the theoretical event horizon $R = r_s$.

IV. GRAPHS.

V404 Cygni:

1. $R = r_s$

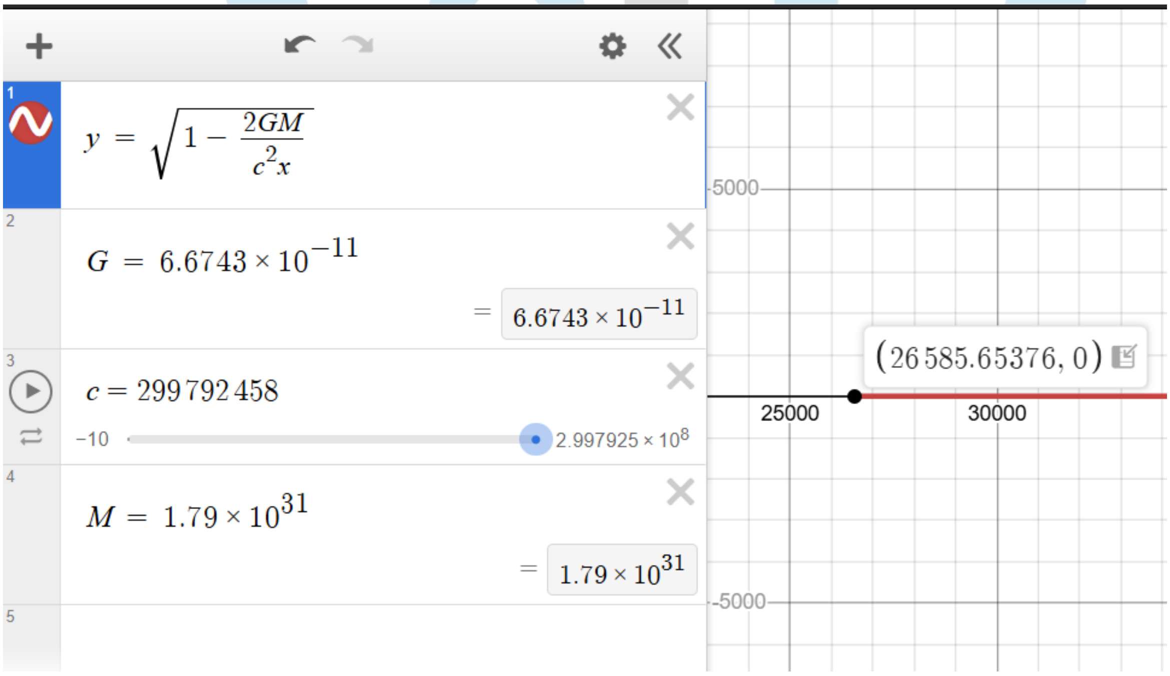

Fig. 1: - Time Dilation factor at the Schwarzschild Radius

The time dilation factor becomes 0 at the Schwarzschild radius value, as observable in figure 1.

2. $R \rightarrow \infty$

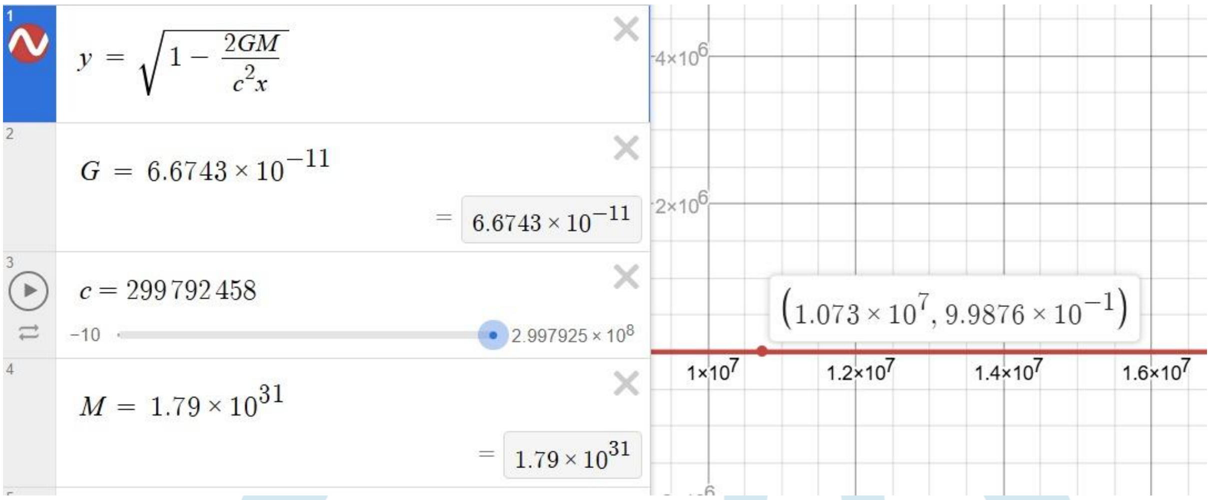

Fig. 2: - Time Dilation factor at Radius $\rightarrow \infty$

This graph illustrates the weak-field limit at immense radial distances. The time dilation curve asymptotically approaches 1, visually confirming that far from the mass, spacetime curvature is negligible and local proper time synchronizes with distant coordinate time.

PSR J0030+0451:

1. Near-Field Regime

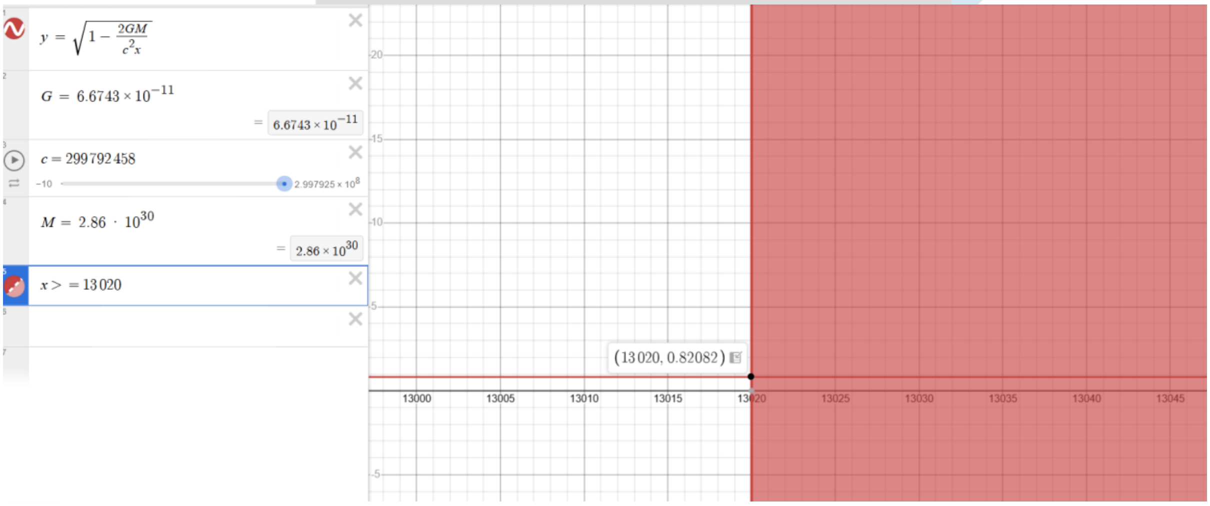

Fig. 3: - Time Dilation factor for Near-Field Regime

This graph captures the near-field regime, where the curve is forced to terminate at the neutron star's verified physical crust.

2. $R \rightarrow \infty$

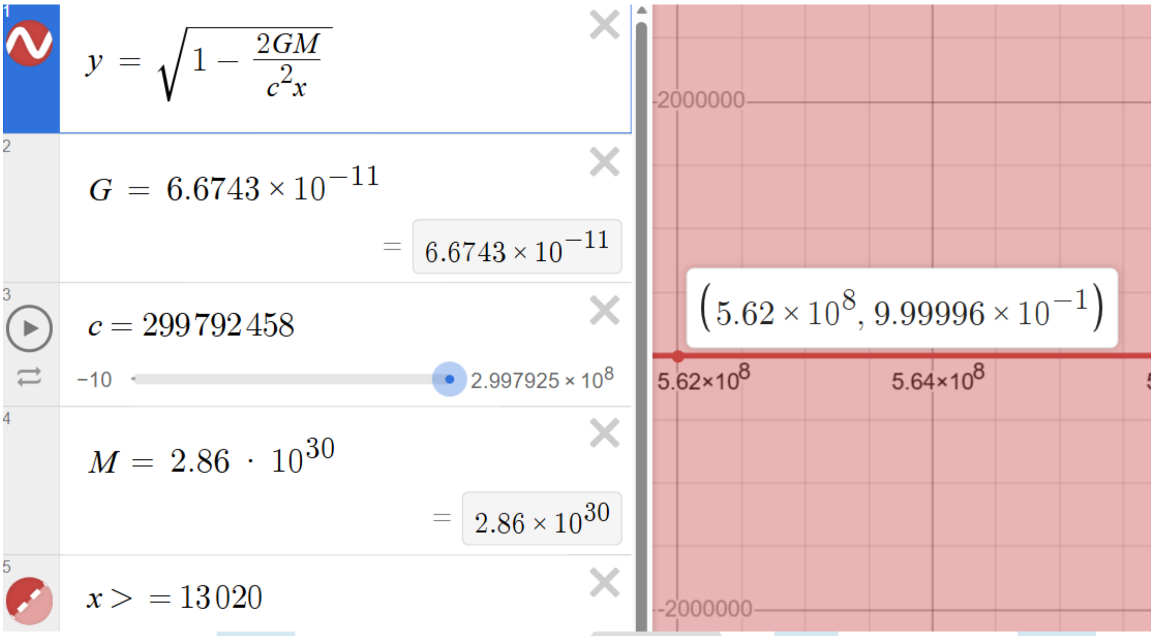

Fig. 4: - Time Dilation factor at Radius → ∞

This graph illustrates the weak-field limit at immense radial distances. The time dilation curve asymptotically approaches 1, visually confirming that far from the mass, spacetime curvature is negligible and local proper time synchronizes with distant coordinate time.

IV.1. GRAPHICAL OBSERVATION:

Upon plotting the comparative radial functions for both verified remnants, three distinct graphical observations emerge:

The Far-Field Overlap: When the radial distance is much greater than the scale radius ($R \gg r_s$), the PSR curves J0030+0451 and V404 Cygni show the same asymptotic behavior. This remark visually confirms that in weak-field limits, the specific structural nature of the mass source (a robust physical surface compared to a singularity) has minimal influence on the curvature of spacetime in the vicinity.

The Physical Cutoff (PSR J0030+0451): The time dilation curve for the neutron star is told to stop at the 13,020 m mark on the spatial x-axis, deliberately, like right there. This visual break is a strict physical necessity, well, mostly. The equation that we plotted is supposed to be a model of the exterior Schwarzschild metric, but it really only applies to the empty vacuum around a mass. The vacuum ends at 13,020 meters, and the solid crust of the neutron star begins, or rather, starts to appear in the model. Now the external relation is not valid because the background changes from empty spacetime to ultra-dense nuclear matter. So the graph is mathematically modified (by a domain restriction) to match this reality, and the curve is left hanging at $D \approx 0.821$.

The Asymptotic Plunge (V404 Cygni): There's this curve, the one showing the black hole, and it keeps going, pretty much with no change at all. When the x-value gets close to 26,590 m, right at the event horizon mark, the slope starts getting so steep it almost feels like it just blows up. And it touches the x-axis exactly where $D = 0$. So the picture is basically the hard limit of the event horizon, like an absolute threshold, and it also shows that proper time effectively halts, compared with what the faraway observer would say.

V RESULT:

V.1 Calculated Mathematical Results:

● PSR J0030+0451: the theoretical Schwarzschild radius comes out at $r_s \approx 4{,}248$ m. But since there is a real physical surface at $R = 13{,}020$ m, the outer metric is, kind of, strictly limited. From that, the lowest time dilation you could see coming from the surface ends up $D \approx 0.821$.

● V404 Cygni: the theoretical event horizon is placed at $r_s \approx 26{,}590$ m. And because there is no physical surface there to slow the infall or stop the descent the way it has to, the math boundary of the dilation function when R gets close to r_s turns out to be exactly 0.

V.2 Graphical Observations:

● Far-Field convergence: At immense radial distances ($R \gg r_s$) both curves kind of overlap, and they asymptotically head toward D = 1, so it basically shows that structural differences become hard to tell apart in weak gravitational fields.
● The physical cutoff: the pulsar's time dilation curve is, in a mathematical sense, limited to stop right at its physical surface (13,020 m on the x-axis). It cuts off rather sharply, and then the curve stays hanging there, sort of suspended at the $D \approx 0.821$ point.
● The asymptotic plunge: the black hole curve keeps going without any real change until the horizon. And as the spatial coordinate comes near 26,590 m, the curve plunges straight downward to hit the x-axis at D = 0 which makes the absolute termination of proper time feel visually clear.

VI. CONCLUSION:

This paper tries to map, sort of, the absolute spatial boundaries for gravitational time dilation, using verified mass and radius data from known stellar remnants. Instead of doing it indirectly, we look at the Schwarzschild metric as a direct function of radial distance, and we provide an analytical, graphical proof that sort of separates stellar structural classes. The outcomes suggest that observed neutron stars like PSR J0030+0451 come with a mathematical truncation, but only because there is a physical surface where $R > r_s$. Meanwhile, black holes like V404 Cygni are described by an uninterrupted spatial curve, one that keeps going until it reaches the event horizon, where $R = r_s$.

If we strip away temporal dynamics and focus purely on radial dependence tied to real-world data, the model shows how structural geometry constrains relativistic effects in our actual universe. The truncation point for the PSR J0030+0451 curve acts like a direct geometric imprint of the nuclear equation of state that governs real pulsars. With that in mind, careful measurements of the "terminal" gravitational time dilation can be treated as a direct indicator of ultra-dense matter boundaries. On the other hand, the continuous descent of the V404 Cygni curve reinforces the special character of an event horizon, and it argues that only complete gravitational collapse lets proper time reach absolute zero in coordinate space. Ultimately, this side-by-side approach checks the geometric scale of the metric against what we see, and it successfully illustrates the spatial limits of observable proper time.

VII. REFERENCES.

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